The Non-traditional Coriolis force drives Westward tilts in Multiscale Theories and Laboratory Experiments of Tropical Dynamics.

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Abstract

Twenty years ago, Andy Majda and Rupert Klein developed a multiscale asymptotic theory of tropical dynamics which they named IPESD. IPESD is a linear, forced, dynamical theory on the tropical synoptic scales coupled to a linear, slower, weak temperature gradient theory on the tropical planetary scales. The two scales interact with one another through upscale fluxes of momentum and temperature (the Reynolds stresses) and downscale advection from the planetary scale (developed by B. and Majda as the IMMD theory in 2010). Majda and myself used these theories in the following years to develop a multiscale kinematic model of the Madden Julian oscillation. Criticisms of this theory have missed the essential fact that the mathematics of the asymptotic analysis are inexorable, and verified by observation - the forcing is stronger on small scales and weaker on large scales; that's all that is really needed to derive IMMD. In our MJO models, we showed that, in order to capture the westerly wind burst structure of the MJO, it was sufficient to force the planetary scales with momentum fluxes (Reynolds stresses) from westward tilted convective structures on the synoptic scales. Left unanswered was the cause of the westward tilted convection, though westward tilts had been observed repeatedly in MJO observing campaigns. The non-traditional terms in the Coriolis force (NCT) seem to provide an excellent candidate for westward tilts, but it is well known that these terms are too weak on scales greater than the mesoscale to affect the dynamics. However, the multiscale theories provide a distinct route for the NCT to affect the planetary scales - through upscale fluxes of momentum. We will show analytically that the net-NCT affects tropical convection by generating zero force in the vertical direction, a westward velocity field in regions of upward flow, and a recirculation in regions of downward flow, around a convective structure. The upscale fluxes from these circulations drive the vertical/westward tilt that was necessary to generate the westerly wind burst in the Majda/Biello models of the MJO. We will also show how these results, westward tilt and the westerly wind burst, can be created in simple laboratory experiments. We sincerely hope that Andy would have been gratified by these results, and excited by our experiment.

