Default priors cause biases in orbital parameter estimates for directly-imaged exoplanets.

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Abstract

The eccentricity of the planet's orbit and the inclination of the orbital plane carry important information about its formation and history. However, exoplanets detected via direct-imaging are often only observed for a very small fraction of their period, making it challenging to reliably estimate their orbits. The aim of this project is to investigate biases in the estimation of orbital parameters of directly-imaged exoplanets, particularly their eccentricities, and to define general guidelines to perform better estimations. For this, we constructed various orbits, and generated mock data for each spanning around 0.5% of their orbital period. We the Orbits For The Impatient (OFTI) algorithms to get orbit posteriors, and compared those to the true orbital parameters. We found that the inclination of the orbital plane is the parameter that most affects our estimations of eccentricity, with orbits that appear near-edge-on producing highly biased eccentricity posteriors. We also found a degeneracy between eccentricity and inclination that makes it ifficult to distinguish circular, edge-on orbits from eccentric, face-on orbits. For the exoplanet-imaging community, we propose practical recommendations, guidelines and warnings relevant to orbit-fitting.

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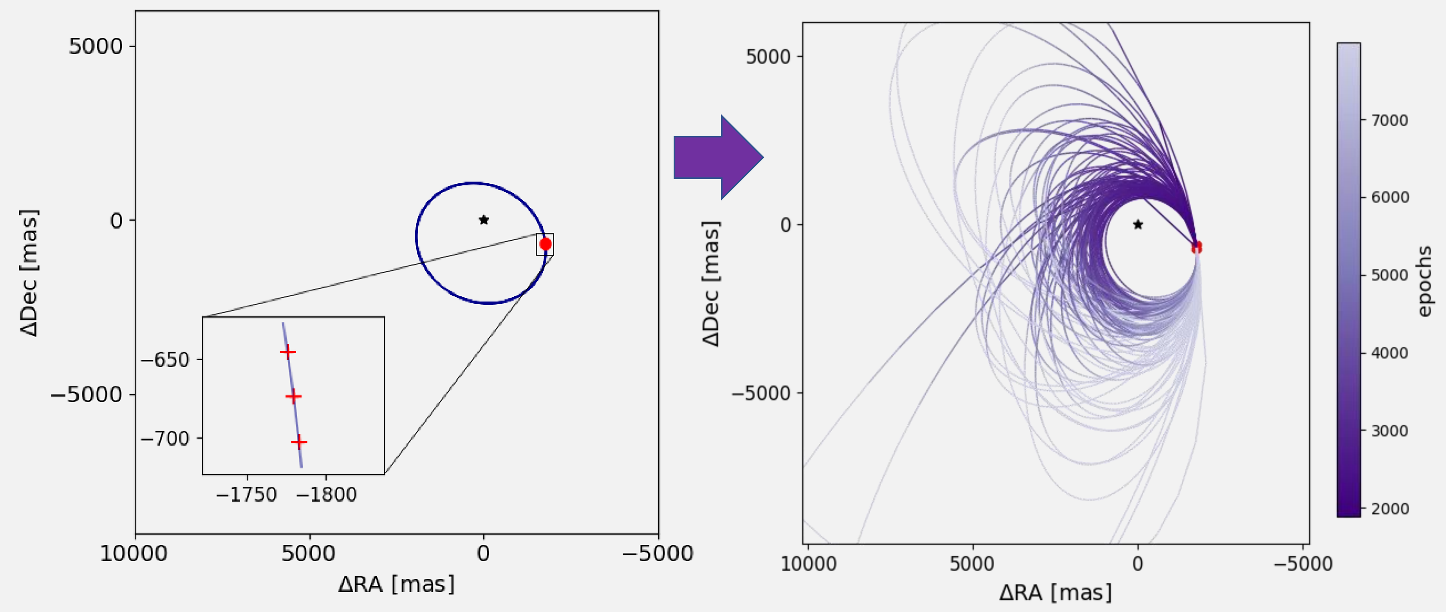
orbitize!

Preamble and method overview

Orbit-fitting of DI exoplanets

- Orbits contain valuable information about their formation and dynamics.
- Direct-imaging (DI) detects planets at wide separations and long periods, so often times we observe only a small fraction of the orbit.
- A small orbital coverage can produce biased posteriors.



