

Comparison of techniques for coupled earthquake and tsunami modeling

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Abstract

From interpreting data to scenario modeling of subduction events, numerical modeling has been crucial for studying tsunami generation by earthquakes. Seafloor instruments in the source region feature complex signals containing a superposition of seismic, ocean acoustic, and tsunami waves. Rigorous modeling is required to interpret these data and use them for tsunami early warning. However, previous studies utilize separate earthquake and tsunami models, with one-way coupling between them and approximations that might limit the applicability of the modeling technique. In this study, we compare four earthquake-tsunami modeling techniques, highlighting assumptions that affect the results, and discuss which techniques are appropriate for various applications. Most techniques couple a 3D Earth model with a 2D depth-averaged shallow water tsunami model. Assuming the ocean is incompressible and that tsunami propagation is negligible over the earthquake duration leads to technique (1), which equates earthquake seafloor uplift to initial tsunami sea surface height. For longer duration earthquakes, it is appropriate to follow technique (2), which uses time-dependent earthquake seafloor velocity as a time-dependent forcing in the tsunami mass balance. Neither technique captures ocean acoustic waves, motivating newer techniques that capture the seismic and ocean acoustic response as well as tsunamis. Saito et al. (2019) propose technique (3), which solves the 3D elastic and acoustic equations to model the earthquake rupture, seismic wavefield, and response of a compressible ocean without gravity. Then, sea surface height is used as a forcing term in a tsunami simulation. A superposition of the earthquake and tsunami solutions provides the complete wavefield, with one-way coupling. The complete wavefield is also captured in technique (4), which utilizes a fully-coupled solid Earth and ocean model with gravity (Lotto & Dunham, 2015). This technique, recently incorporated into the 3D code SeisSol, simultaneously solves earthquake rupture, seismic waves, and ocean response (including gravity). Furthermore, we show how technique (3) follows from (4) subject to well-justified approximations.

Comparison of techniques for coupled earthquake and tsunami modeling

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One-Way Coupled Techniques

Pass information from an
earthquake simulation

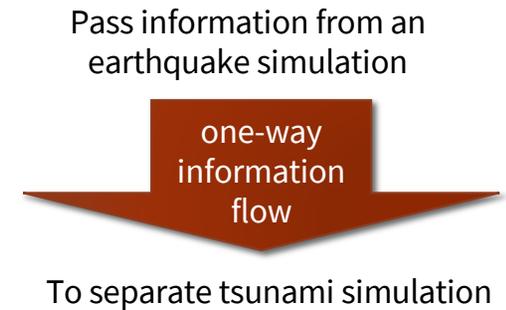
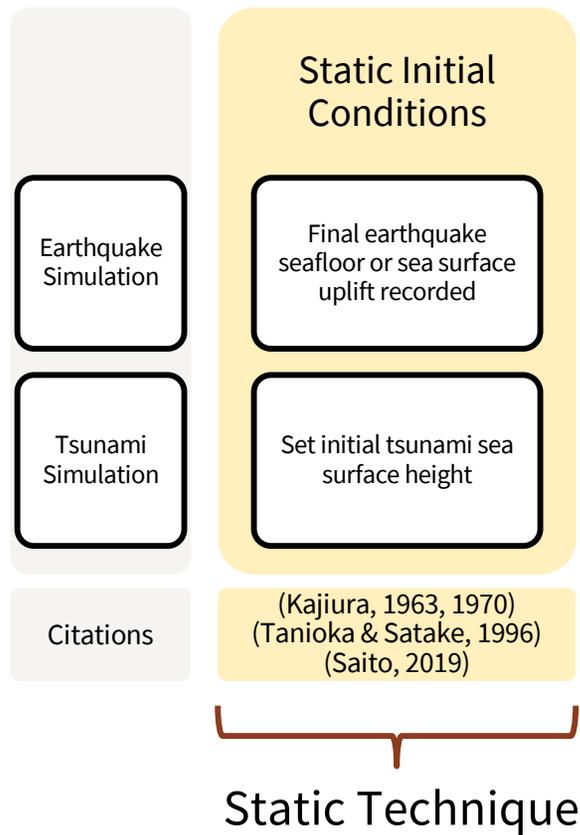


