Improving Bayesian Evidential Learning 1D imaging (BEL1D) accuracy through iterative prior resampling

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Abstract

Bayesian Evidential Learning 1D Imaging (BEL1D) has been recently introduced as a new computationally efficient tool for the interpretation of 1D geophysical datasets in a Bayesian framework. Applications have already been demonstrated for Surface Nuclear Magnetic Resonance (SNMR) data and surface waves dispersion curves. The case of SNMR is particularly relevant in hydrogeophysics, as it directly sounds the water content of the subsurface. BEL1D relies on the constitution of statistical relationships in a reduced dimension space between model parameters and simulated data using prior model samples that replicate the field experiment. In BEL1D, this relationship is deduced through Canonical Correlation Analysis (CCA). When using large prior distributions, CCA may lead to numerous poorly correlated distributions for higher dimensions. Those poorly correlated distributions are resulting in a low reduction of uncertainty on some parameters, even if the experiment is supposed to be sensitive to them. This phenomenon is related to the aggregation of multiple parameters in the same dimension, hence the possible aggregation of sensitive and insensitive parameters. However, arbitrarily reducing the extent of the prior will lead to biased estimations. To overcome this impediment, we introduce an iterative procedure, using the posterior model space of the previous iteration as prior model of the current iteration. This approach frequently reveals higher correlations between the datasets and the model parameters, while still using large unbiased priors. It enables BEL1D to produce better estimations of the posterior probability density functions of the model parameters. Nonetheless, iterating on BEL1D presents several challenges related to the presence of insensitive parameters, that will always mitigate the capacity to reduce at once the uncertainty on the whole set of parameters describing the models. On noise-free synthetic datasets, this method leads to near-exact estimation of the sensitive parameters after few (two to three) iterations. On noisy datasets, the resulting distributions bear some uncertainty, arising directly from the presence of noise, but to a lesser extent than the non-iterative approach. The procedure remains more computationally efficient than McMC.

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1. Bayesian Evidential Learning 1D imaging

Uncertainty appraisal is a key concern to geophysicists when imaging the subsurface. This issue is classically handled by stochastic inversion (costly CPU) or by error propagation (unrealistic uncertainty).

Bayesian Evidential Learning 1D imaging (BEL1D) is a Bayesian method that enables the stochastic interpretation of 1D geophysical data, with a reasonable CPU cost and realistic uncertainty estimations. The framework is based on Bayesian Evidential Learning *(e.g. Scheidt et al., 2018; Hermans et al., 2016)*.

The method relies on the constitution of statistical relationships between model parameters and the associated data-sets from prior realizations (**Fig. 1**). It offers the advantage not to require the input of biasing information through regularization parameters as is often the case in classical inversion processes. However, the consistent definition of a prior model space is still required. Nonetheless, the method handles efficiently large priors, the impediment being the difficulty to properly constitute representative correlations.

Above all, the method enables the quantification of uncertainty for the model parameters.