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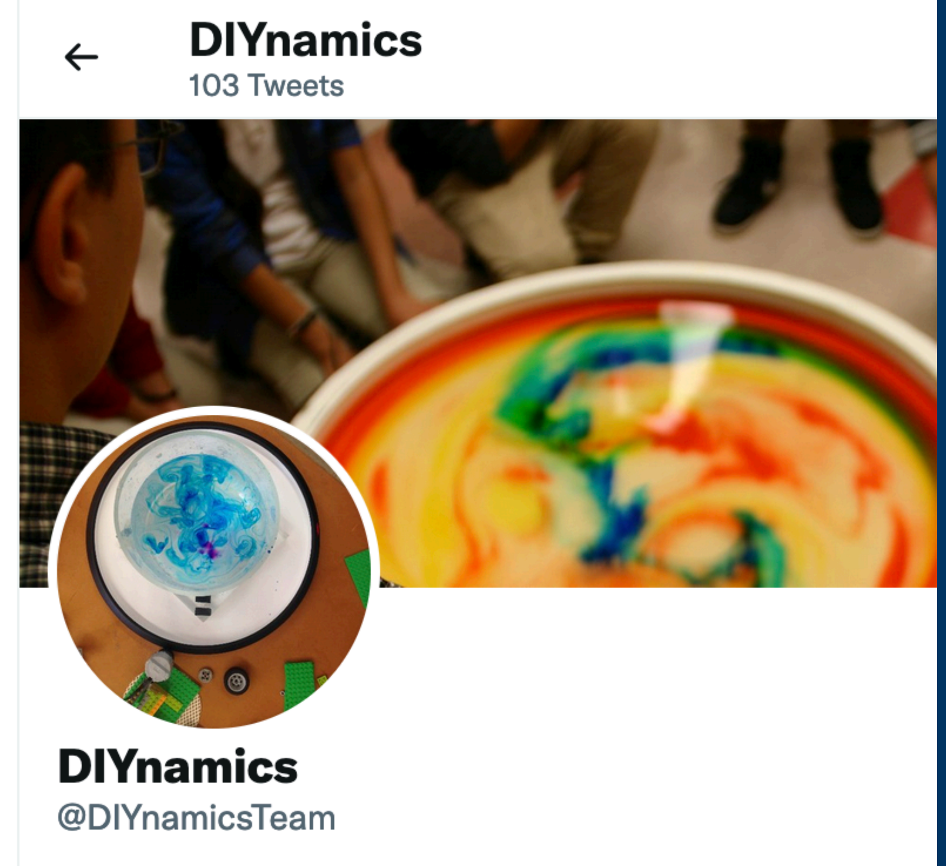
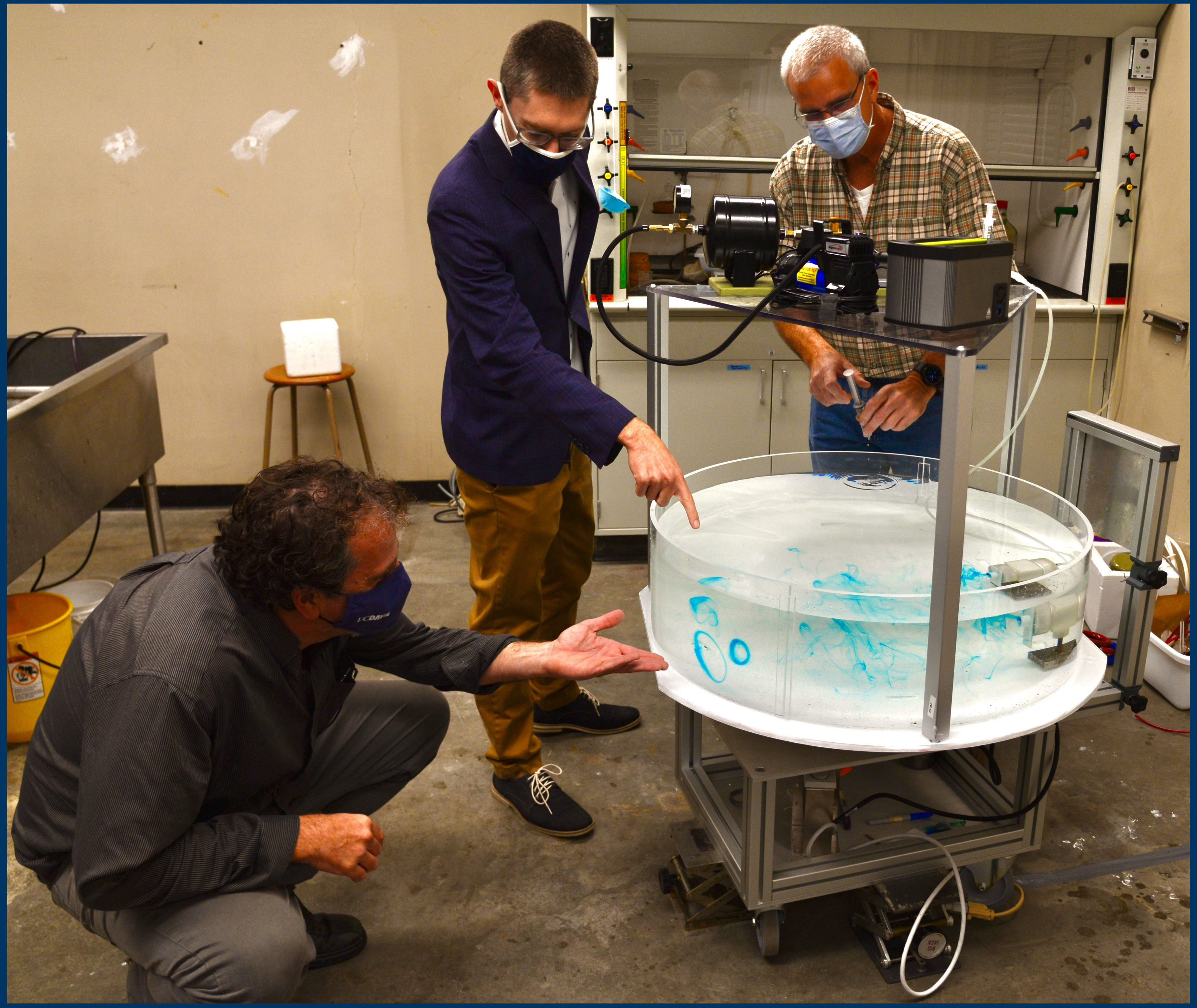
AGGIES