To separate tsunami simulation



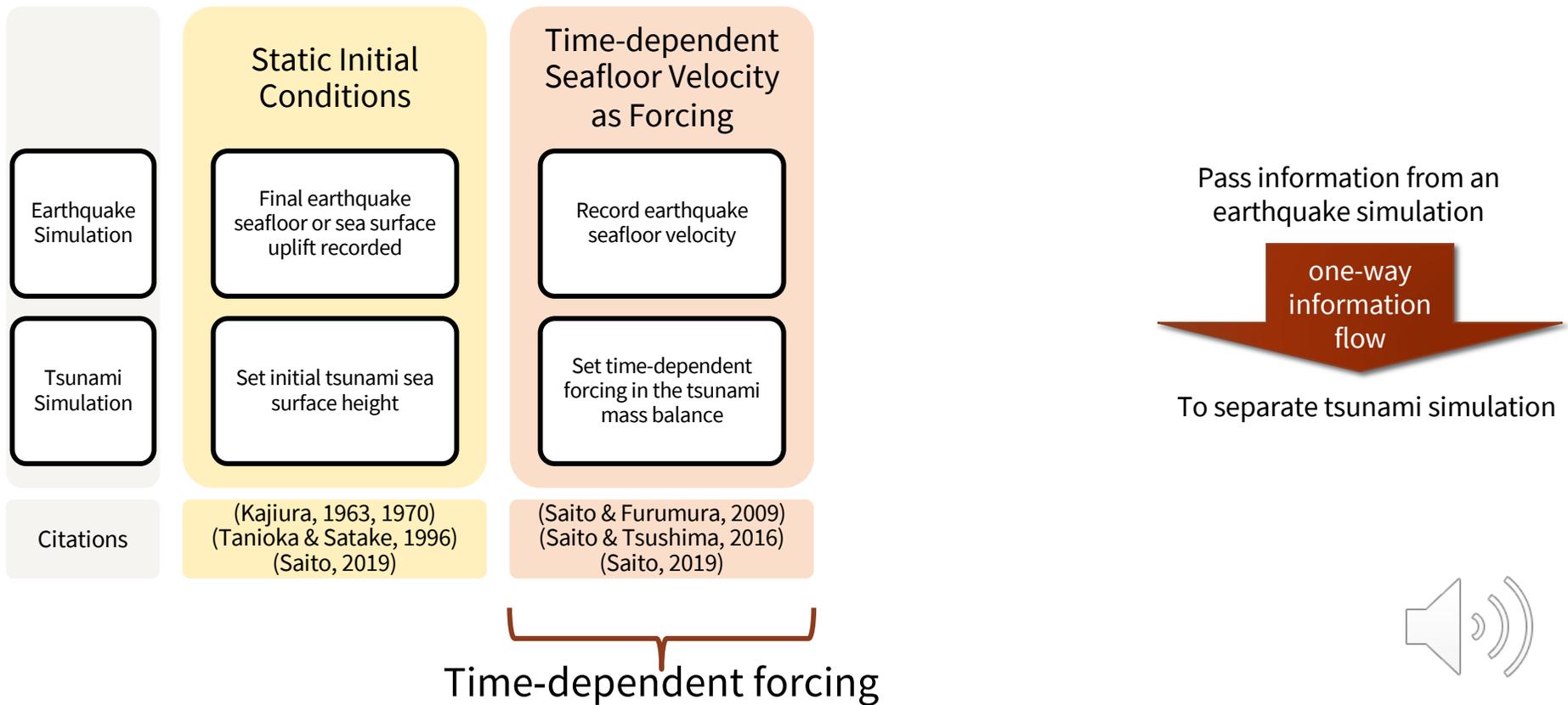
Coupled earthquake and tsunami modeling

Comparison of Model Techniques



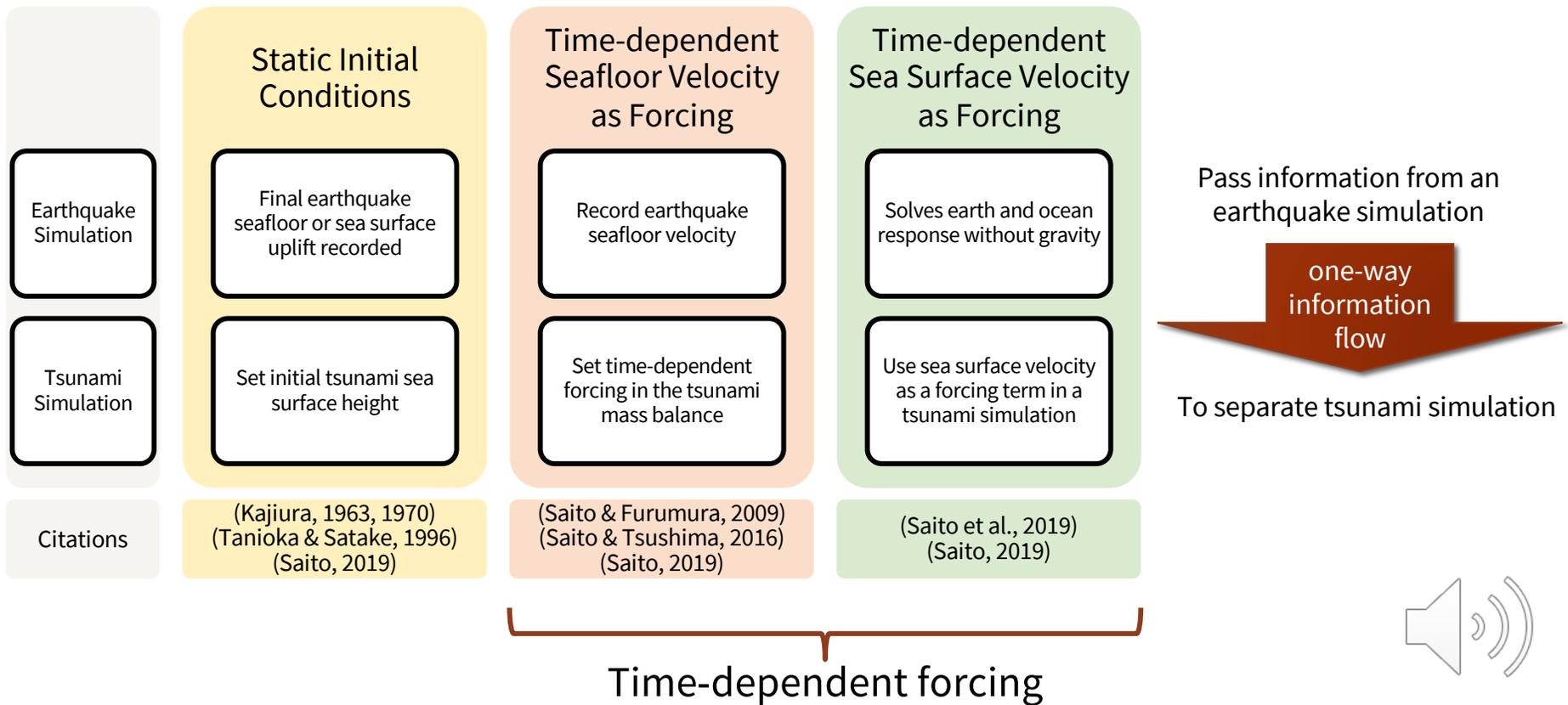
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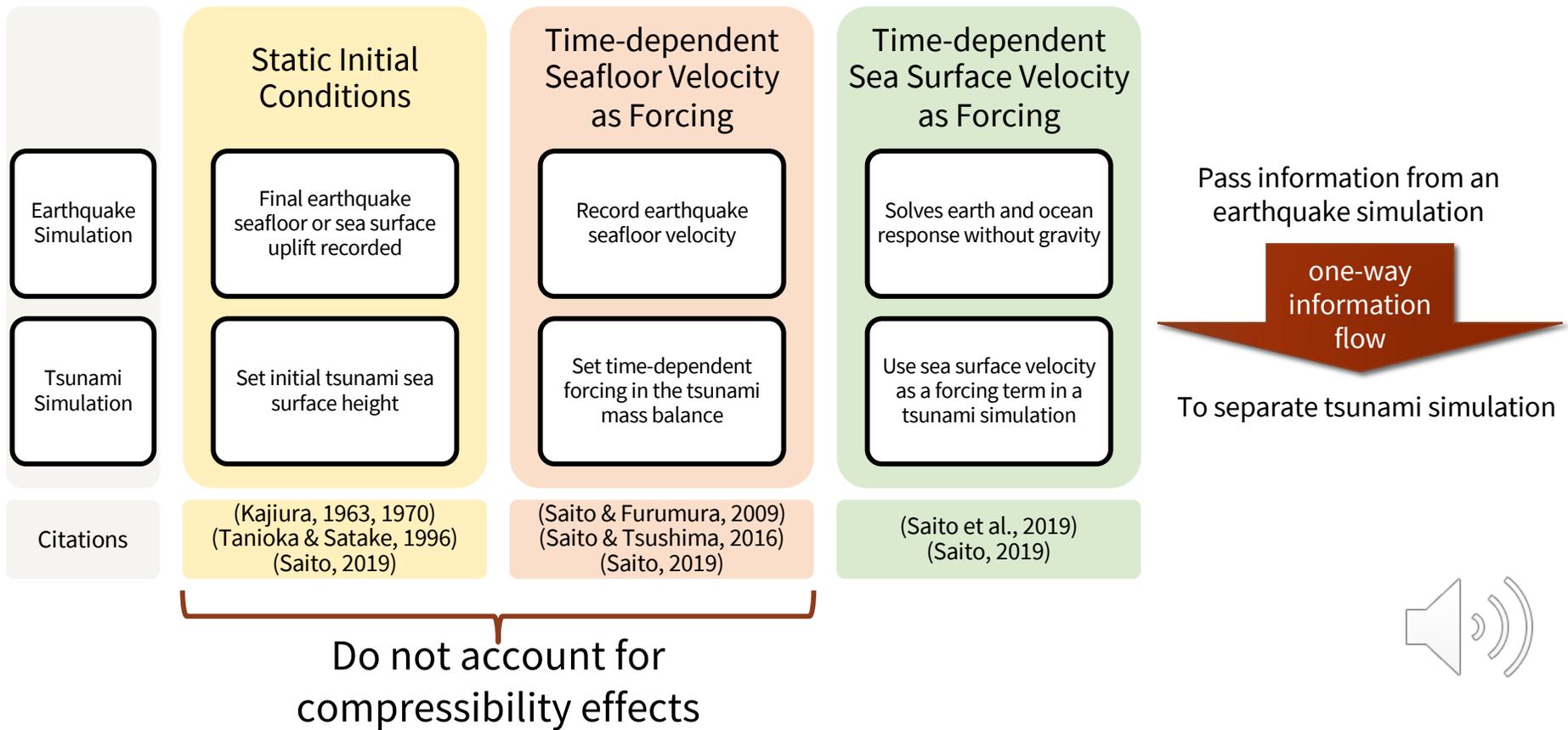
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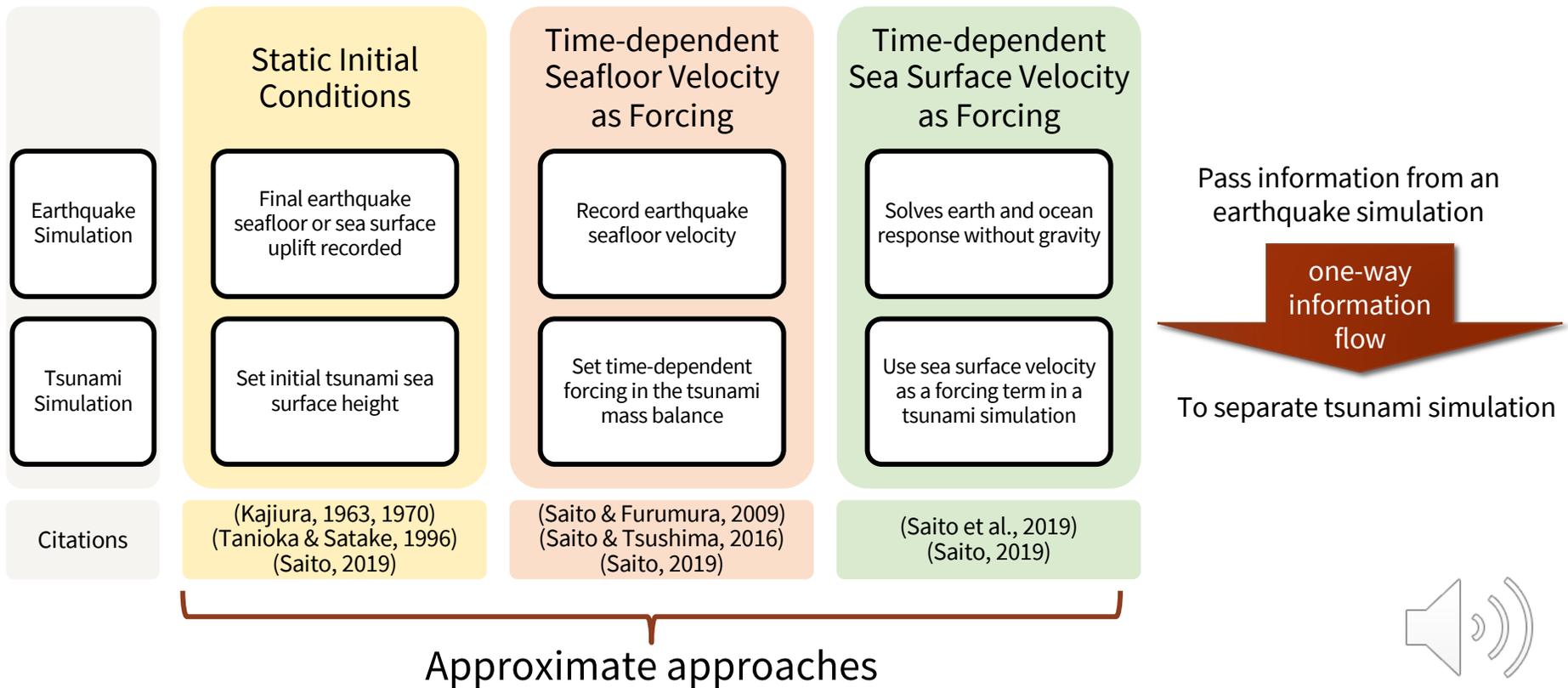
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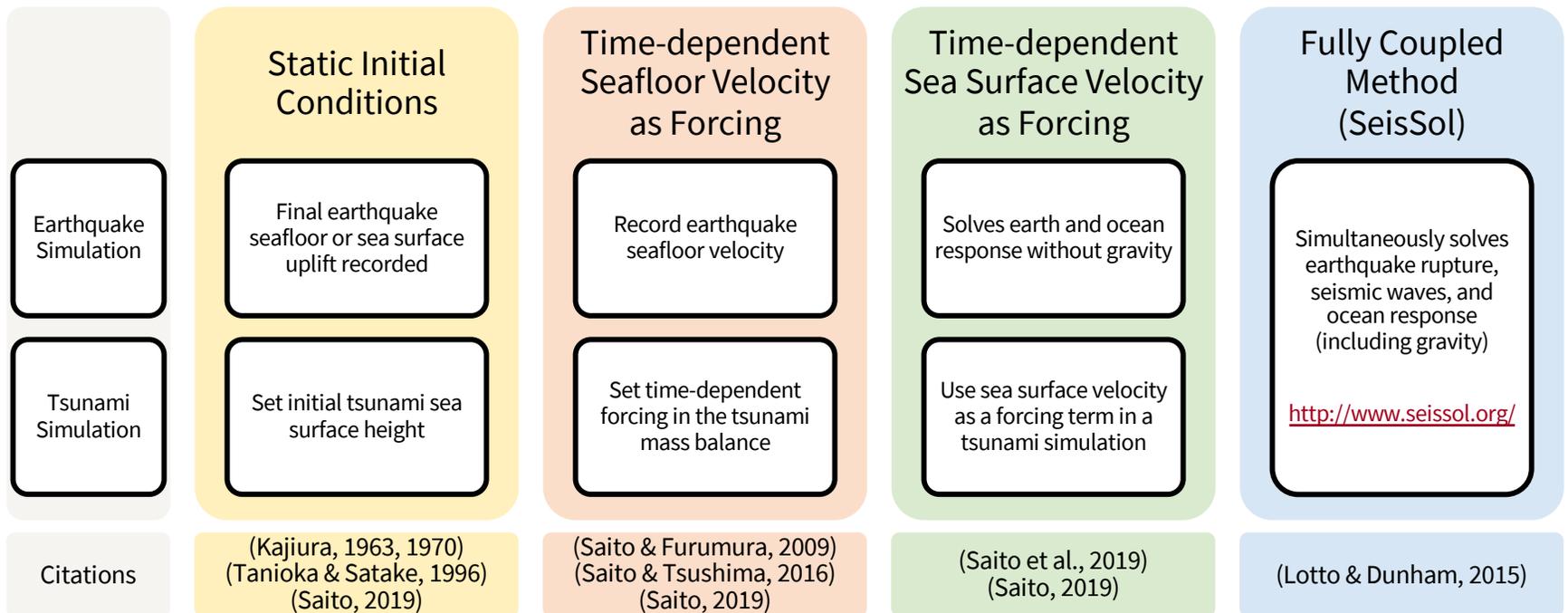
Coupled earthquake and tsunami modeling

Comparison of Model Techniques



Coupled earthquake and tsunami modeling

Comparison of Model Techniques



Approximate approaches

Considered “ground truth”
Newly implemented in 3D



Coupled earthquake and tsunami modeling

Comparison of Model Techniques

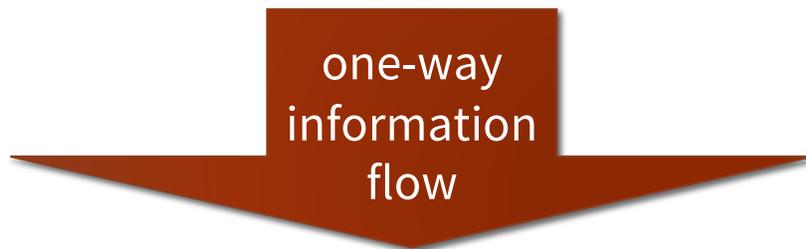
New in 3D

	Static Initial Conditions	Time-dependent Seafloor Velocity as Forcing	Time-dependent Sea Surface Velocity as Forcing	Fully Coupled Method (SeisSol)
Earthquake Simulation	Final earthquake seafloor or sea surface uplift recorded	Record earthquake seafloor velocity	Solves earth and ocean response without gravity	Simultaneously solves earthquake rupture, seismic waves, and ocean response (including gravity) http://www.seissol.org/
Tsunami Simulation	Set initial tsunami sea surface height	Set time-dependent forcing in the tsunami mass balance	Use sea surface velocity as a forcing term in a tsunami simulation	
Citations	(Kajiura, 1963, 1970) (Tanioka & Satake, 1996) (Saito, 2019)	(Saito & Furumura, 2009) (Saito & Tsushima, 2016) (Saito, 2019)	(Saito et al., 2019) (Saito, 2019)	(Lotto & Dunham, 2015)

Model the full-wave field 

One-Way Coupled Techniques

Pass information from an
earthquake simulation



To separate tsunami simulation

Model techniques:

- Static initial conditions vs time-dependent forcing
- Incompressible vs compressible ocean

How is modeled data affected by:

1. long rupture duration
2. acoustic wave generation



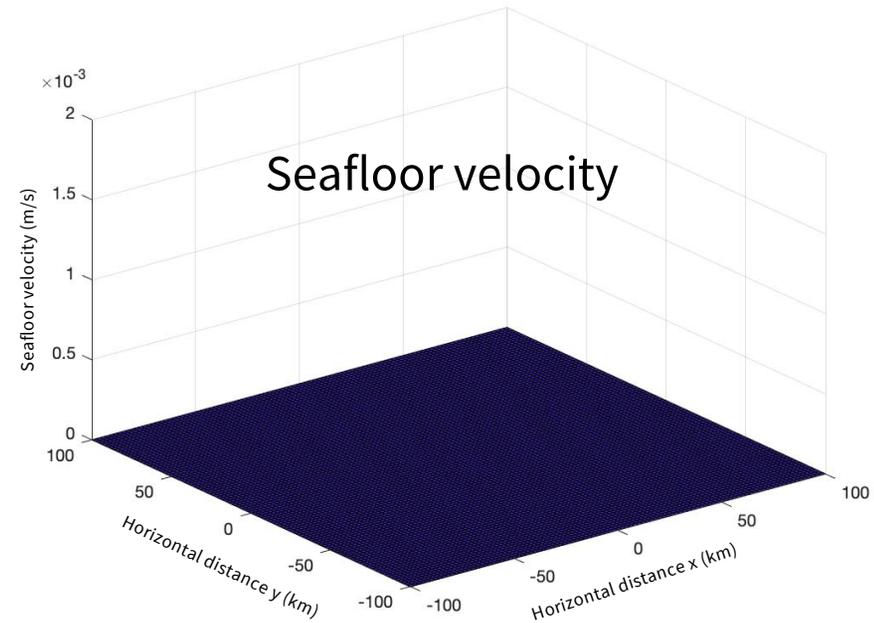
Problem setup

- We want to examine how **long duration rupture** and **compressibility** affect wave generation in the four modeling techniques