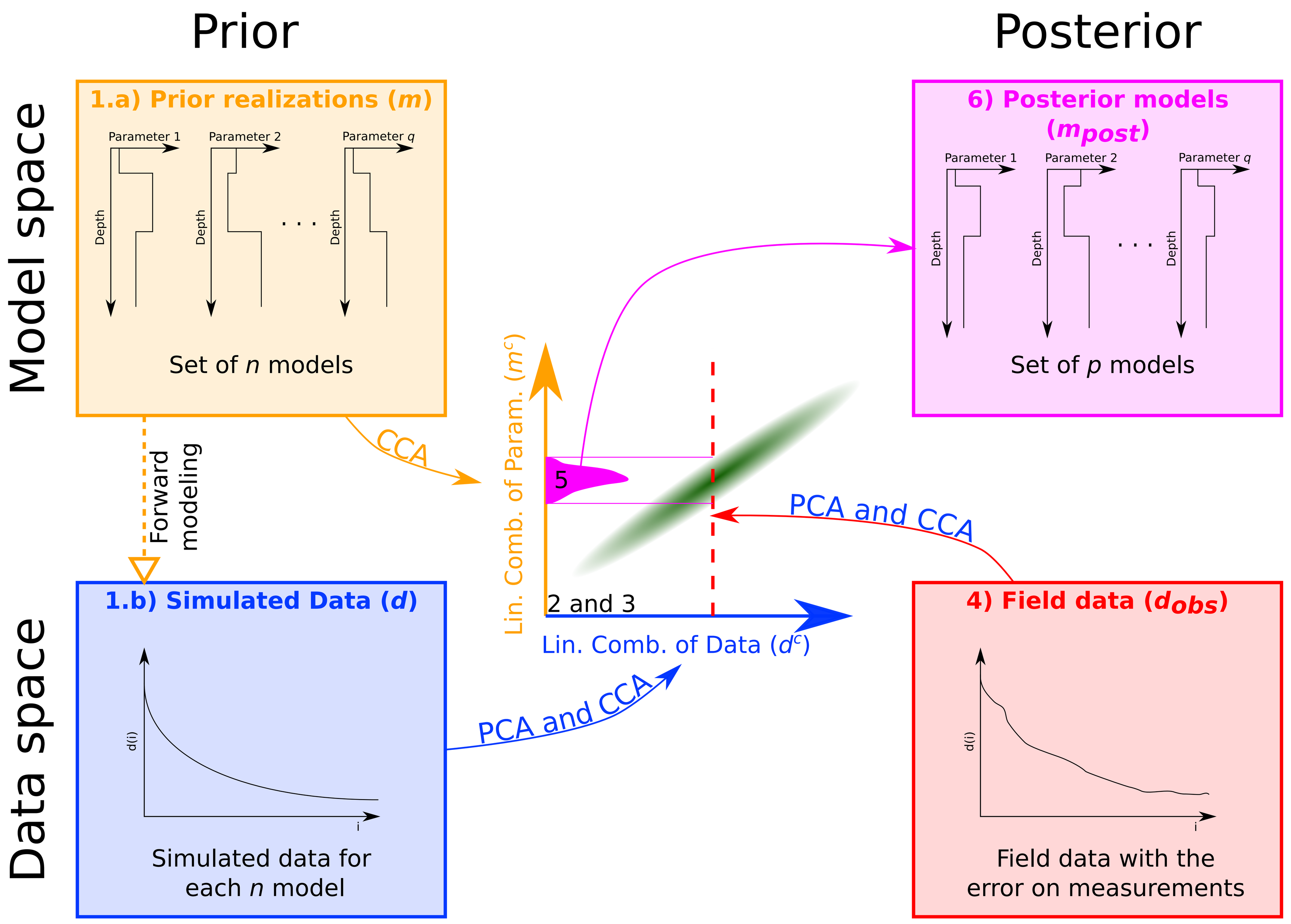

Fig. 1: Schematic illustration of BEL1D

2. Iterative prior resampling

Iterative prior resampling *(e.g. Cheng et al., 2019)* is relatively simple and may contribute to better estimate the uncertainty. The algorithm (**Fig. 2**) is:

Iteration 0: Build the initial prior model space ($prior_0$)
Iteration 1: Run BEL1D with $prior_0 \rightarrow post_1$
Iteration 2: Run BEL1D with $prior_1=post_1 \rightarrow post_2$
Etc.

It enables to better constrain models since higher correlations between models parameters and data may be observed.