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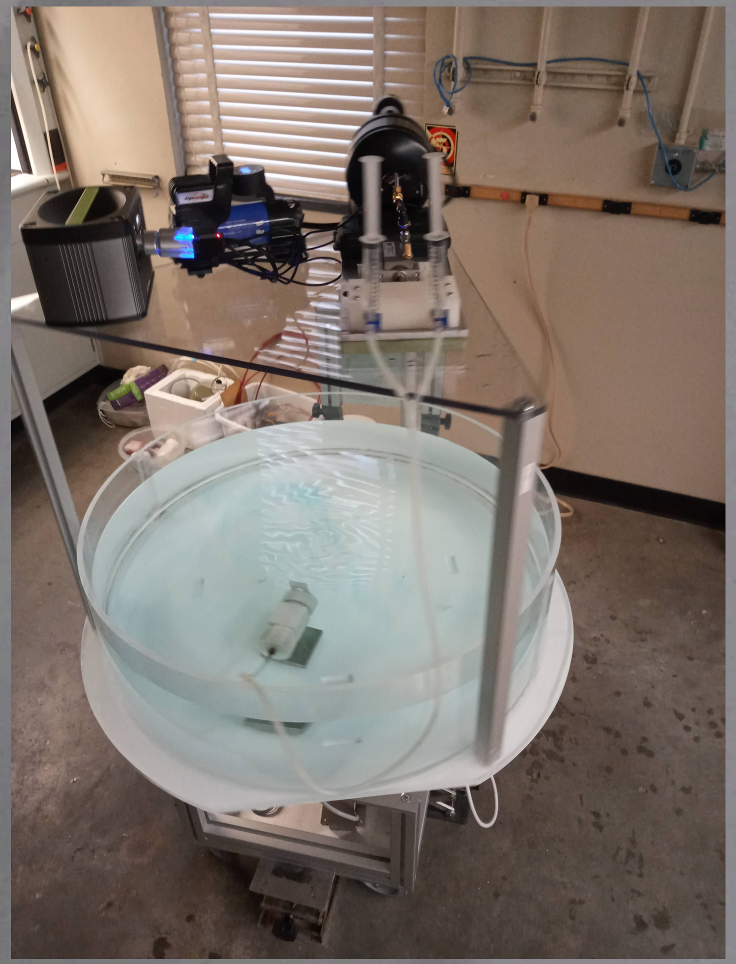


- Counterclockwise rotating tank
- 36 second period
- Seen from above
- 12 cm depth of water
- Horizontal injection of toroidal vortices
- Water + Methyl blue
- 4 pairs of vortex rings

IPESD + MJO

The Multiscale Models of Tropical Dynamics

- J.A.B. & Majda (early 2000s) on the MJO
- Majda & Klein (2003)

