Ephemeris calibration of Laplace's tidal equation model for ENSO

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Abstract

Conventional tidal prediction models typically combine constituent lunisolar forcing factors harmonically fit to sets of collected tidal gauge data. A harmonics analysis is favored over a precise orbital ephemeris-based gravitational forcing model, as tides are localized in scope and sensitive to a particular volume geometry. But what happens when the dynamic behavior is much larger in scale? As we have demonstrated previously [1], lunisolar tidal constituents forced by a strong biennial-modulated annual signal will provide a high-quality fit to ENSO – albeit subject to over-fitting of the numerous constituent factors available. Yet as ENSO is a large-scale phenomenon, it should be more amenable to applying a precise set of ephemeris data as the forcing to a Laplace's tidal equation formulation. This should reflect the underlying physics governing the dynamics more realistically, while severely constraining the degrees of freedom (DOF) in factors which lead to the possibility of over-fitting. We used the NASA JPL Horizons (https://ssd.jpl.nasa.gov/horizons.cgi) ephemeris data for the Sun and Moon as a parametric input to the well-known 1/Rˆ3 gravitational forcing function and verify as good a quality fit as that available from a high-DOF harmonics approach. This extends over the modern-day instrumental record of ENSO but also covers the coral proxy records that span the years from 1650 to 1880. The approach works effectively because the extra DOF (including phases and elliptical nonlinearities in the orbits) needed to precisely define the gravitational forcing are accurately tracked by the Horizons ephemeris algorithm. Importantly, the results are highly sensitive to the relative forcing amplitudes, which is not surprising, since the fast lunisolar cycles are projected over spans of hundreds of years. The challenge is equivalent to attempting to perform a conventional tidal analysis over a similarly lengthy time span, while also dealing with noise and a limited resolution time-series. [1] Pukite, P.R. "Biennial-Aligned Lunisolar-Forcing of ENSO: Implications for Simplified Climate Models." AGU Fall Meeting, 2017.

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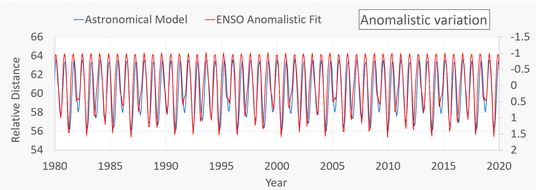
http://GeoEnergyMath.com

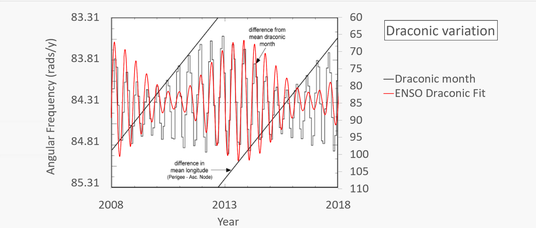
ENSO model described in monograph Mathematical GeoEnergy, AGU/Wiley (2019)
Developed via an analytical formulation and solution to Laplace's Tidal Equations (LTE)
Forcing provided by three lunar tidal factors and annual impulse, modulated bienially
Detailed orbital paths calibrated from NASA JPL Horizons ephemeris data
Final fitting performed by scaling of standing wave solutions of a forced LTE formulation

Overall, the approach resembles a conventional tidal analysis, but over a longer time scale

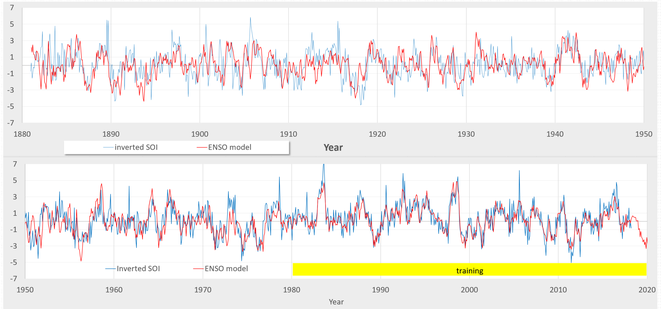
Several approaches used to validate the analysis
1. Temporal-domain cross-validation
2. Frequency-domain cross-validation
3. Multiple time-scale validation (monthly and daily)
4. Common-mode validation (other indices QBO, PDO, AMO, NAO)
5. Geophysics validation (Ephemeris, Length-of-Day, Chandler wobble)
6. Out-of-band coral proxy validation