Orbits as seen in the sky plane. Observations of exoplanets with DI usually span a small fraction of the orbit (left), so parameter space of possible orbits for those observations is often large (right). This results in wide posteriors and the risks of bias. Biases can come from using certain priors, from degeneracy between orbital elements, fromusing data from different sources, etc. Plot on the right made with Orbitize! (Blunt et al, 2020).

Goal: Explore eccentricity bias as a function of true orbital parameters

- We constructed a grid of orbits individually varying eccentricities (e), inclinations (i), arguments of periastron (ω) and epochs of periastron (τ).
- We generated mock datafor each orbit spanning ~ 0.5 % of its period.
- Orbit-fits were performed with those data (as in the figure above) using the OFTI algorithm (Blunt et al, 2017).

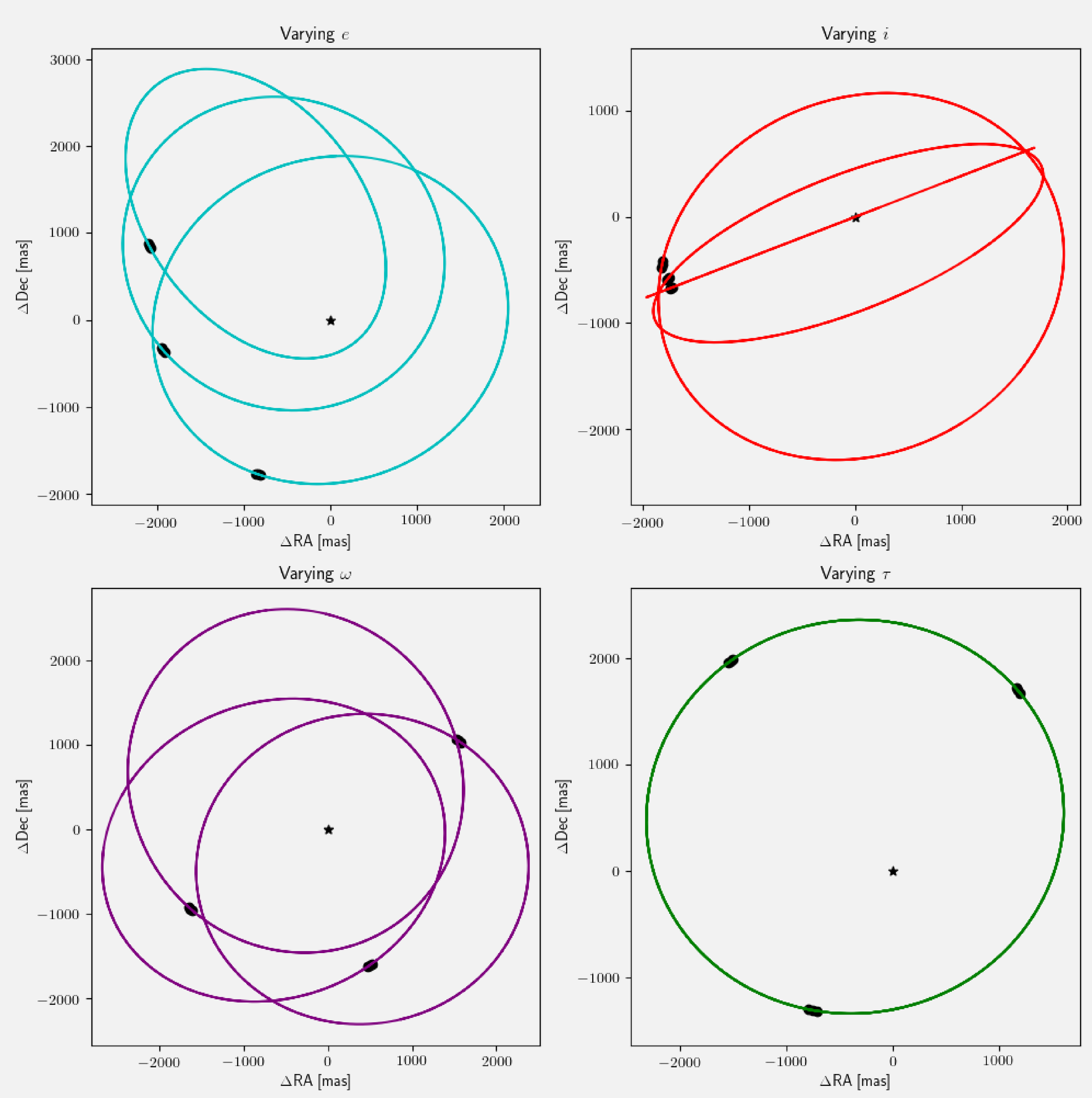
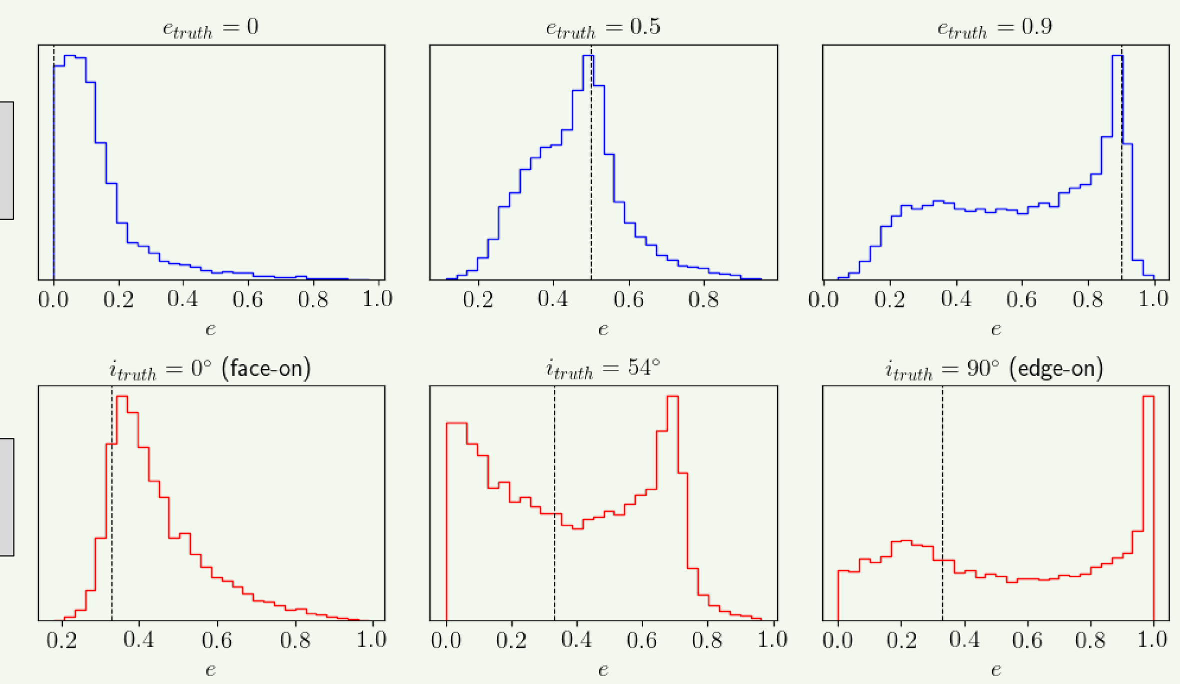