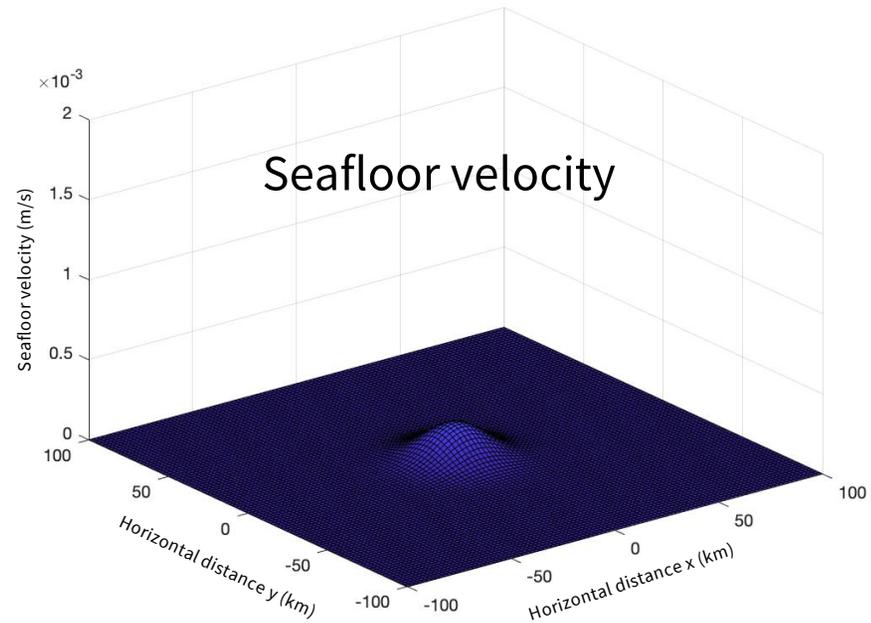
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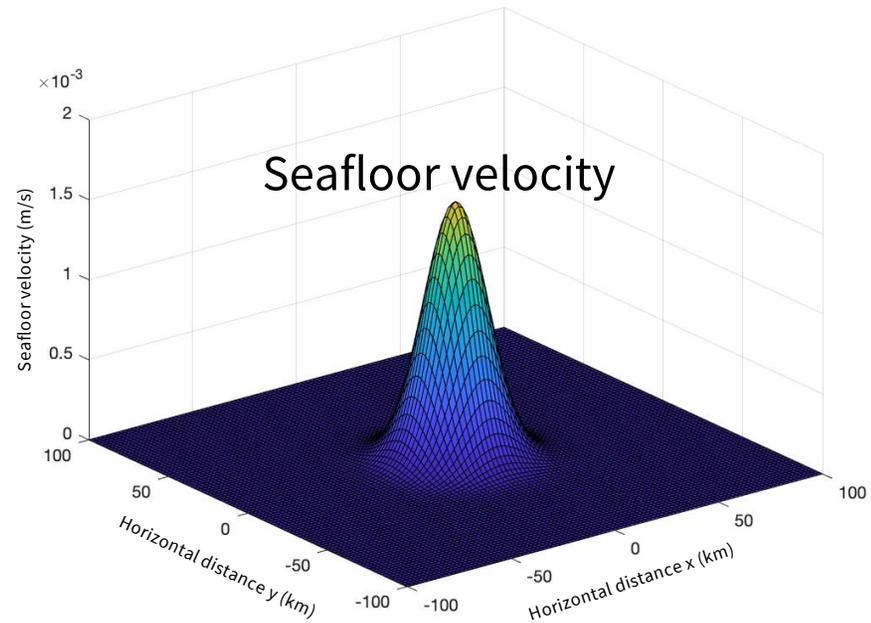
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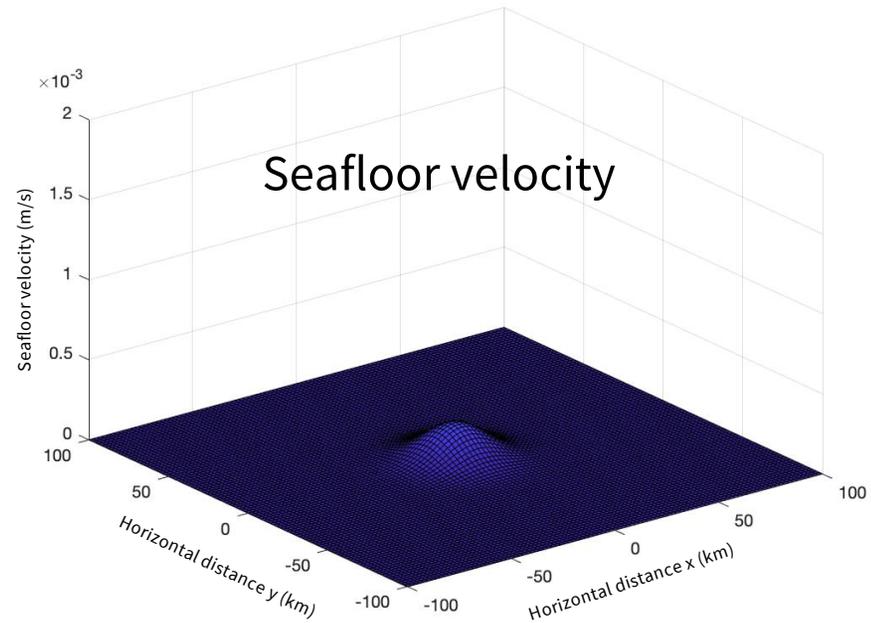
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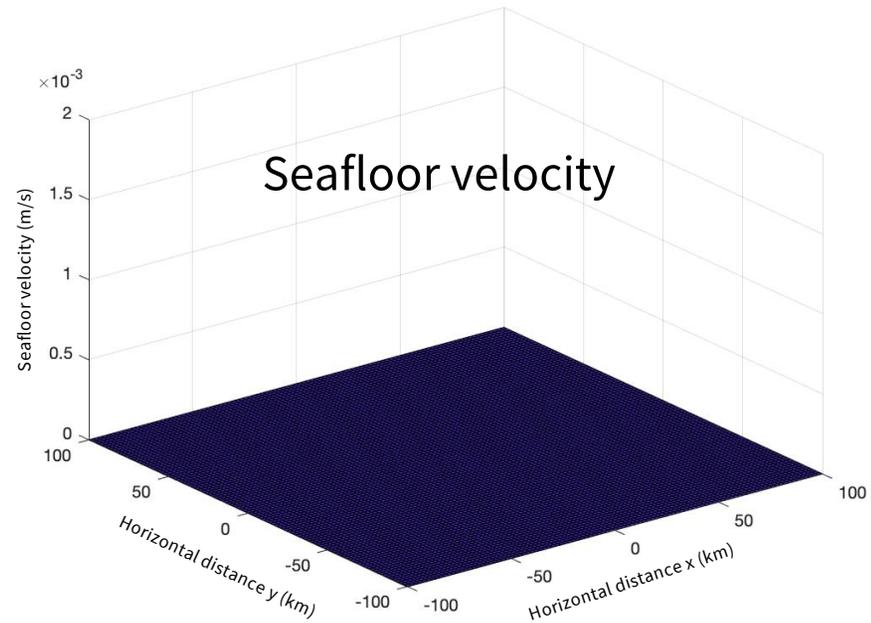
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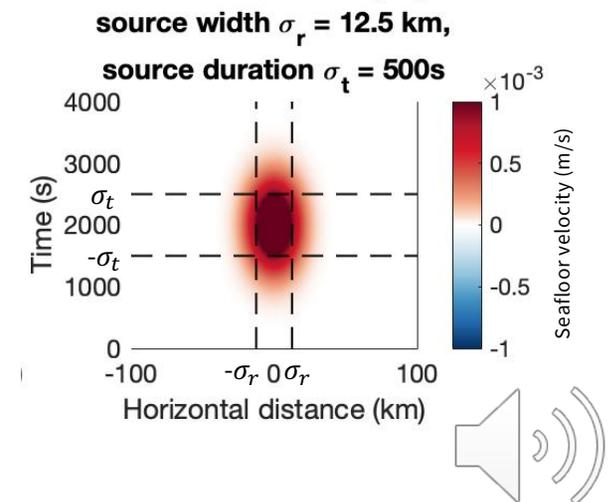
Problem setup

- We want to examine how **long duration rupture** and **compressibility** affect wave generation in the four modeling techniques



Problem setup

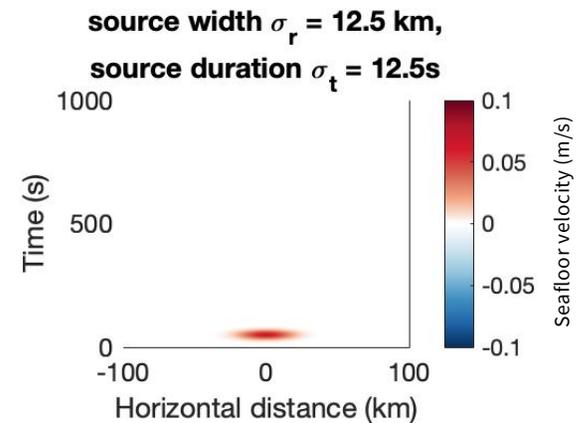
- We want to examine how **long duration rupture** and **compressibility** affect wave generation in the four modeling techniques
- We vary source width (σ_r) and duration (σ_t) in the earthquake simulation to test different scenarios setups



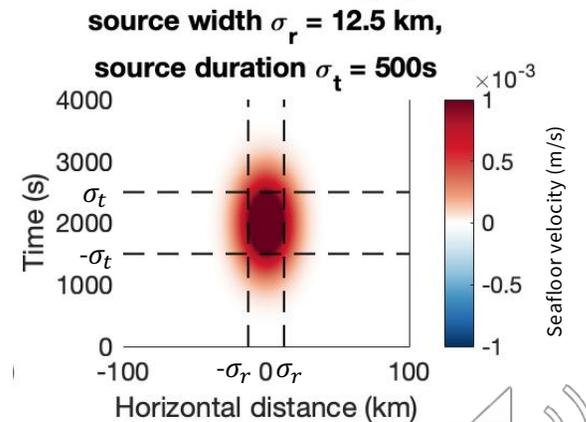
Problem setup

- We want to examine how **long duration rupture** and **compressibility** affect wave generation in the four modeling techniques
- We vary source width (σ_r) and duration (σ_t) in the earthquake simulation to test different scenarios setups

Short rupture duration σ_t

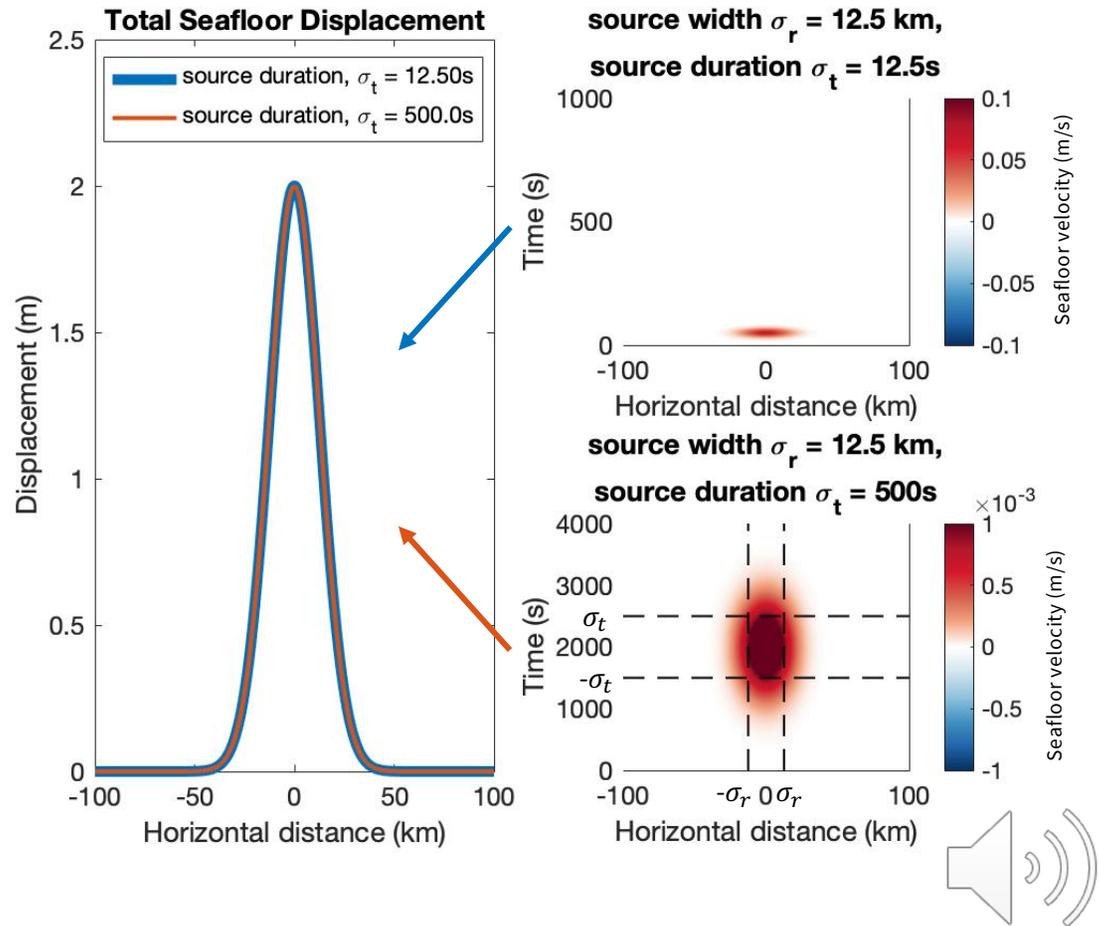


Long rupture duration σ_t



Problem setup

- We want to examine how **long duration rupture** and **compressibility** affect wave generation in the four modeling techniques
- We vary source width (σ_r) and duration (σ_t) in the earthquake simulation to test different scenarios setups



Problem setup

- We want to examine how **long duration rupture** and **compressibility** affect wave generation in the four modeling techniques
- We vary source width (σ_r) and duration (σ_t) in the earthquake simulation to test different scenarios setups

Within shallow water limit?

No if:

$$\frac{H}{\sigma_r} \gg 1$$

Tsunami propagates over source duration?

No if:

$$\frac{\sigma_r}{\sigma_t \sqrt{gH}} \gg 1$$

Acoustic waves significant?

No if:

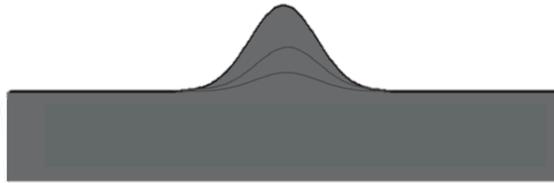
$$\frac{c\sigma_t}{H} \gg 1$$

Where, the ocean depth is $H = 4\text{km}$, tsunami wave speed is \sqrt{gH} , and acoustic wavespeed c



Problem setup

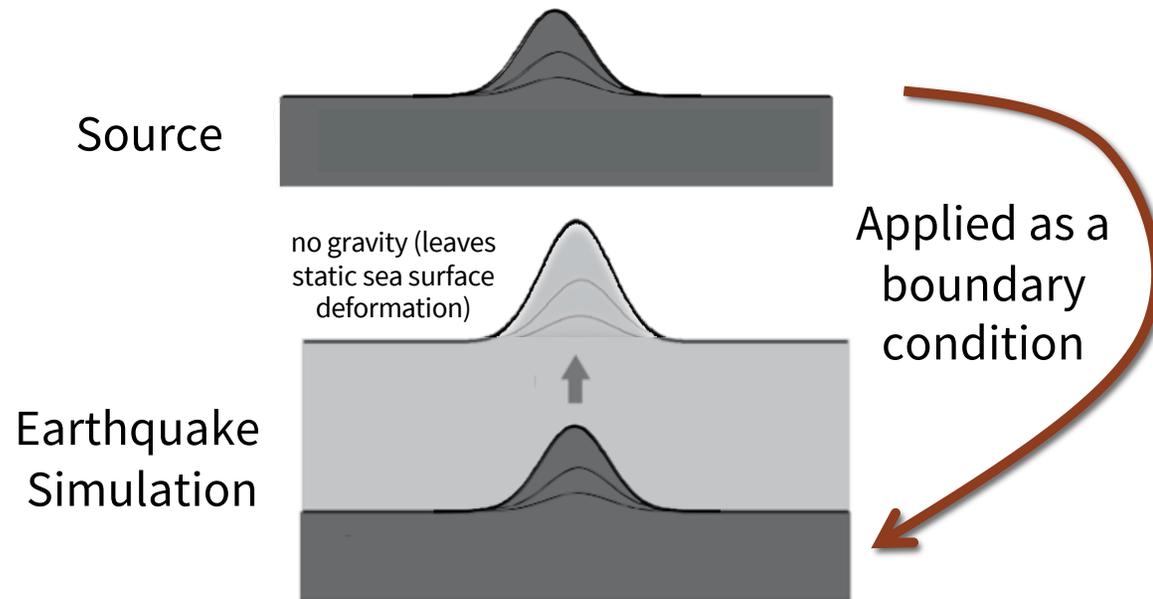
Source



(modified from Saito et al., 2019)