Lunar Ephemeris Calibration



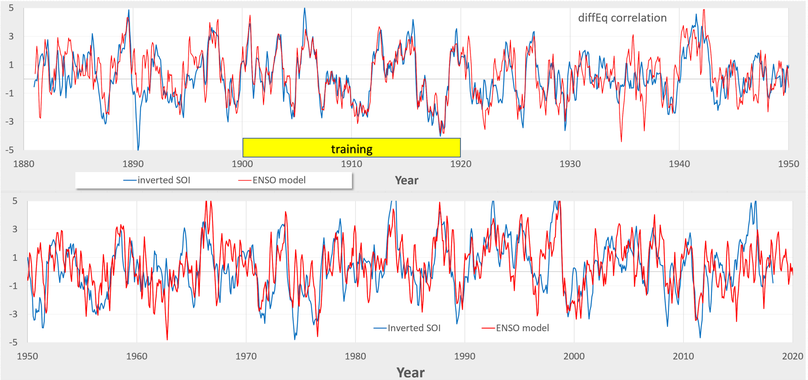


(1) Temporal-domain Cross-Validation of ENSO Model



Cross-validation in the time domain uses one interval for training and an orthogonal interval for validation.

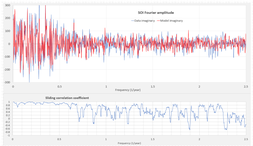
If there is over-fitting on the training interval and the test fit is still good, the #DOF is adequate.

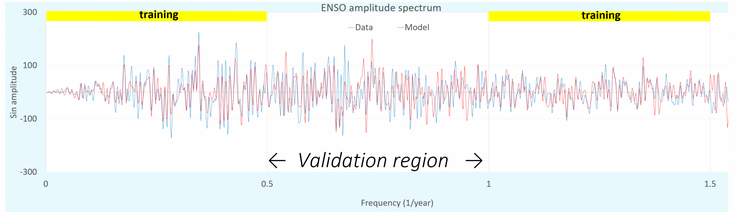


(2) Frequency-domain Cross-Validation

Cross-validation in the frequency domain uses a similar approach but is only valid for non-linear models. It validates the existence of harmonics that span the frequency spectrum and are related by the model details, in this case a biennial factor.

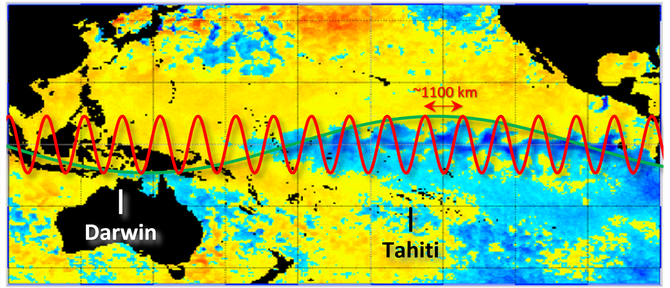
Typical spectral fit



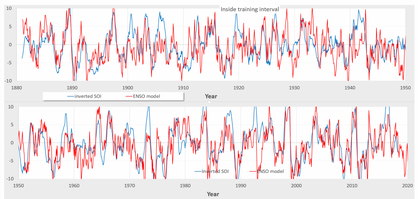


ENSO model standing waves

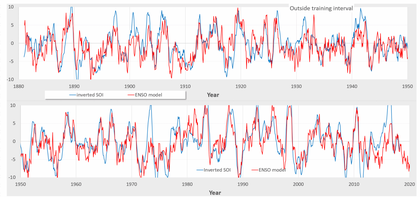
Low Wave Number - Darwin/Tahiti Dipole
High Wave Number - Tropical Instability Wave



Estimated using 0.5 to 1 /year frequency components



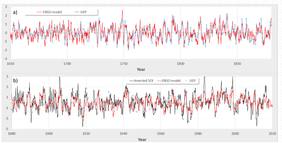
Estimated using <0.5 and 1 to 1.5 /year frequency components