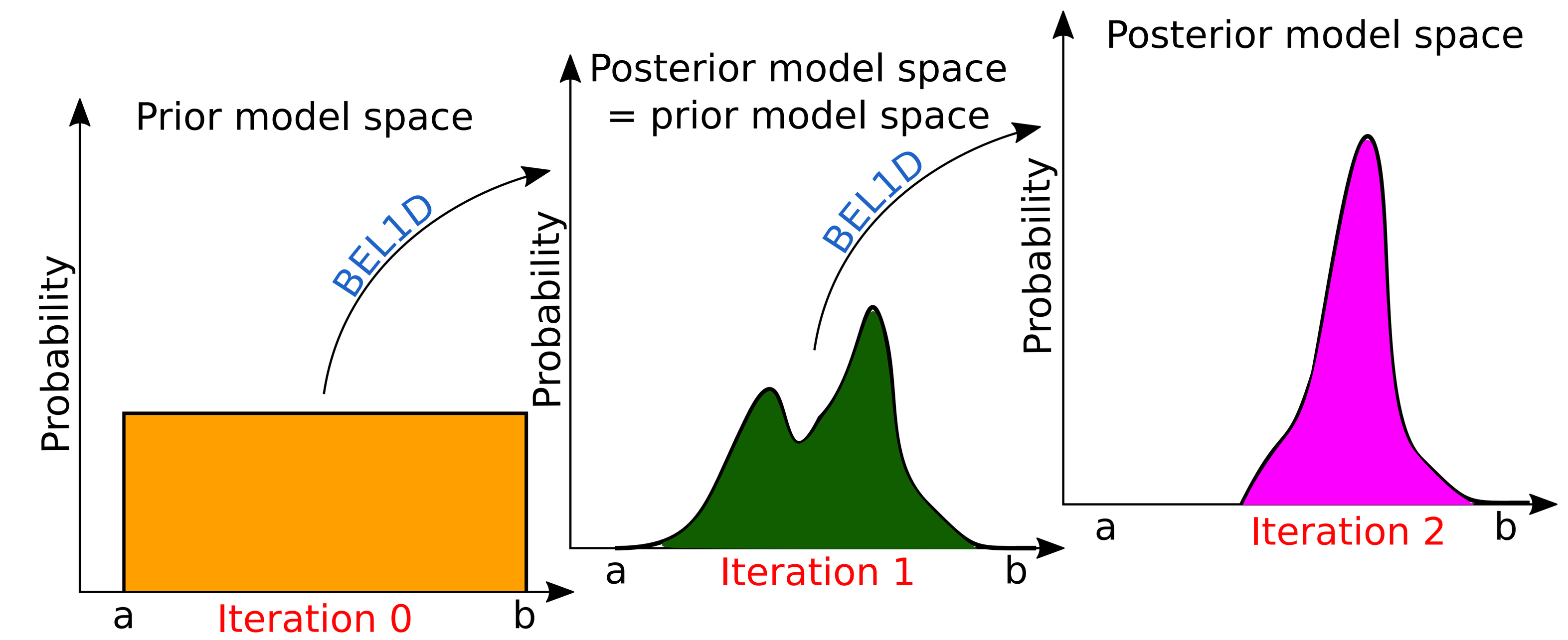

Fig. 2: Illustration of prior resampling

3. SNMR

Surface Nuclear Magnetic Resonance (SNMR) benefits from the quantum properties of protons (H^+) contained in water, hence is directly sounding water in the subsurface. Current is injected/received in an antenna on the ground and interacts with the protons spins as illustrated in **Fig. 3**. The received signal depends on the water content (amplitude) and the way water is linked to the soil particules (relaxation time).