Problem setup

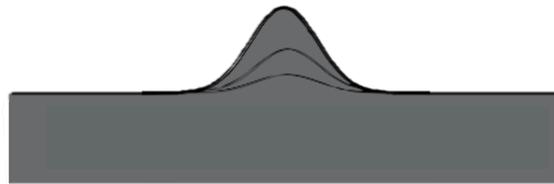


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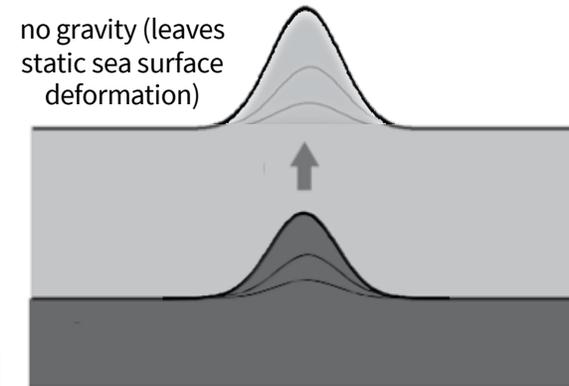


Problem setup

Source



Earthquake Simulation



one-way information flow

Tsunami Simulation

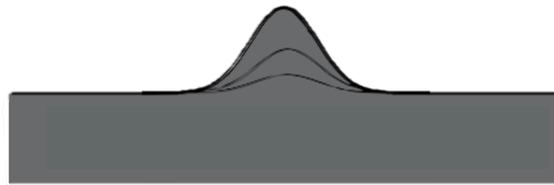


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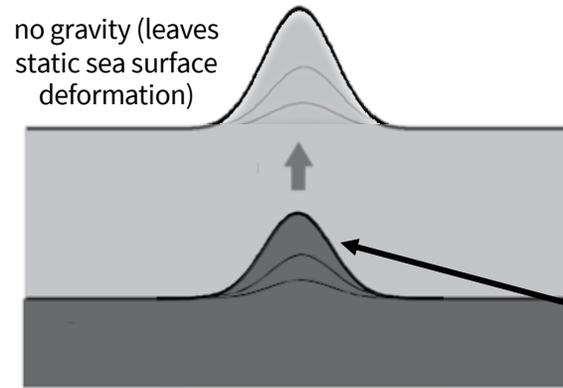


Problem setup

Source



Earthquake Simulation



Static Initial condition

one-way information flow

Tsunami Simulation

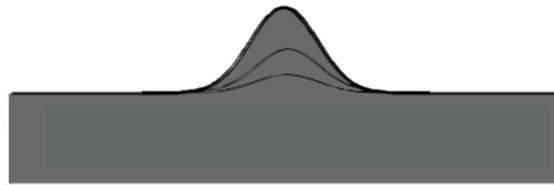


(modified from Saito et al., 2019)

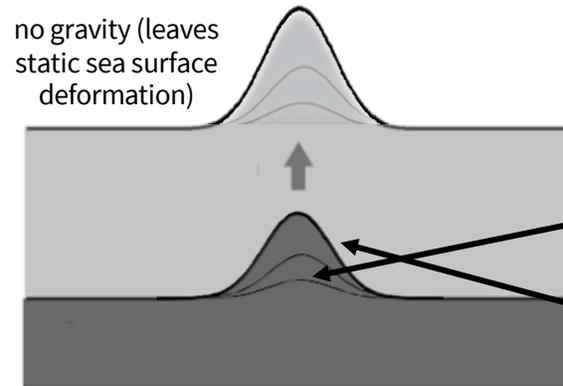


Problem setup

Source



Earthquake Simulation



no gravity (leaves static sea surface deformation)

Time-dependent Seafloor Velocity as Forcing

Static Initial condition

one-way information flow

Tsunami Simulation

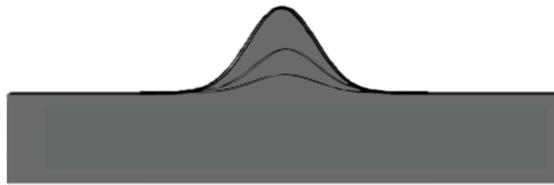


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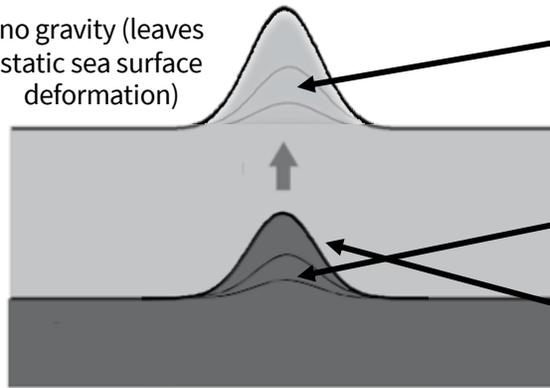
Problem setup

Source



no gravity (leaves static sea surface deformation)

Earthquake Simulation



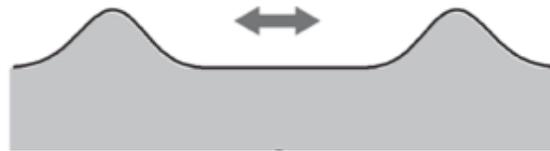
Time-dependent Sea Surface Velocity as Forcing

Time-dependent Seafloor Velocity as Forcing

Static Initial condition

one-way information flow

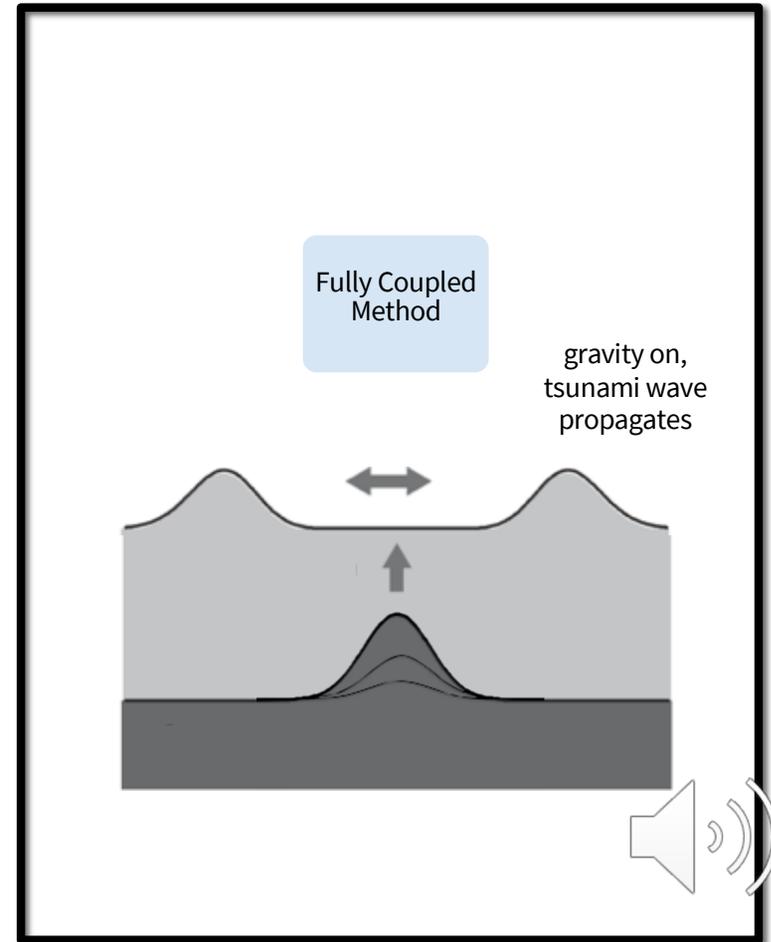
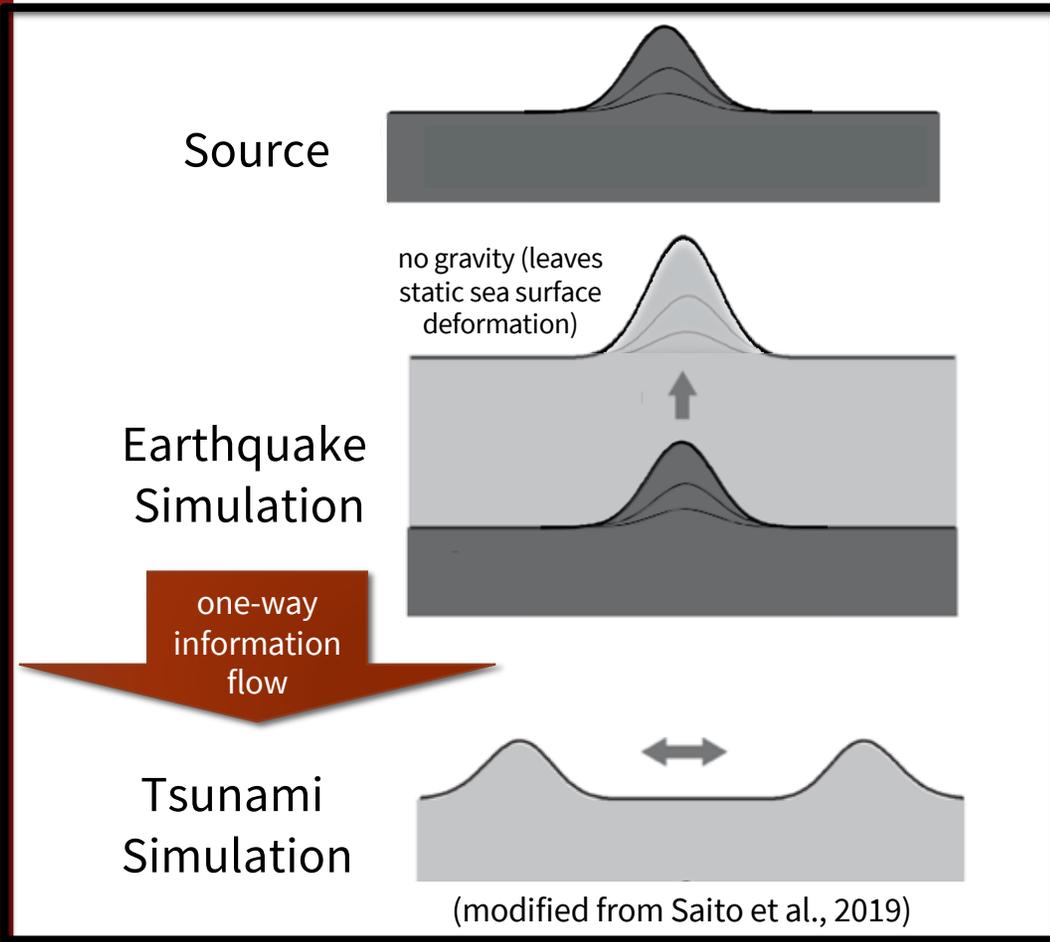
Tsunami Simulation



(modified from Saito et al., 2019)



Problem setup



Revised Slides Begin here

Research is always ongoing, this talk has been revised to show new exciting results!



Example 1:

Source Width : $\sigma_r = 12.5 \text{ km}$

Source Duration : $\sigma_t = 500 \text{ s}$

Within shallow water limit?

Yes: $\frac{H}{\sigma_r} = 0.32 < 1$

Tsunami propagates over source duration?

Yes: $\frac{\sigma_r}{\sigma_t \sqrt{gH}} = 0.13 < 1$

Acoustic waves significant?

No: $\frac{c\sigma_t}{H} = 187.5 > 1$

Method 1.
Static Initial
condition

Method 2.
Time-dependent
Seafloor Velocity
as Forcing

Method 3.
Time-dependent
Sea Surface
Velocity as
Forcing

Method 4.
Fully Coupled
Method

We can anticipate
method 1 (using a
static initial
condition) will be
incorrect



Source Width : $\sigma_r = 12.5$ km
Source Duration : $\sigma_t = 500$ s
Yes: Within shallow water limit?
Yes: Tsunami propagates over source duration?
No: Acoustic waves significant?

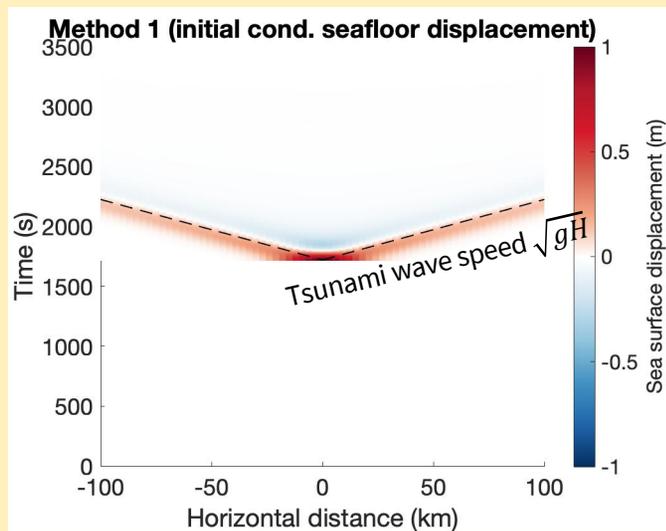
Method 1.
Static Initial
condition

Method 2.
Time-dependent
Seafloor Velocity
as Forcing

Method 3.
Time-dependent
Sea Surface
Velocity as
Forcing

Method 4.
Fully Coupled
Method

Seafloor displacement as initial conditions



Source Width : $\sigma_r = 12.5$ km
 Source Duration : $\sigma_t = 500$ s
 Yes: Within shallow water limit?
 Yes: Tsunami propagates over source duration?
 No: Acoustic waves significant?