Illustration of the orbit-generating process. Three of the orbits for each orbital element tested are shown, relative to the primary's location in the sky plane. The black dots are the position at which we generated mock data.

Results

Univariate analysis

- Different values of ω and τ did not affect the e posteriors significantly, but different values of e and i did.



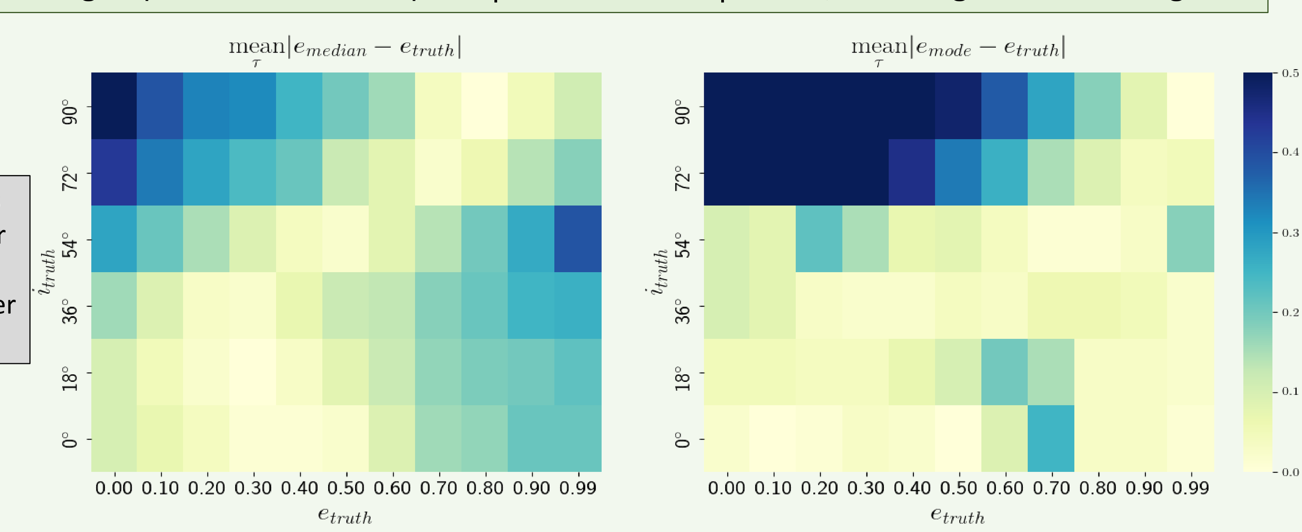
Moderate and high eccentricities were biased towards lower values (blue plots).

While varying e, the peak of the posterior remained aligned with the true e (blue plots).

Varying i resulted in large biases at near $90°$ (edge-on) inclinations (red plots).

Multivariate analysis

- We performed tests with different combinations of parameters (e, i, τ) using 11 different values of e, 6 of i and 6 of τ, resulting in 396 different orbits.
- We quantified bias as the displacement of the median and the mode of the posteriors from the true values (i.e. $|median - truth|$, $|mode - truth|$, respectively). We did this for both the e and the i posteriors.
- We averaged $|median - truth|$ and $|mode - truth|$ over τ, resulting in a 2D e vs i grid.

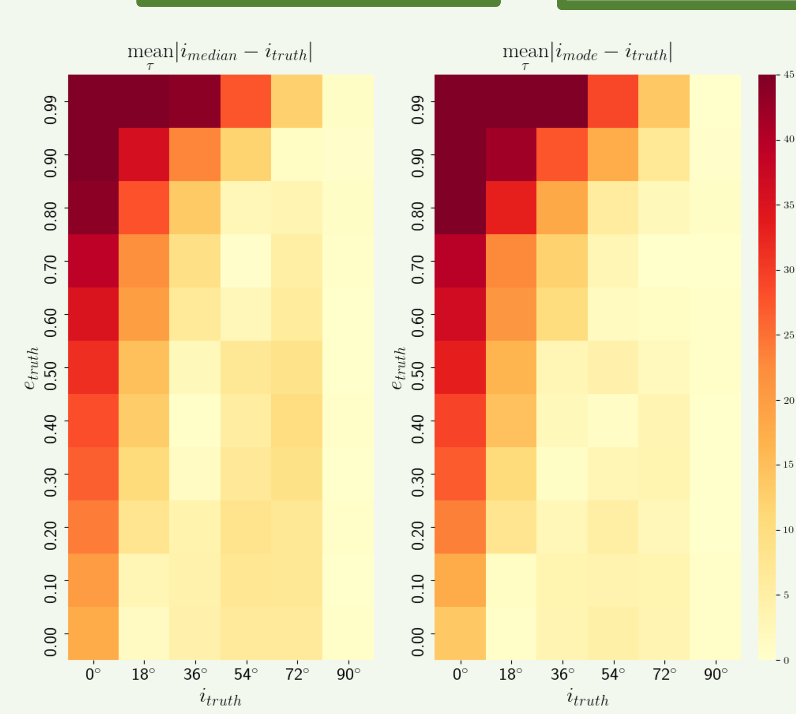


Mean e bias for each pair i, e. Darker color is larger bias.