(3) Multiple Time-Scale Validation of ENSO Model

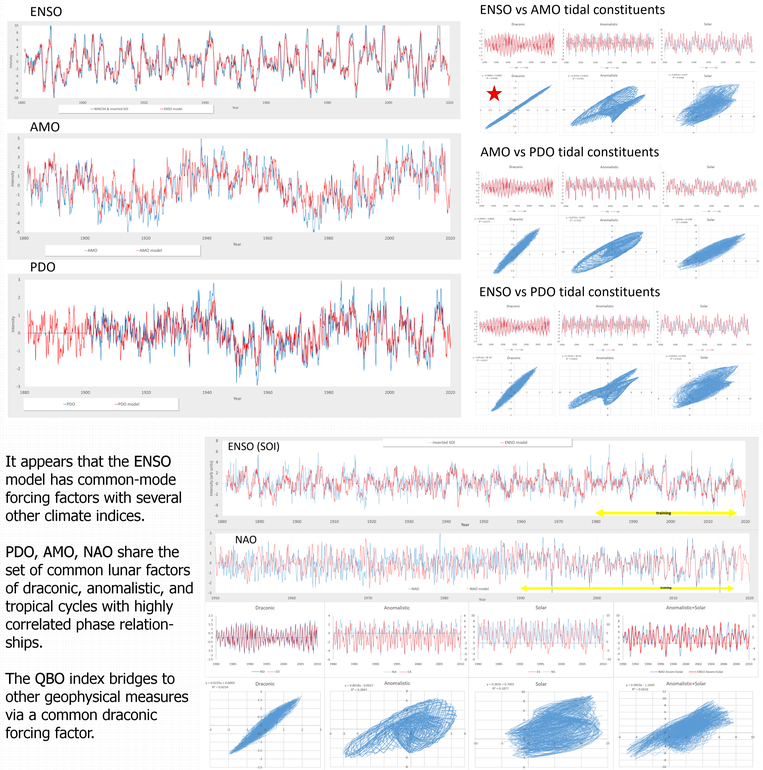
The ENSO measure in terms of the Southern Oscillation Index (SOI) shows significant spectral power both at the low-frequency & low-wavenumber Darwin-Tahiti dipole and at the higher-frequency & high-wavenumber behavior possibly related to Tropical Instability Waves. The ENSO model requires a phase relationship in the dispersion profile between the low-K and and high-K components which is reflected in a multiple time scale fit.

(6) Out-of-Band Proxy Validation of ENSO Model



Using coral proxy measures of ENSO via UEP, we can back extrapolate the modern-day instrumental fit several hundred years. Although not highly discriminatory due to the coarse yearly resolution, the model is not invalidated (see left). On the right is an approach for forward predictive extrapolation.

(4) Common-Mode Validation of ENSO Model

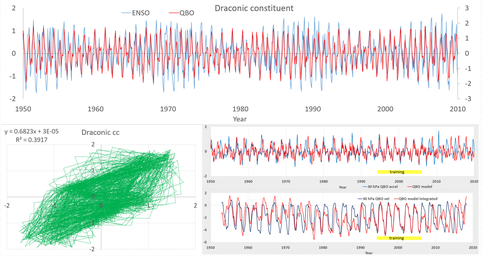


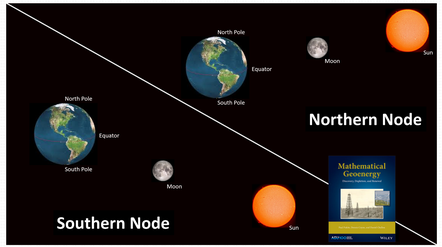
It appears that the ENSO model has common-mode forcing factors with several other climate indices.

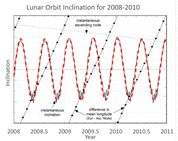
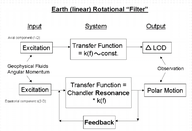
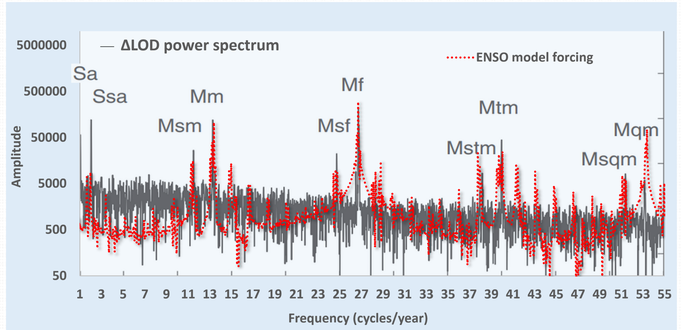
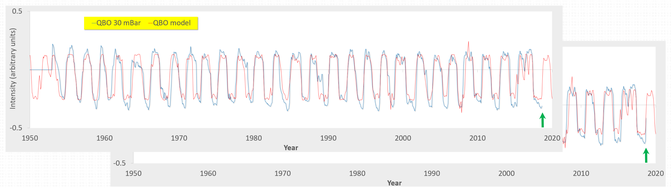
PDO, AMO, NAO share the set of common lunar factors of draconic, anomalistic, and tropical cycles with highly correlated phase relation-ships.

The QBO index bridges to other geophysical measures via a common draconic forcing factor.

(5) Geophysics Validation of ENSO Model







Other geophysical measures show significant phase relationships to the ENSO forcing factors. The lunar nodal crossing in particular impacts QBO and the Chandler wobble similarly.