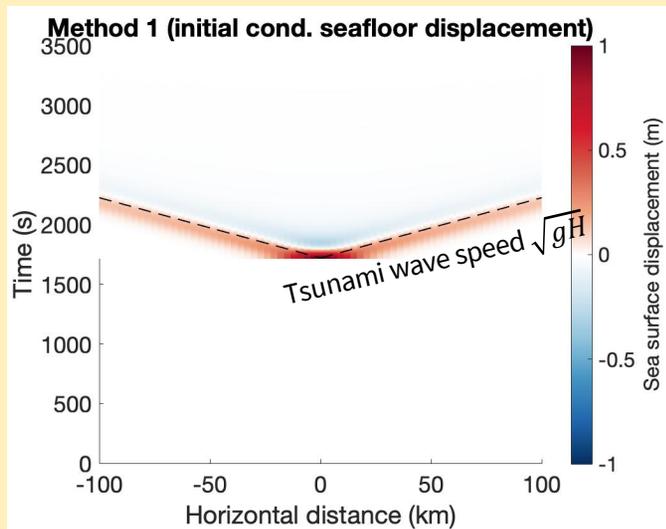
Method 1.
Static Initial
condition

Method 2.
Time-dependent
Seafloor Velocity
as Forcing

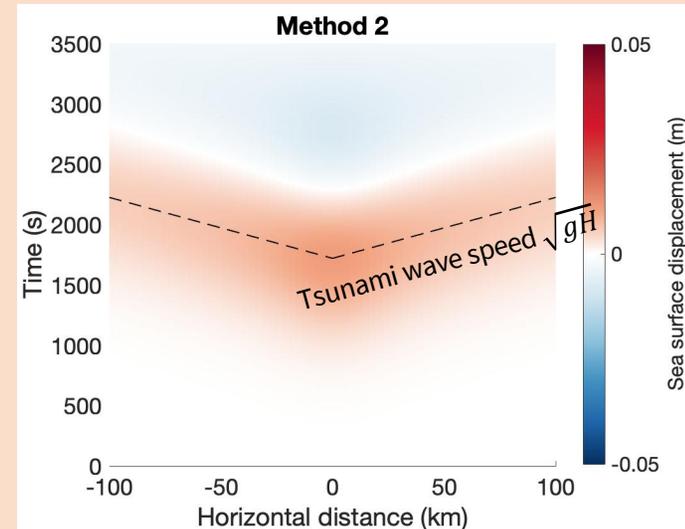
Method 3.
Time-dependent
Sea Surface
Velocity as
Forcing

Method 4.
Fully Coupled
Method

Seafloor displacement as initial conditions



Seafloor velocity as forcing



Source Width : $\sigma_r = 12.5$ km
 Source Duration : $\sigma_t = 500$ s
 Yes: Within shallow water limit?
 Yes: Tsunami propagates over source duration?
 No: Acoustic waves significant?

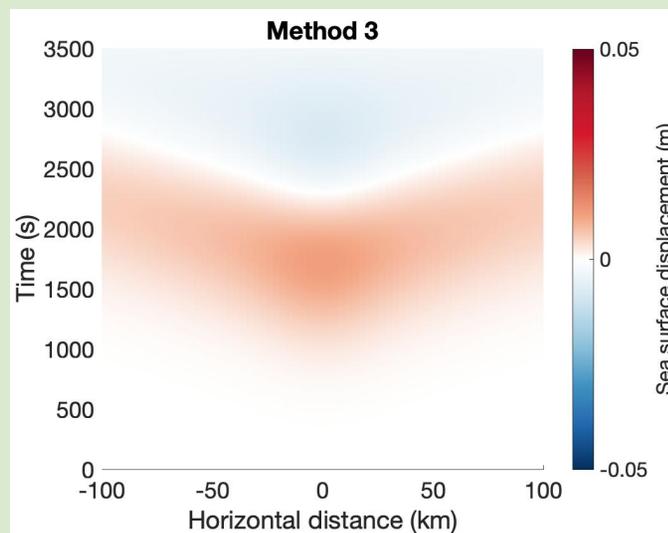
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Static Initial
condition

Method 2.
Time-dependent
Seafloor Velocity
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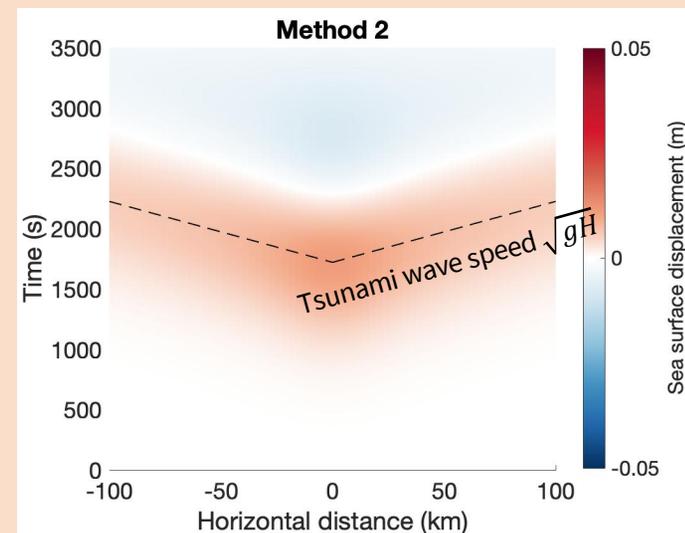
Method 3.
Time-dependent
Sea Surface
Velocity as
Forcing

Method 4.
Fully Coupled
Method

Sea surface velocity as forcing



Seafloor velocity as forcing



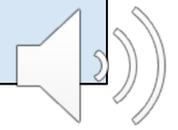
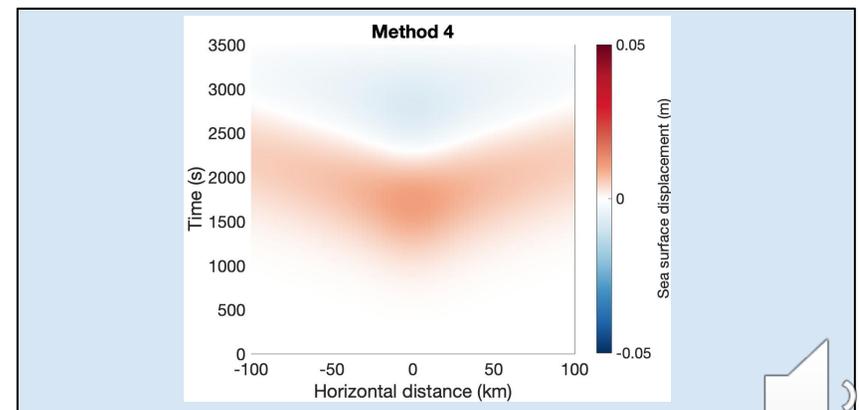
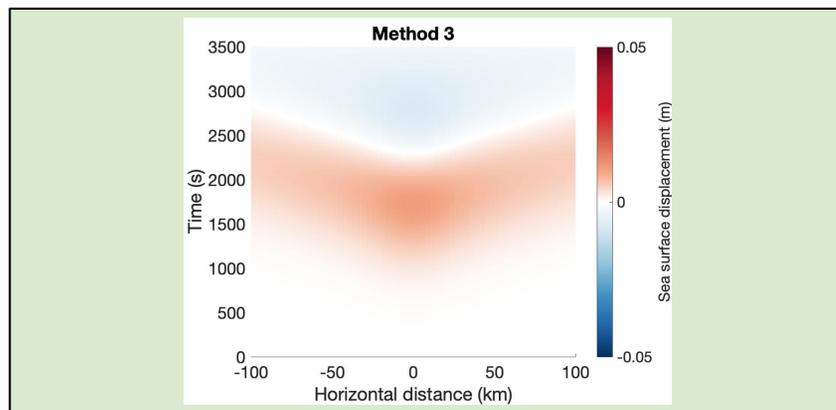
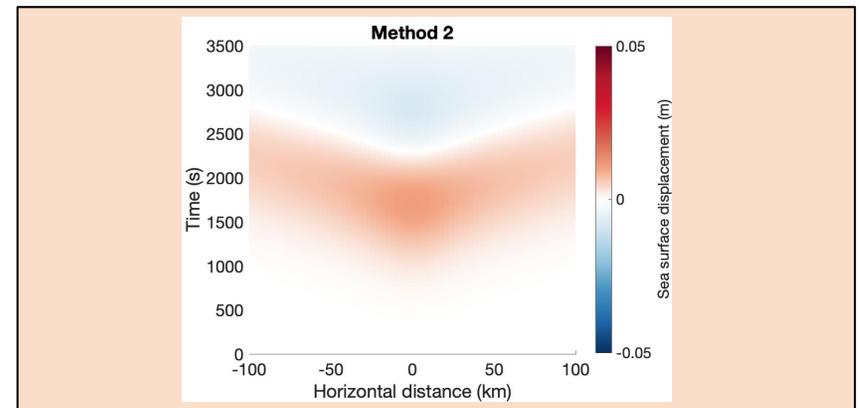
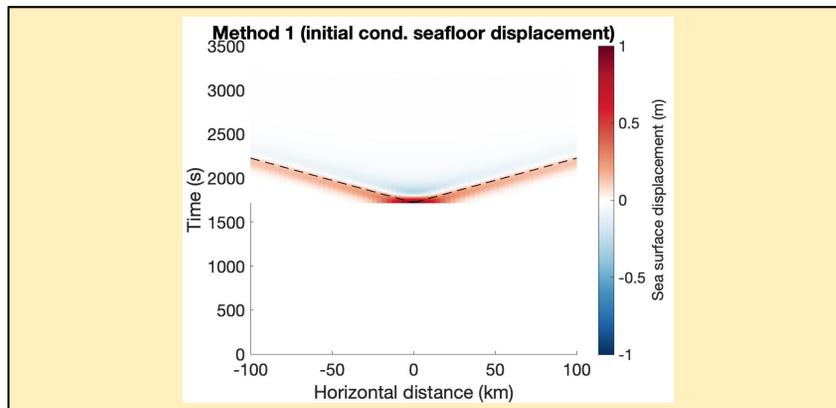
Source Width : $\sigma_r = 12.5$ km
 Source Duration : $\sigma_t = 500$ s
 Yes: Within shallow water limit?
 Yes: Tsunami propagates over source duration?
 No: Acoustic waves significant?

Method 1.
Static Initial
condition

Method 2.
Time-dependent
Seafloor Velocity
as Forcing

Method 3.
Time-dependent
Sea Surface
Velocity as
Forcing

Method 4.
Fully Coupled
Method



Example 2:

Source Width : $\sigma_r = 12.5$ km

Source Duration : $\sigma_t = 1.25$ s

Within shallow water limit?

Yes: $\frac{H}{\sigma_r} = 0.32 < 1$

Tsunami propagates over source duration?

No: $\frac{\sigma_r}{\sigma_t \sqrt{gH}} = 50.5 > 1$

Acoustic waves significant?

Yes: $\frac{c\sigma_t}{H} = 0.47 < 1$

Method 1.
Static Initial
condition

Method 2.
Time-dependent
Seafloor Velocity
as Forcing

Method 3.
Time-dependent
Sea Surface
Velocity as
Forcing

Method 4.
Fully Coupled
Method

Method 1 and 2 do not model acoustic wave generation, we anticipate the results will differ compared to methods 3 and 4



Source Width : $\sigma_r = 12.5$ km
 Source Duration : $\sigma_t = 1.25$ s
 Yes: Within shallow water limit?
 No: Tsunami propagates over source duration?
 Yes: Acoustic waves significant?

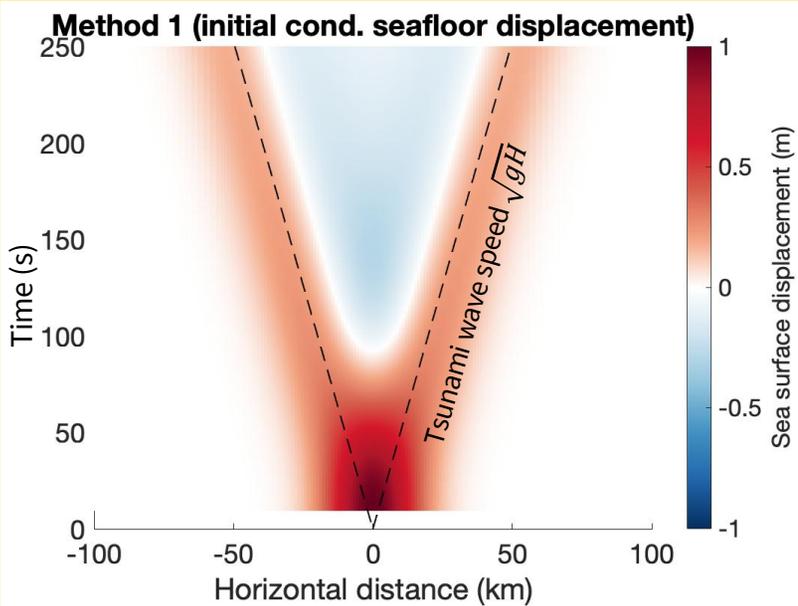
Method 1.
Static Initial
condition

Method 2.
Time-dependent
Seafloor Velocity
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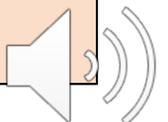
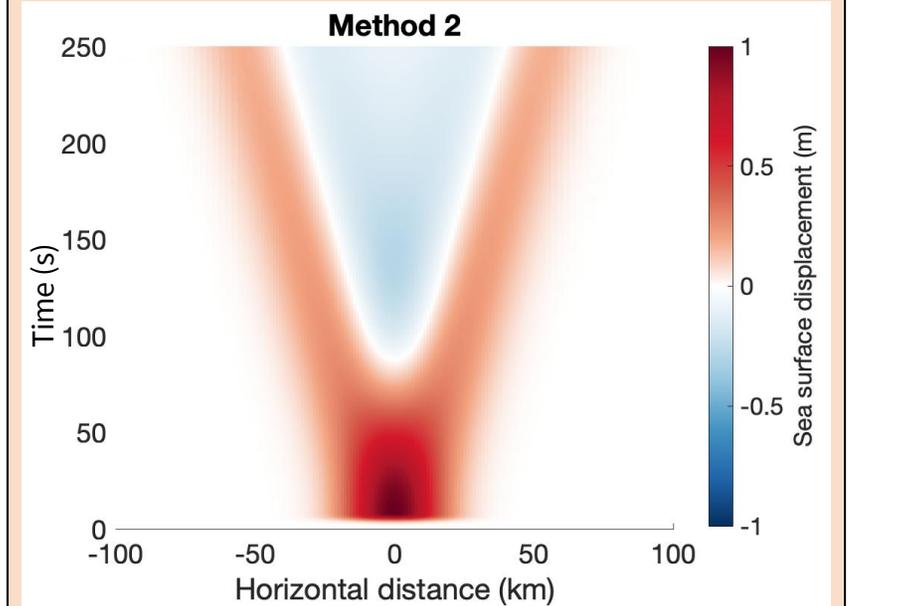
Method 3.
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Method 4.
Fully Coupled
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Seafloor displacement as initial conditions



Seafloor velocity as forcing



Source Width : $\sigma_r = 12.5$ km
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Yes: Within shallow water limit?
No: Tsunami propagates over source duration?
Yes: Acoustic waves significant?