Near $90°$ (edge-on) inclinations heavily bias eccentricity in all cases.

In high eccentricities the median performs poorly, but the mode does not.

Near $0°$ (face-on) inclinations and low eccentricities behave well overall.



Mean i bias for each pair i, e. Darker color is larger bias.

Median and mode show the same pattern, so they most likely point to estimates near each other. This can translate into misleading "well-constrained" i posteriors that do not reflect the often large biases.

Biases against $0°$ (face-on) inclinations exist for all eccentricities, understandable by Lucy-Sweeney bias (Lucy & Sweeney, 1971).

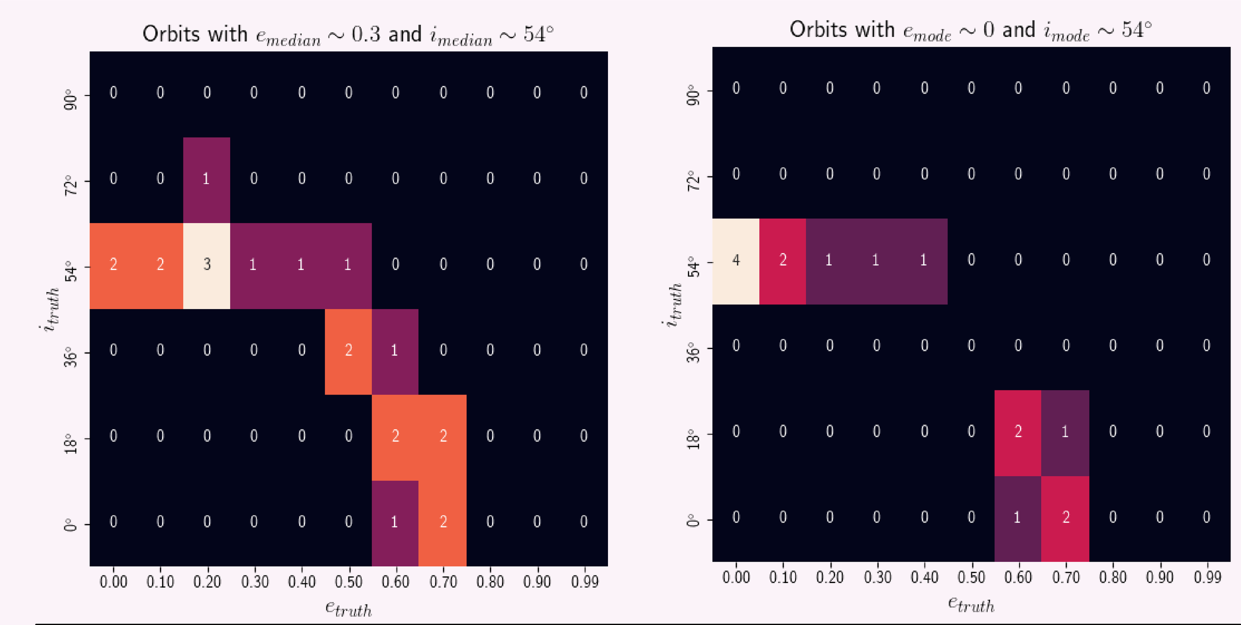
Inclination biases are minimal at nea $90°$ (edge-on) true inclinations.

The more eccentric the orbit, the more inclination bias there is across inclinations.