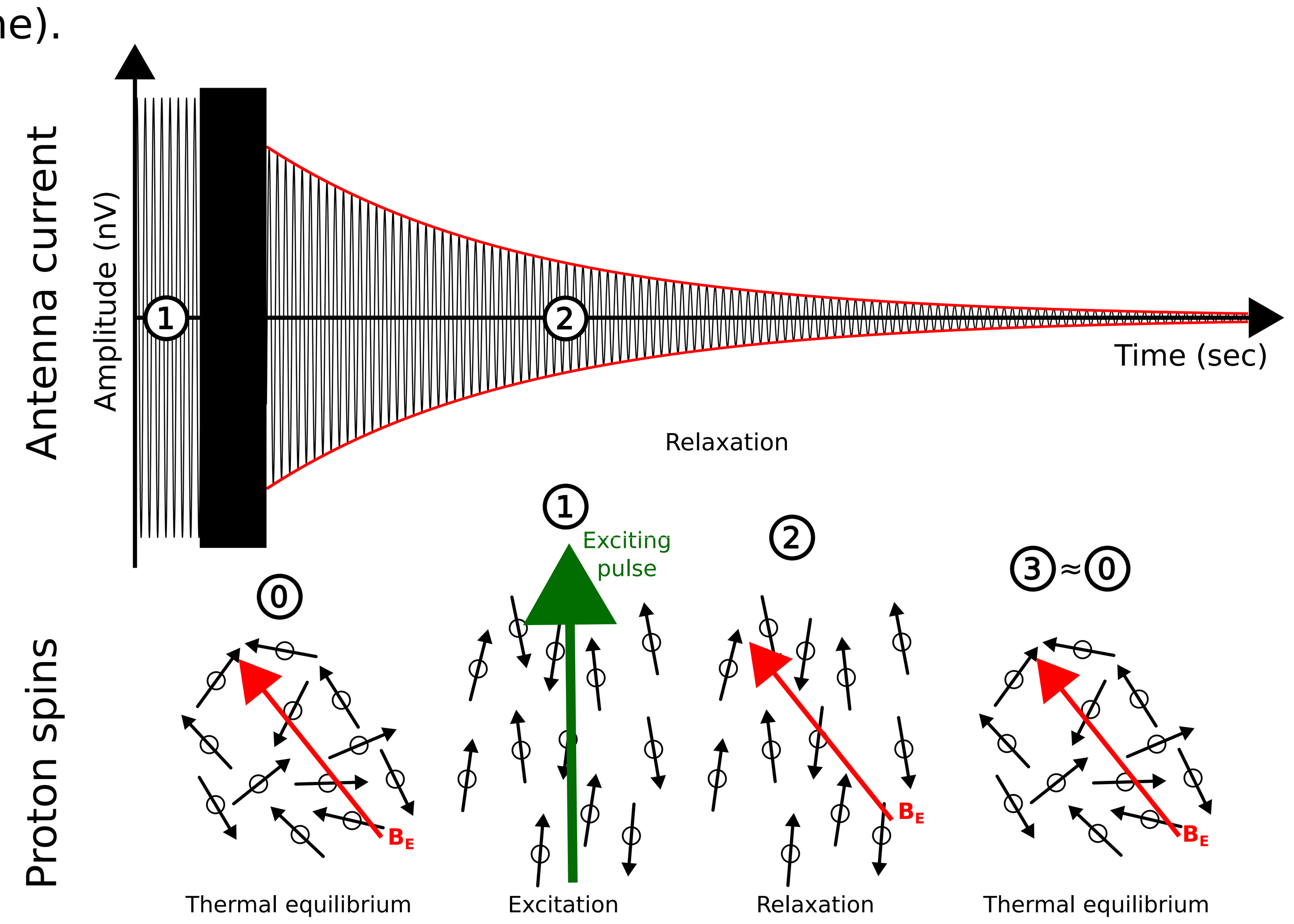

Fig. 3: Principle of NMR (FID pulse sequence)

4. Results

Prior resampling is applied to a simple 2-layers model (**Fig. 4**):
- As expected, we obtain a better estimation of the model parameters
- Trends in the model are discovered (increasing T_2^*)
- RMSE are lower