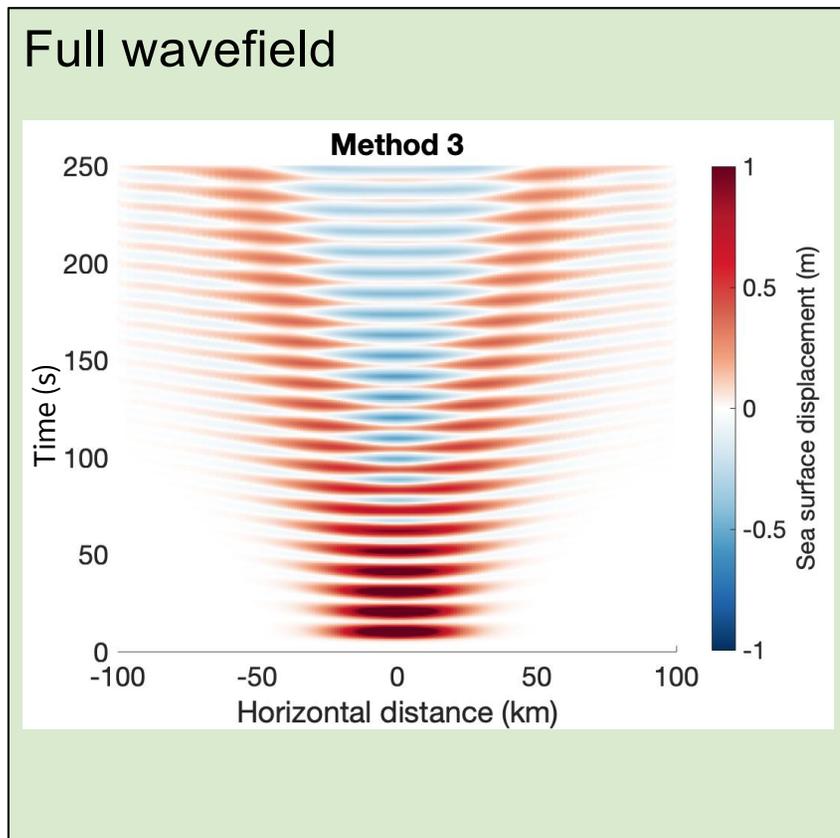
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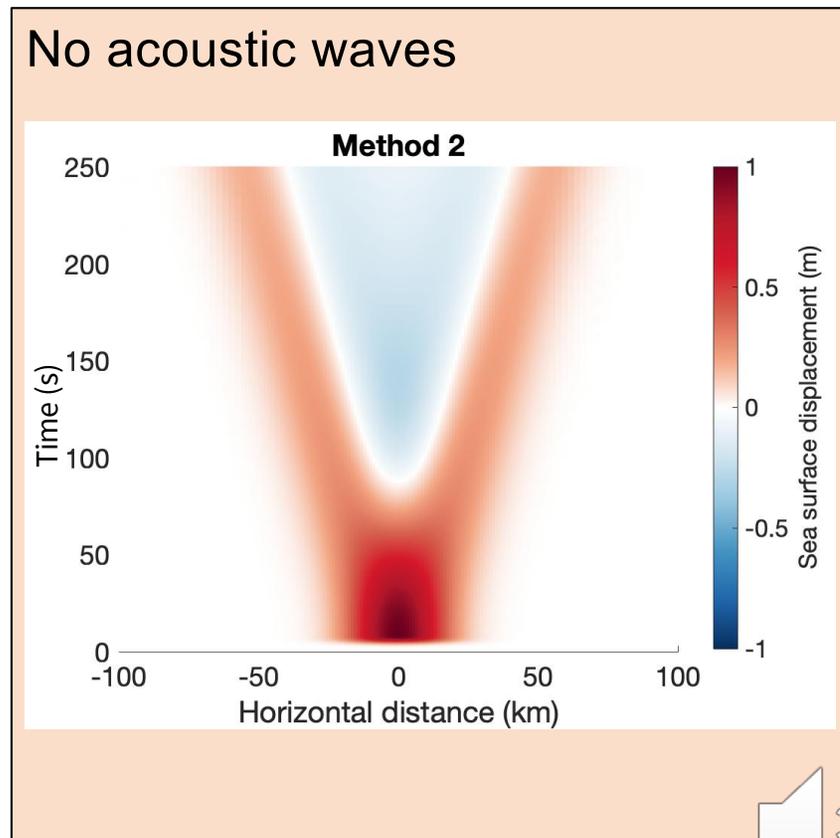
Method 3.
Time-dependent
Sea Surface
Velocity as
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Method 4.
Fully Coupled
Method

Full wavefield



No acoustic waves



Source Width : $\sigma_r = 12.5$ km
Source Duration : $\sigma_t = 1.25$ s
Yes: Within shallow water limit?
No: Tsunami propagates over source duration?
Yes: Acoustic waves significant?

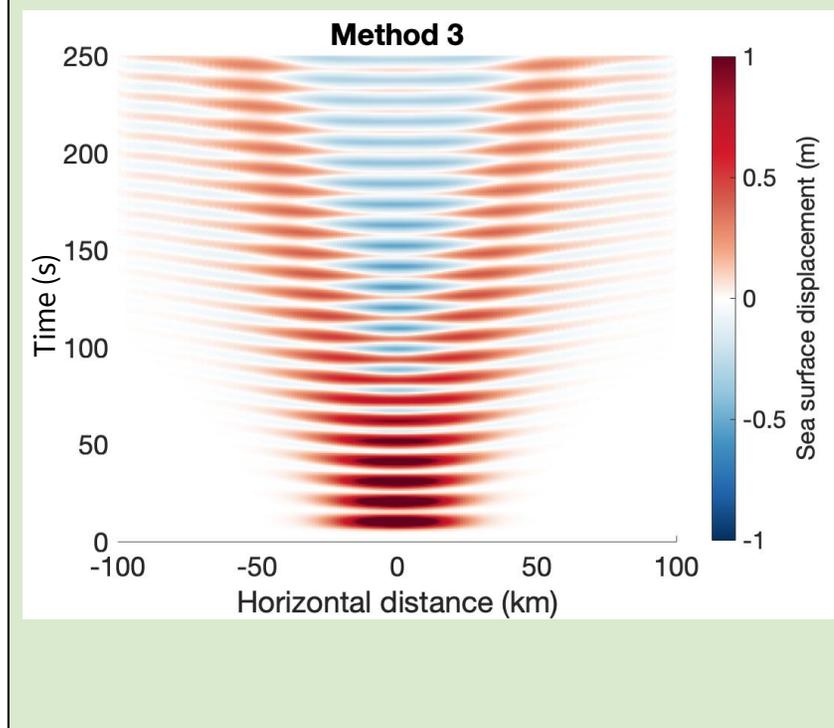
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Method 2.
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Seafloor Velocity
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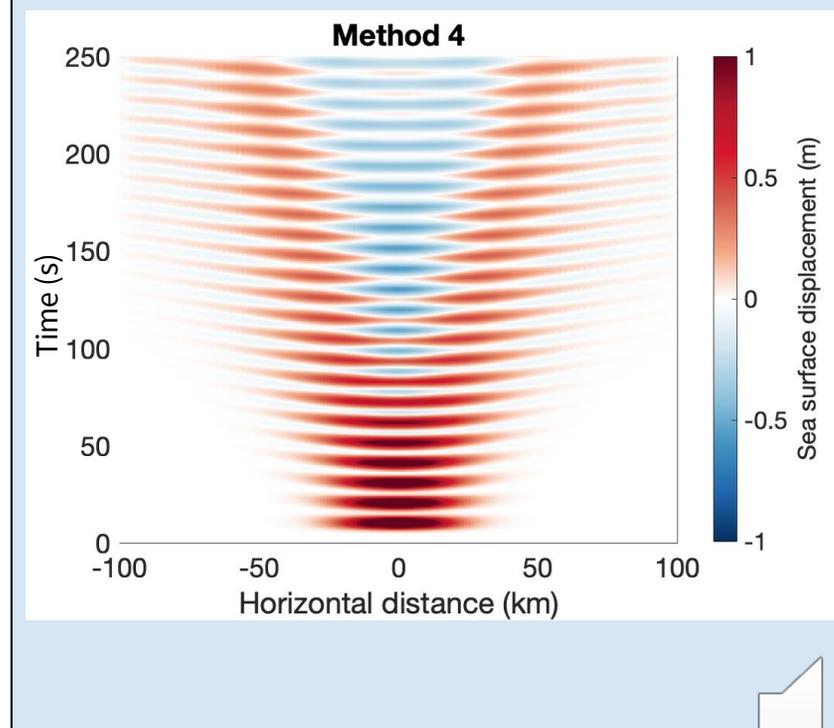
Method 3.
Time-dependent
Sea Surface
Velocity as
Forcing

Method 4.
Fully Coupled
Method

Full wavefield (one-way coupling)



Full wavefield (fully coupled)



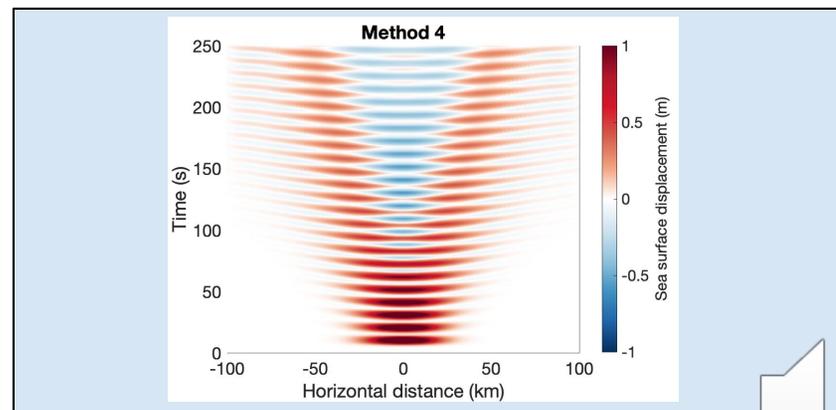
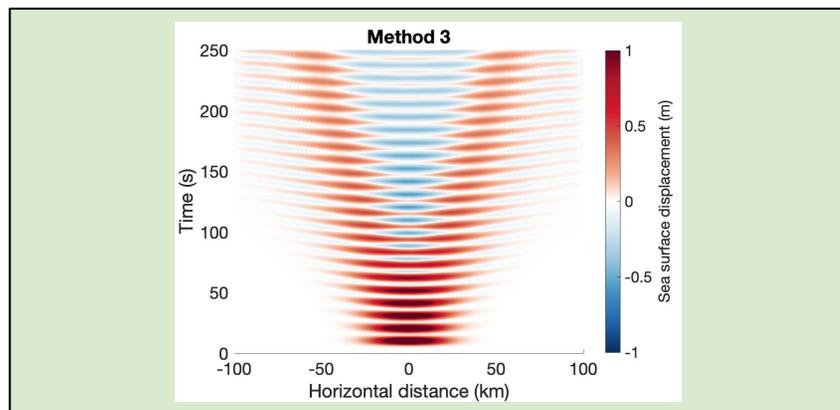
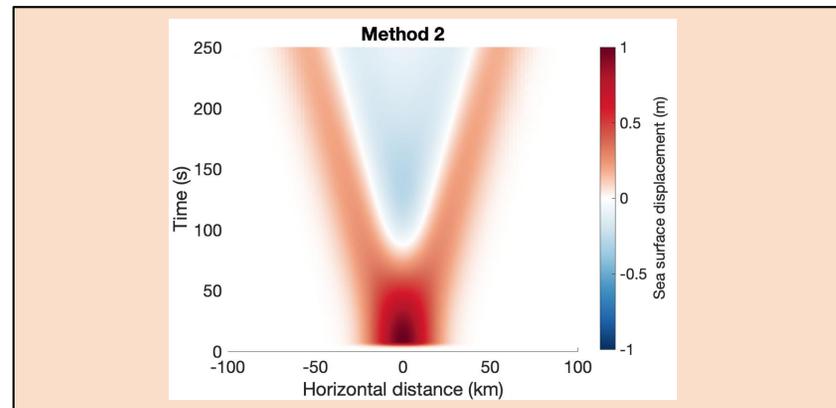
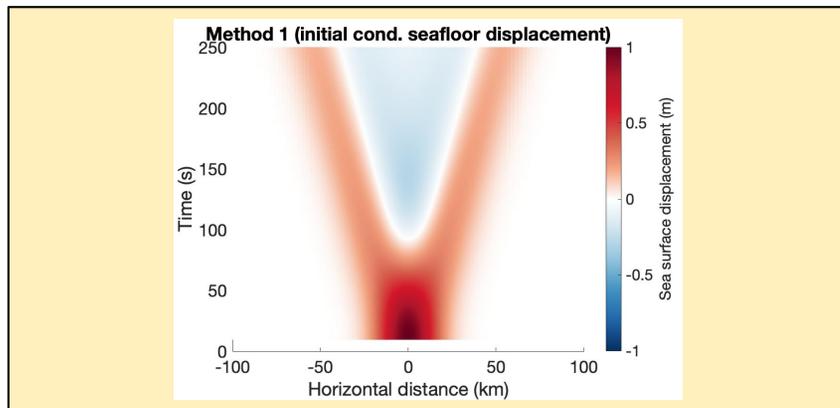
Source Width : $\sigma_r = 12.5$ km
Source Duration : $\sigma_t = 1.25$ s
Yes: Within shallow water limit?
No: Tsunami propagates over source duration?
Yes: Acoustic waves significant?