Applications

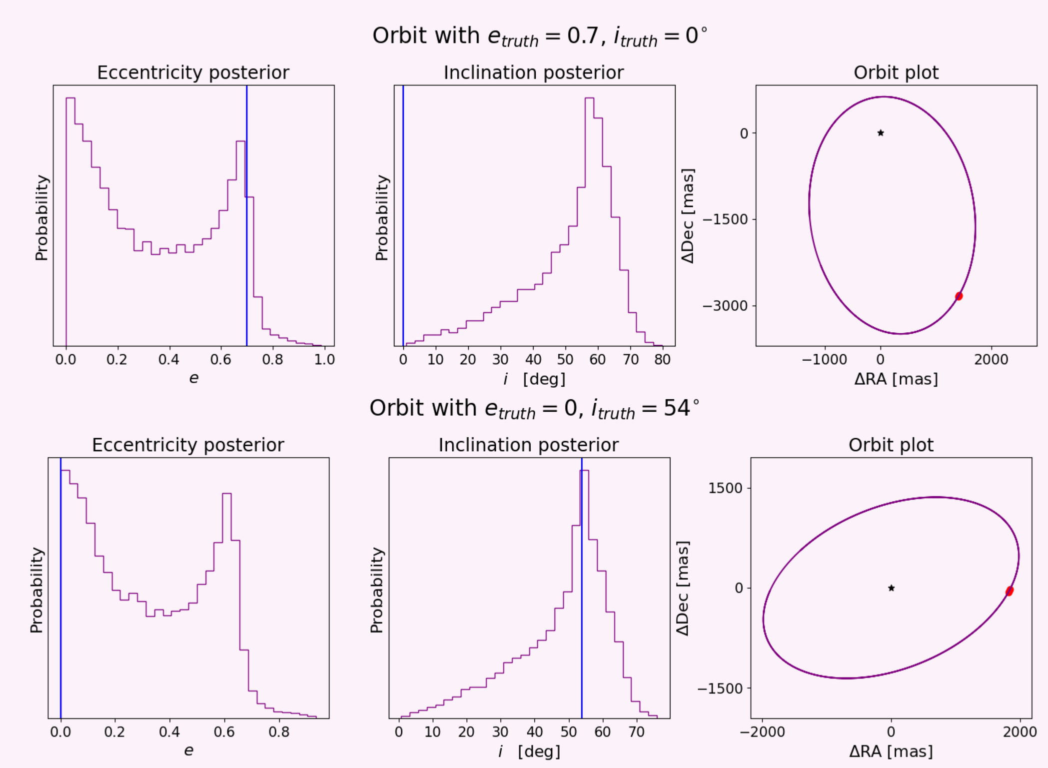
Case study: the e — i degeneracy

- In real life we do not know the true values of orbital parameters, only their posteriors.
- If we have the medians and modes of e and i, how certain can we be of those estimates depending of the region of parameter space they land on?
- Say you performed at orbit-fit that yielded posteriors with $e_{median} \sim 0.3, i_{median} \sim 54°$, $e_{mode} \sim 0, i_{mode} \sim 54°$. What orbits match those estimates?
- We took all 396 orbits from our multivariate analisys and selected the ones that matched the median or the mode estimates (within ± 0.05 for e and within $\pm 9°$ for i).



True orbital parameters of tested orbits that match the same median or mode estimates for e and i. The number in each bin is the number of orbits from that cell that matched the estimates (we used 6 values of τ for each pair e, i, so each cell contains exactly 6 orbits).

- **There is a degeneracy between *eccentric, near-edge-on* (i_{truth} >~ $45°$) orbits and *near-circular, near-face-on* (i_{truth} <~ $45°$) orbits.**
- How similar can the actual posteriors be?



Very similar! The degeneracy between inclination and eccentricity greatly affects the shapes of the posteriors.

This could explain the posteriors of 51 Eridani b (De Rosa et al, 2019).

After repeating this process for different pairs of estimates of e and i, **we found that this *$e - i$* degeneracy takes place for most values of *measured* eccentricity at near-edge-on *measured* inclinations (i.e. i_{median} or i_{mode} >~ $45°$), and that they are significantly reduced for most values of *measured* eccentricity when we have near-face-on *measured* inclinations (i.e. i_{median} or i_{mode} <~ $45°$).**

Conclusions

- True inclinations and true eccentricities are the orbital elements that can bias the most the eccentricity posterior, especially at near-edge-on values of i_{truth} and high values of e_{truth}.
- There is evidence to suggest that the mode is a better estimator of eccentricity than the median for near-face-on (<~ $45°$) true inclinations, and even measured inclinations.
- Inclination posteriors can be "well-behaved" but still be very biased.
- There is a degeneracy between eccentricity and inclination that makes it difficult to distinguish highly eccentric, face-on orbits from near-circular, near-edge-on orbits.
- The $i - e$ degeneracy is mostly present at near-edge-on measured inclinations.

Future work: Studying biases with greater orbital coverage; investigating the effects of RV data in mitigating biases; constructing less-biasing priors.