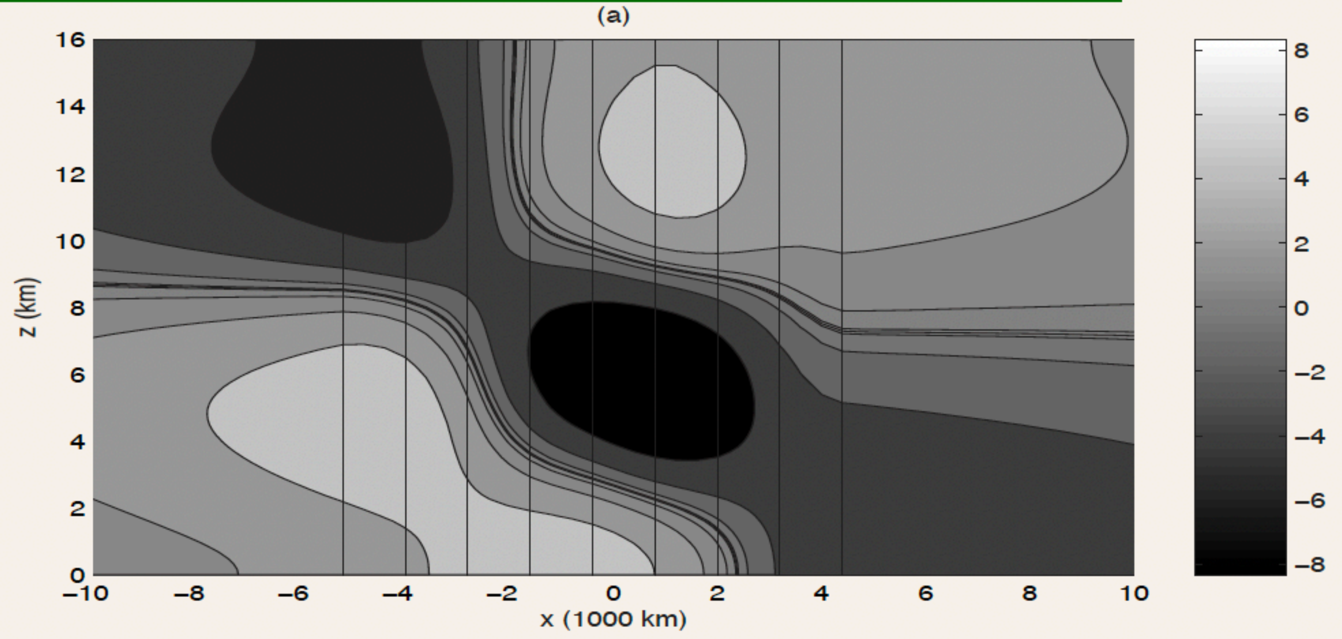
- The multi scale models are a multiscale system of PDEs wherein diabatic heating on the synoptic scale drives the large scale circulation

$$
\begin{aligned}
\overline{U}_t - y\overline{V} + \overline{P}_X &= F^U \quad - d_0\,\overline{U} \\
y\overline{U} + \overline{P}_y &= 0 \\
\overline{\Theta}_t + \overline{W} &= F^\theta \quad - d_\theta\,\overline{\Theta} + \overline{S}_\theta \\
\overline{P}_z &= \overline{\Theta} \\
\overline{U}_X + \overline{V}_y + \overline{W}_z &= 0
\end{aligned}
$$

$$
\begin{aligned}
F^U &= -\overline{(v'\,u')}_y - \overline{(w'\,u')}_z \\
F^\theta &= -\overline{(v'\,\theta')}_y - \overline{(w'\,\theta')}_z
\end{aligned}
$$

Equatorial MJO model: Winds above the equator

- Lower troposphere congestus heating in the east
- Westward tilted anvil superclusters in the west