Method 1.
Static Initial
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Method 2.
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Seafloor Velocity
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Method 4.
Fully Coupled
Method



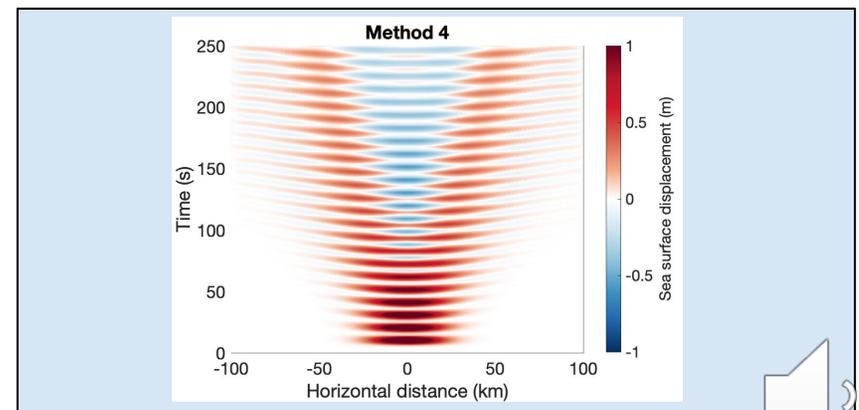
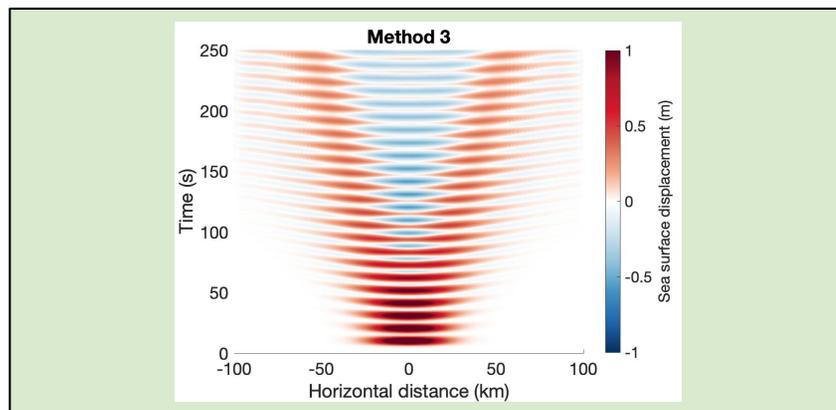
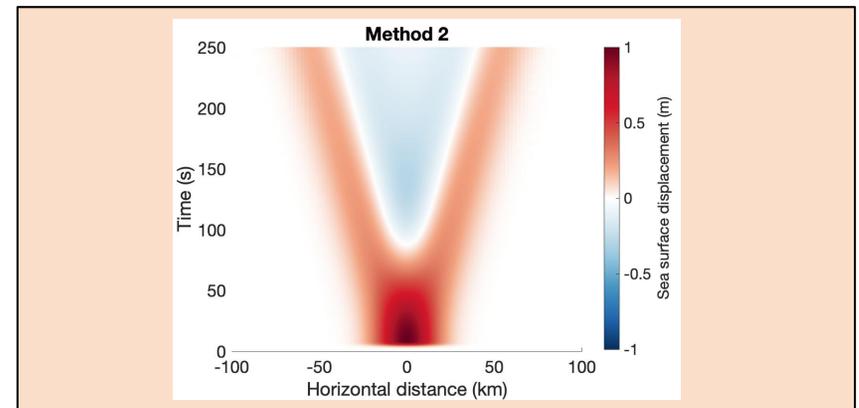
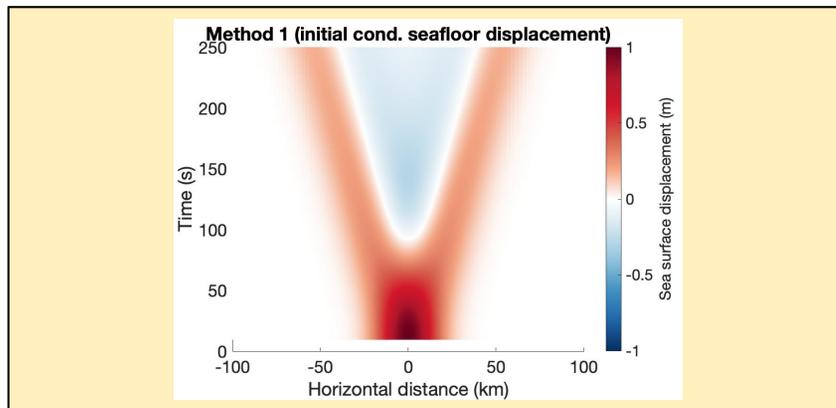
Source Width : $\sigma_r = 12.5$ km
 Source Duration : $\sigma_t = 1.25$ s
 Yes: Within shallow water limit?
 No: Tsunami propagates over source duration?
 Yes: Acoustic waves significant?

Method 1.
Static Initial
condition

Method 2.
Time-dependent
Seafloor Velocity
as Forcing

Method 3.
Time-dependent
Sea Surface
Velocity as
Forcing

Method 4.
Fully Coupled
Method



Example 3:

Source Width : $\sigma_r = 1.25$ km

Source Duration : $\sigma_t = 1.25$ s

Within shallow water limit?

No: $\frac{H}{\sigma_r} = 3.20 > 1$

Tsunami propagates over source duration?

No: $\frac{\sigma_r}{\sigma_t \sqrt{gH}} = 5.05 > 1$

Acoustic waves significant?

Yes: $\frac{c\sigma_t}{H} = 0.47 < 1$

Method 1.
Static Initial
condition

Method 2.
Time-dependent
Seafloor Velocity
as Forcing

Method 3.
Time-dependent
Sea Surface
Velocity as
Forcing

Method 4.
Fully Coupled
Method

Method 1 and 2 do not model acoustic wave generation, we anticipate the results will differ compared to methods 3 and 4

*Note, in this study Method 1, 2, and 3 all use a linear shallow water solver not accounting for dispersion affects



Source Width : $\sigma_r = 1.25$ km
 Source Duration : $\sigma_t = 1.25$ s
 No: Within shallow water limit?
 No: Tsunami propagates over source duration?
 Yes: Acoustic waves significant?

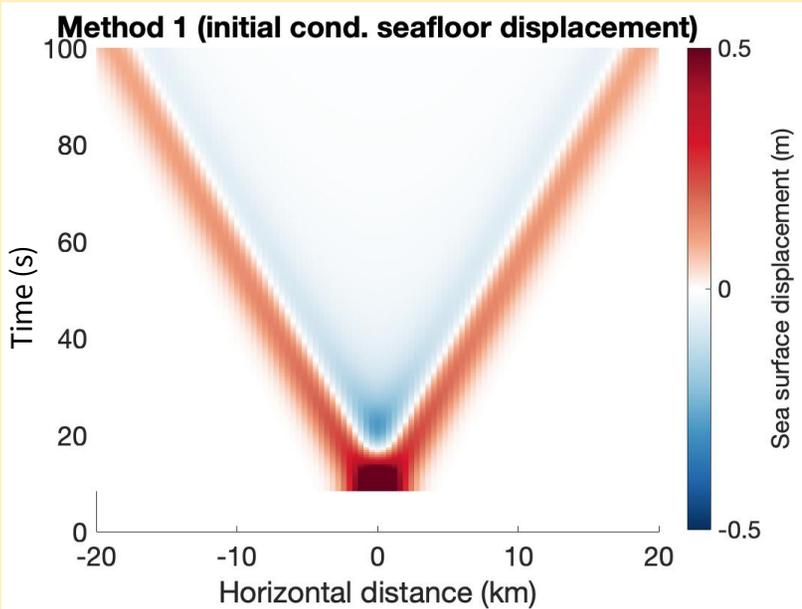
Method 1.
Static Initial
condition

Method 2.
Time-dependent
Seafloor Velocity
as Forcing

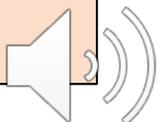
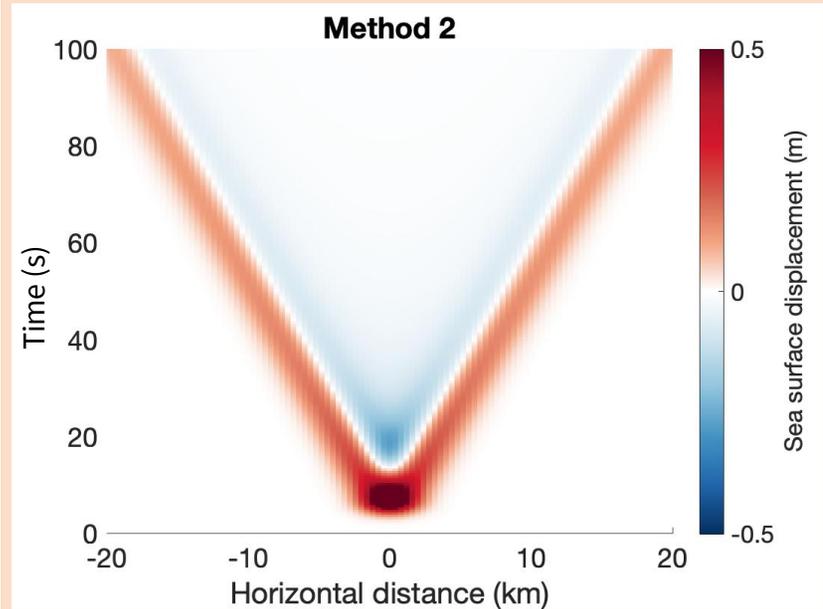
Method 3.
Time-dependent
Sea Surface
Velocity as
Forcing

Method 4.
Fully Coupled
Method

Seafloor displacement as initial conditions



Seafloor velocity as forcing



Source Width : $\sigma_r = 1.25$ km
Source Duration : $\sigma_t = 1.25$ s
No: Within shallow water limit?
No: Tsunami propagates over source duration?
Yes: Acoustic waves significant?

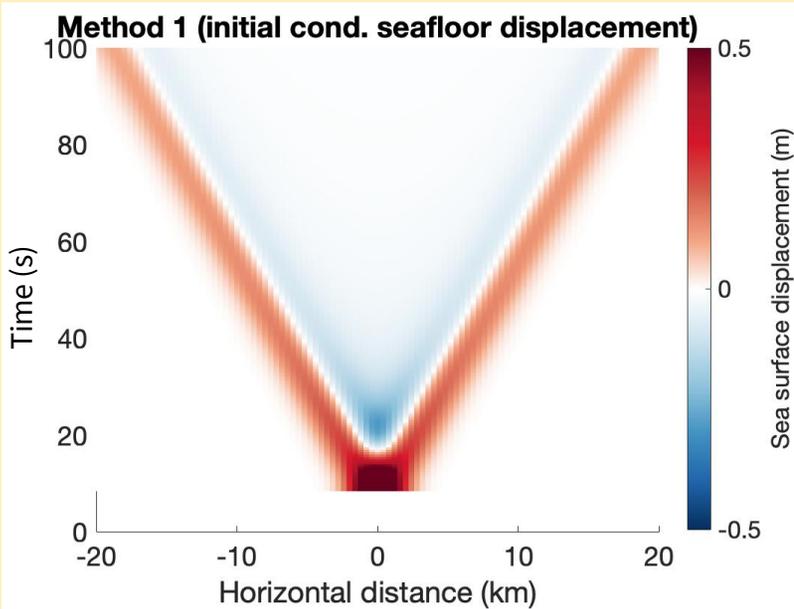
Method 1.
Static Initial
condition

Method 2.
Time-dependent
Seafloor Velocity
as Forcing

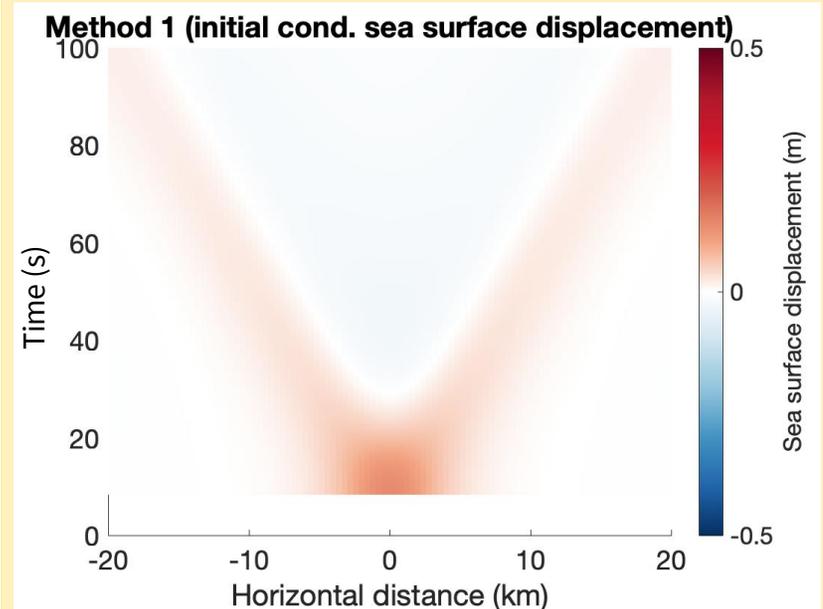
Method 3.
Time-dependent
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Velocity as
Forcing

Method 4.
Fully Coupled
Method

With short wavelengths



Without short wavelengths



Source Width : $\sigma_r = 12.5$ km
 Source Duration : $\sigma_t = 1.25$ s
 Yes: Within shallow water limit?
 No: Tsunami propagates over source duration?
 No: Acoustic waves significant?