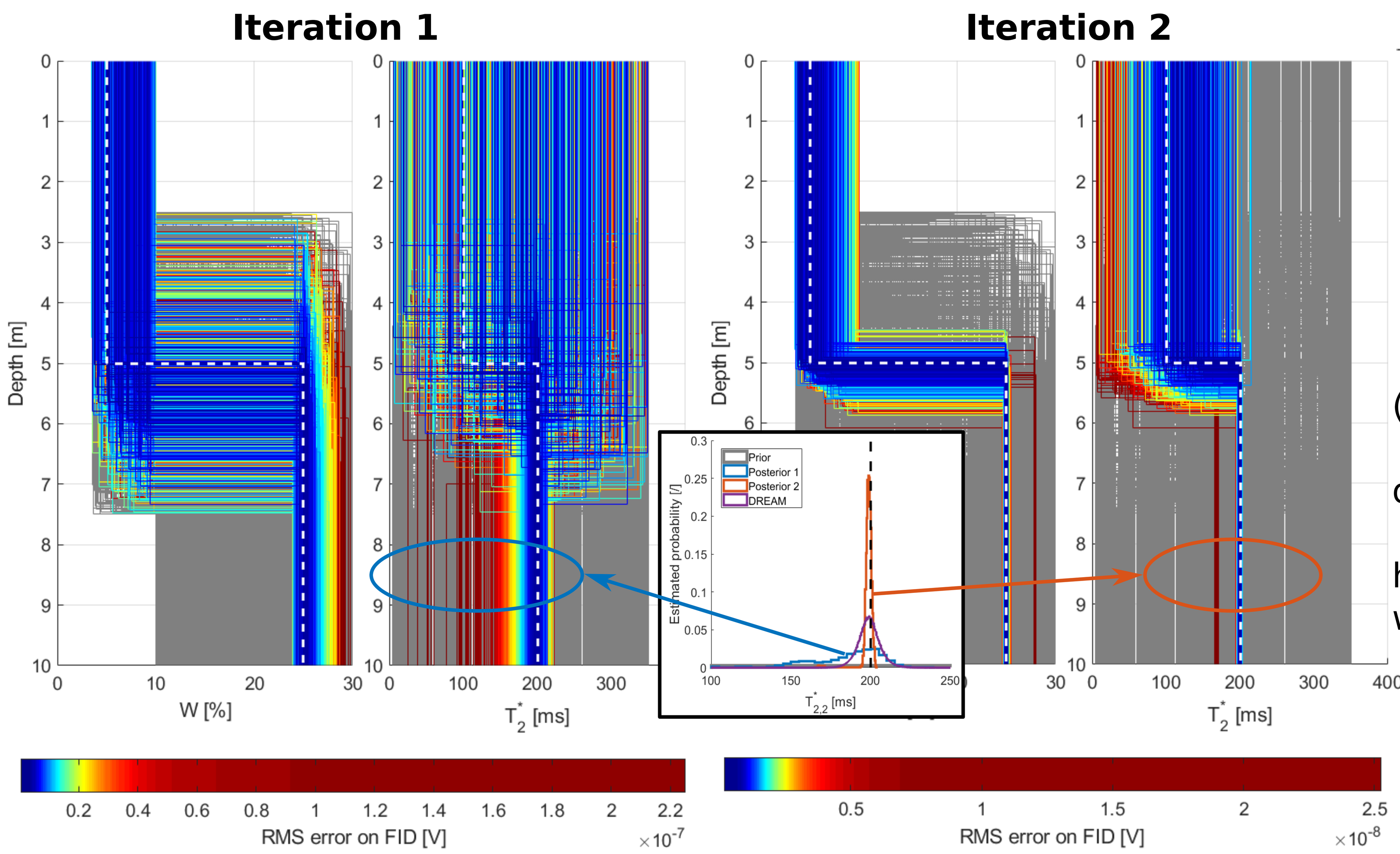

Fig. 4: Prior resampling results

Prior resampling applied to BEL1D:
- Benefits from better correlation between the parameters and the data (**Fig. 5**)

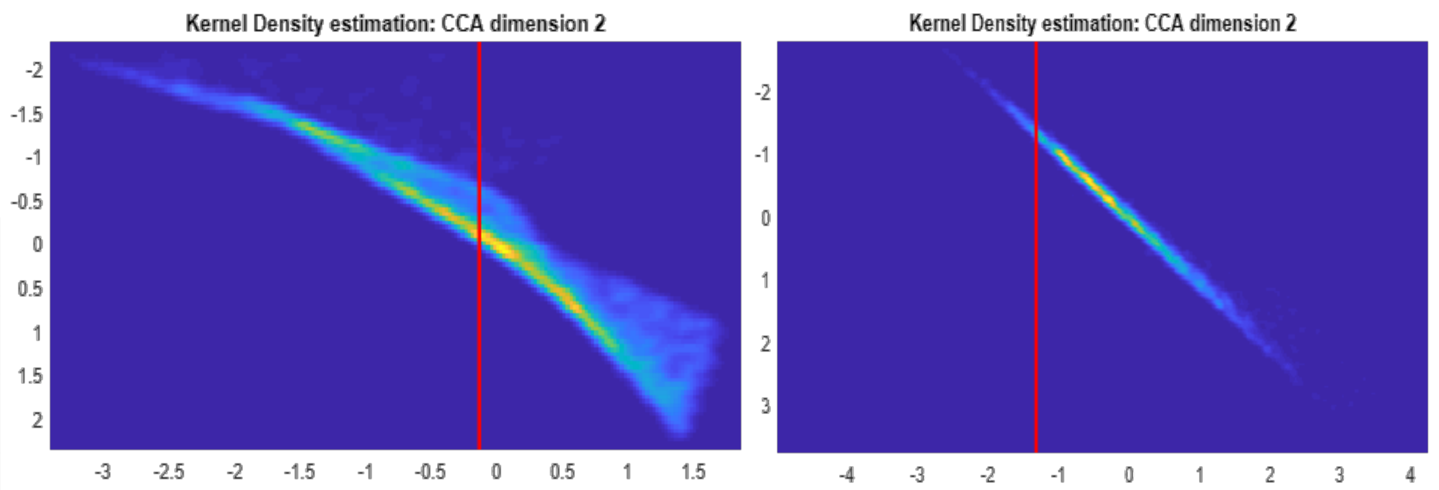

Fig. 5: Exemple of correlation at iteration 1 (left) and 2 (right)

- Enables a better estimation of the models parameters (espacially in the case of large priors)
- Permits to discriminate low probability modes in posterior distribution
- The McMC software DREAM (Vrugt, 2016) took about half an hour to converge towards an acceptable level of uncertainty whereas BEL1D needed about 3 minutes.

Find the BEL1D codes on Github:
github.com/hadrienmichel/BEL1D

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