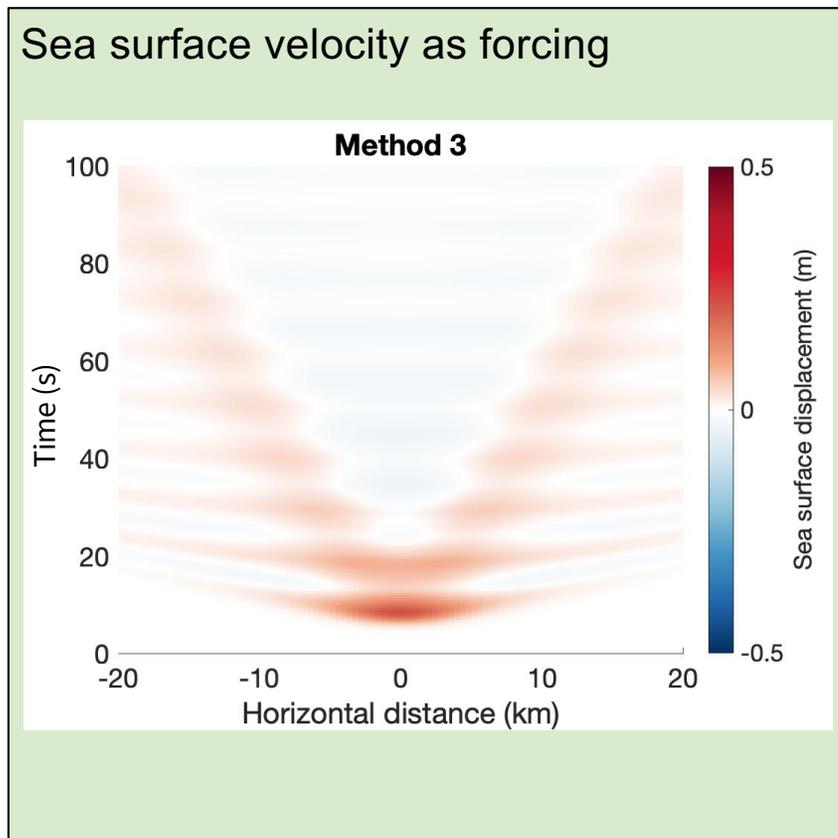
Method 1.
Static Initial
condition

Method 2.
Time-dependent
Seafloor Velocity
as Forcing

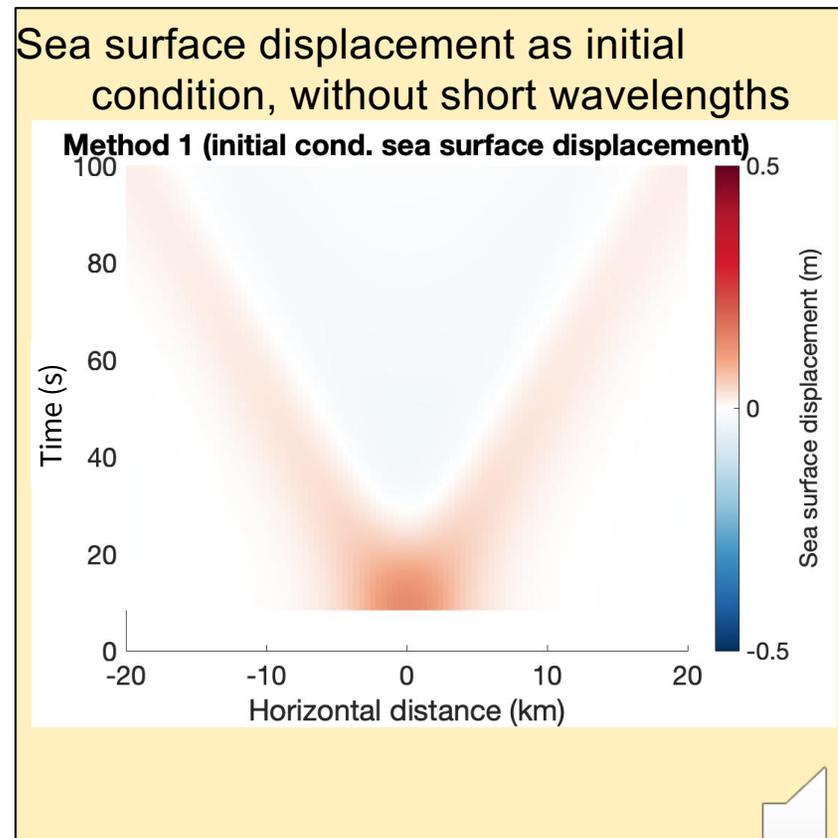
Method 3.
Time-dependent
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Velocity as
Forcing

Method 4.
Fully Coupled
Method

Sea surface velocity as forcing



Sea surface displacement as initial condition, without short wavelengths



Source Width : $\sigma_r = 12.5$ km
Source Duration : $\sigma_t = 1.25$ s
Yes: Within shallow water limit?
No: Tsunami propagates over source duration?
No: Acoustic waves significant?

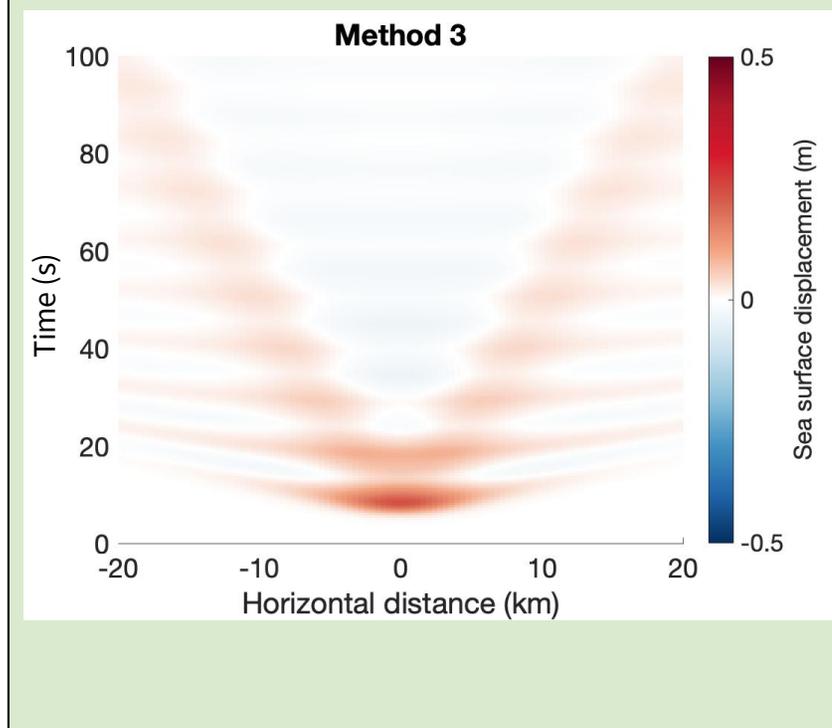
Method 1.
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Method 2.
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Seafloor Velocity
as Forcing

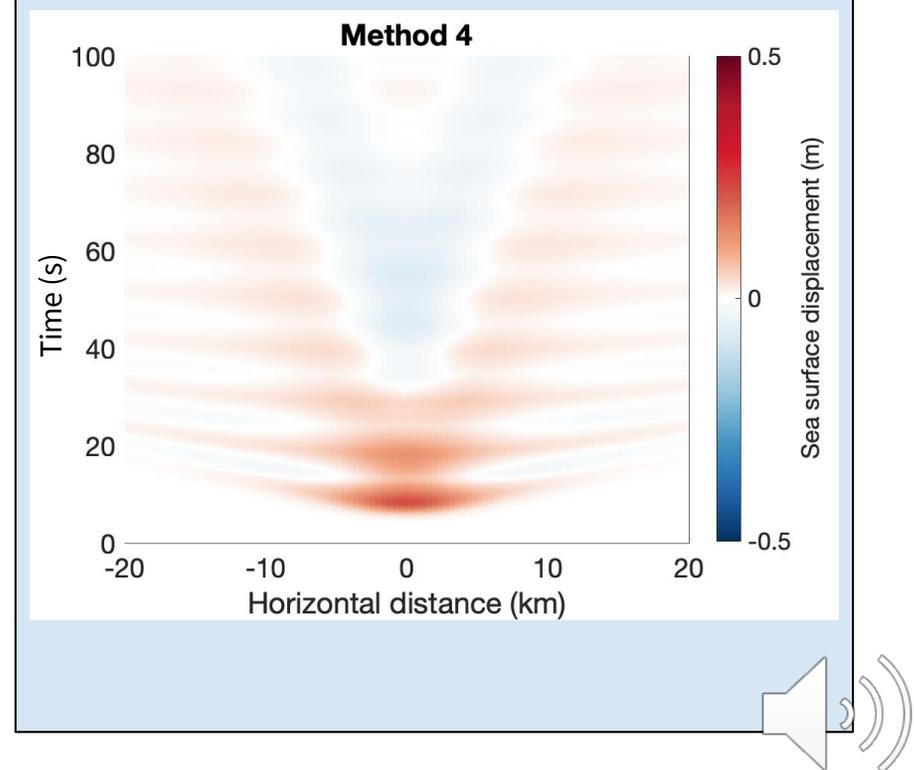
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Time-dependent
Sea Surface
Velocity as
Forcing

Method 4.
Fully Coupled
Method

Full wavefield (one-way coupling)



Full wavefield (fully coupled)



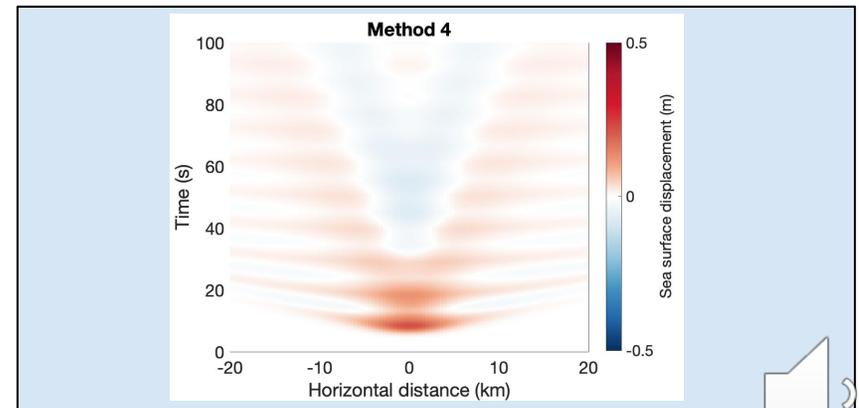
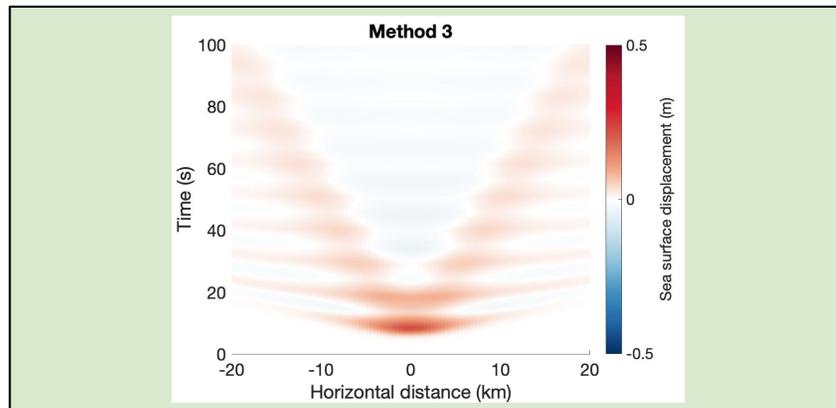
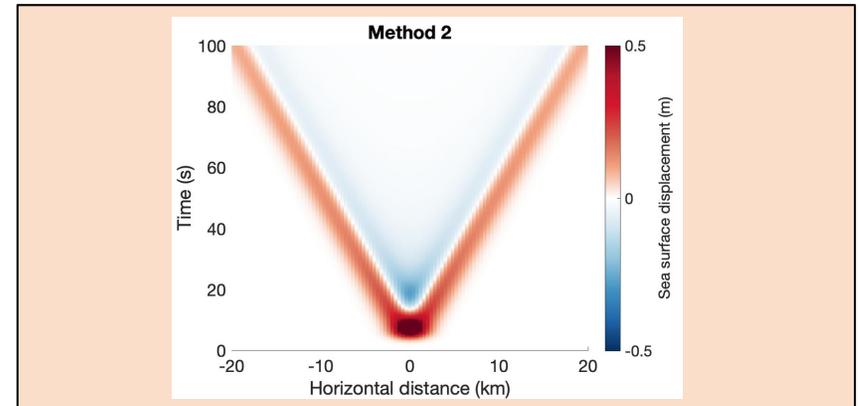
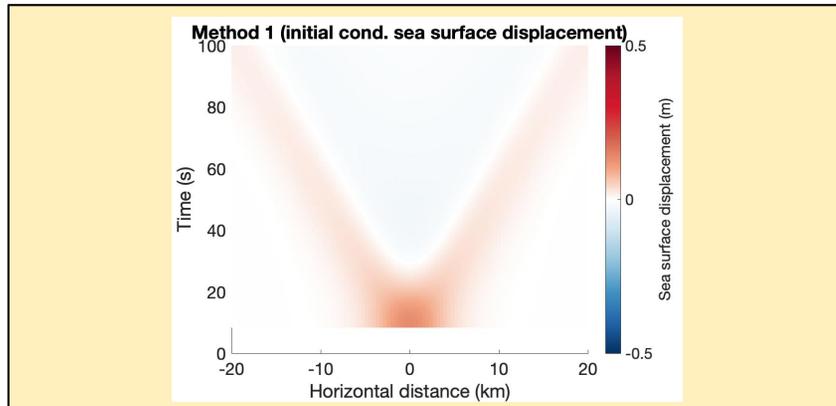
Source Width : $\sigma_r = 1.25$ km
 Source Duration : $\sigma_t = 1.25$ s
 No: Within shallow water limit?
 No: Tsunami propagates over source duration?
 Yes: Acoustic waves significant?

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Method



Static Initial Conditions

Final earthquake seafloor or sea surface uplift recorded

Set initial tsunami sea surface height

Requires:
Shallow water limit (if short wavelength are not filtered)

$$\frac{H}{\sigma_r} \ll 1$$

Tsunami waves do not propagate over source duration

$$\frac{\sigma_r}{\sigma_t \sqrt{gH}} \gg 1$$

And acoustic waves are not generated

$$\frac{c\sigma_t}{H} \gg 1$$

Time-dependent Seafloor Velocity as Forcing

Record earthquake seafloor velocity

Set time-dependent forcing in the tsunami mass balance

Requires:
Shallow water limit

$$\frac{H}{\sigma_r} \ll 1$$

And acoustic waves are not generated

$$\frac{c\sigma_t}{H} \gg 1$$

Time-dependent Sea Surface Velocity as Forcing

Solves earth and ocean response without gravity

Use sea surface velocity as a forcing term in a tsunami simulation

If non-dispersive shallow water solver, requires:
Shallow water limit

$$\frac{H}{\sigma_r} \ll 1$$

If Boussinesq tsunami solver:
We expect valid in more cases

Fully Coupled Method (SeisSol)

Simultaneously solves earthquake rupture, seismic waves, and ocean response (including gravity)

<http://www.seissol.org/>

Valid in all cases

In Summary

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In Summary

Citation

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