- Vertical/Westward tilted convection (w'u') is essential, but unexplained

Dynamics of Non-rotating Updraft Tori

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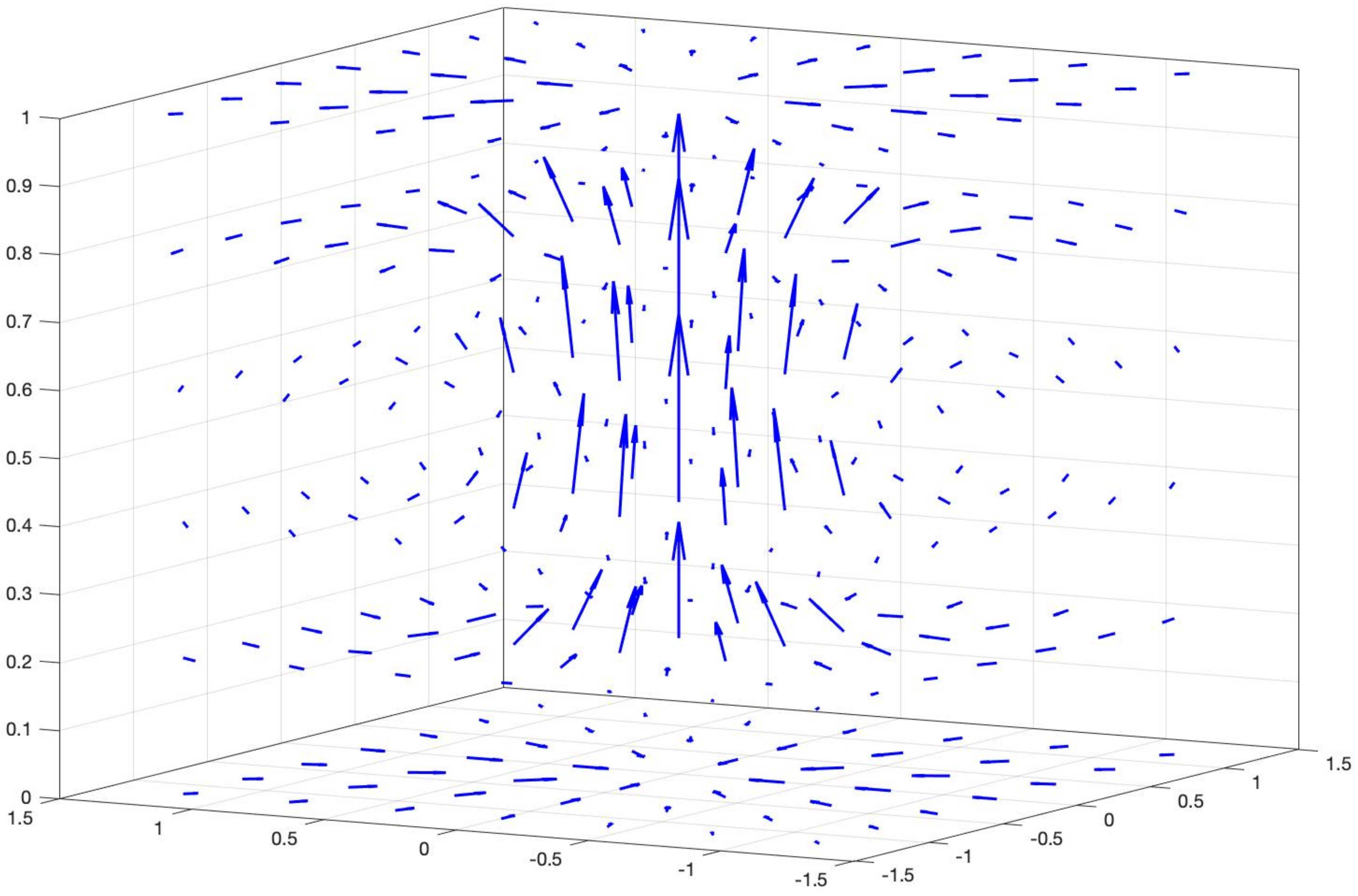
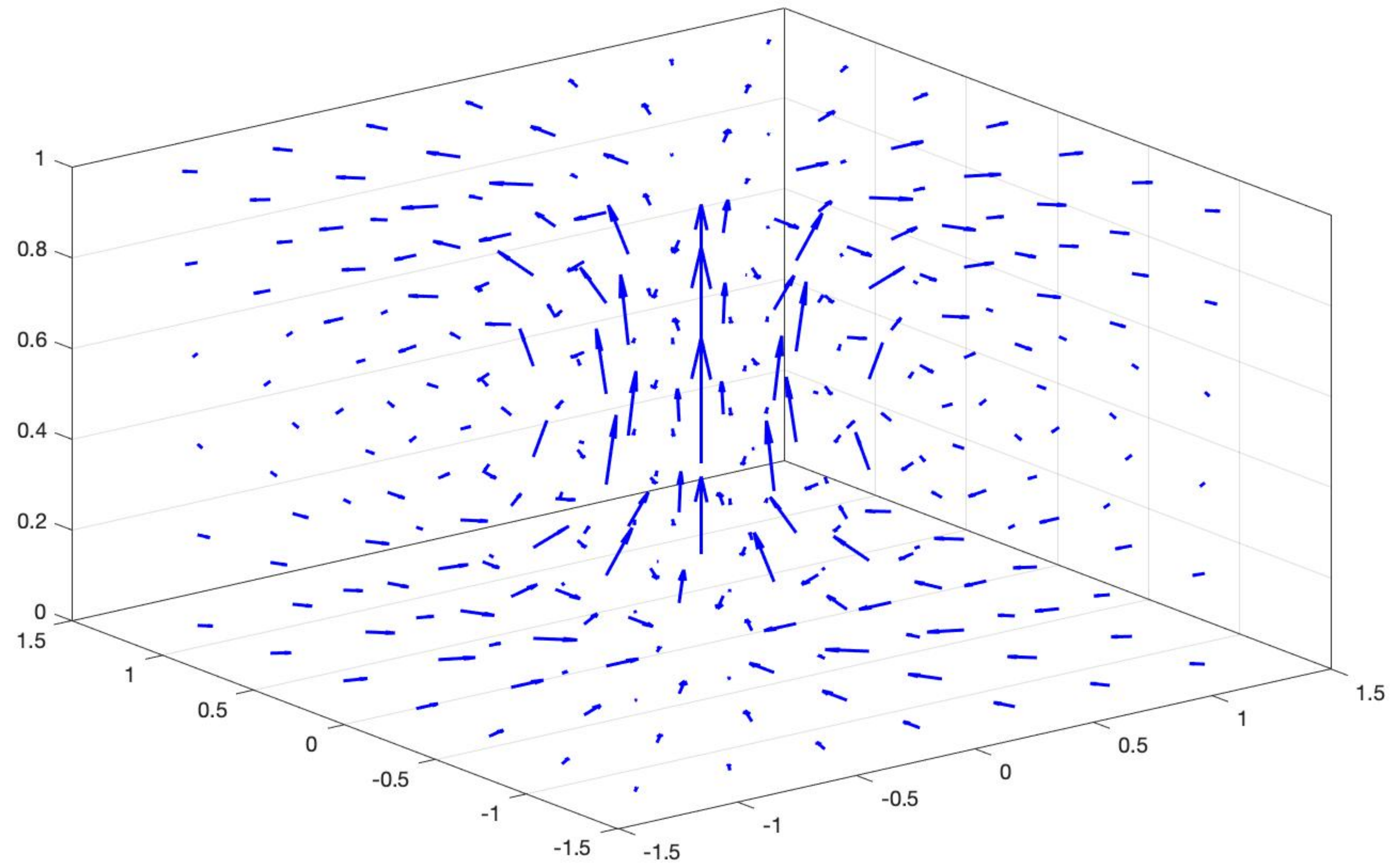
- w/ Igel, we are developing dynamical convective models consisting of poloidal circulations
- poloidal circulations are tori





Some Other DoNUT Talks/Posters

- A25D-1709: The Divergence-free Net Coriolis Force creates Vertical Geostrophic Balance and Westward tilted Atmospheric Convection
- NG33A-06: The Non-Traditional Coriolis Terms and Their Impact on the Convective Weak Temperature Gradient

* A22A-08: Modeling Tropical Convective Clouds and Circulations with the DoNUT

- Focus in these talks is in using the DoNUT (or a conceptually simplified version) to understand the impact of physics on convective circulations.

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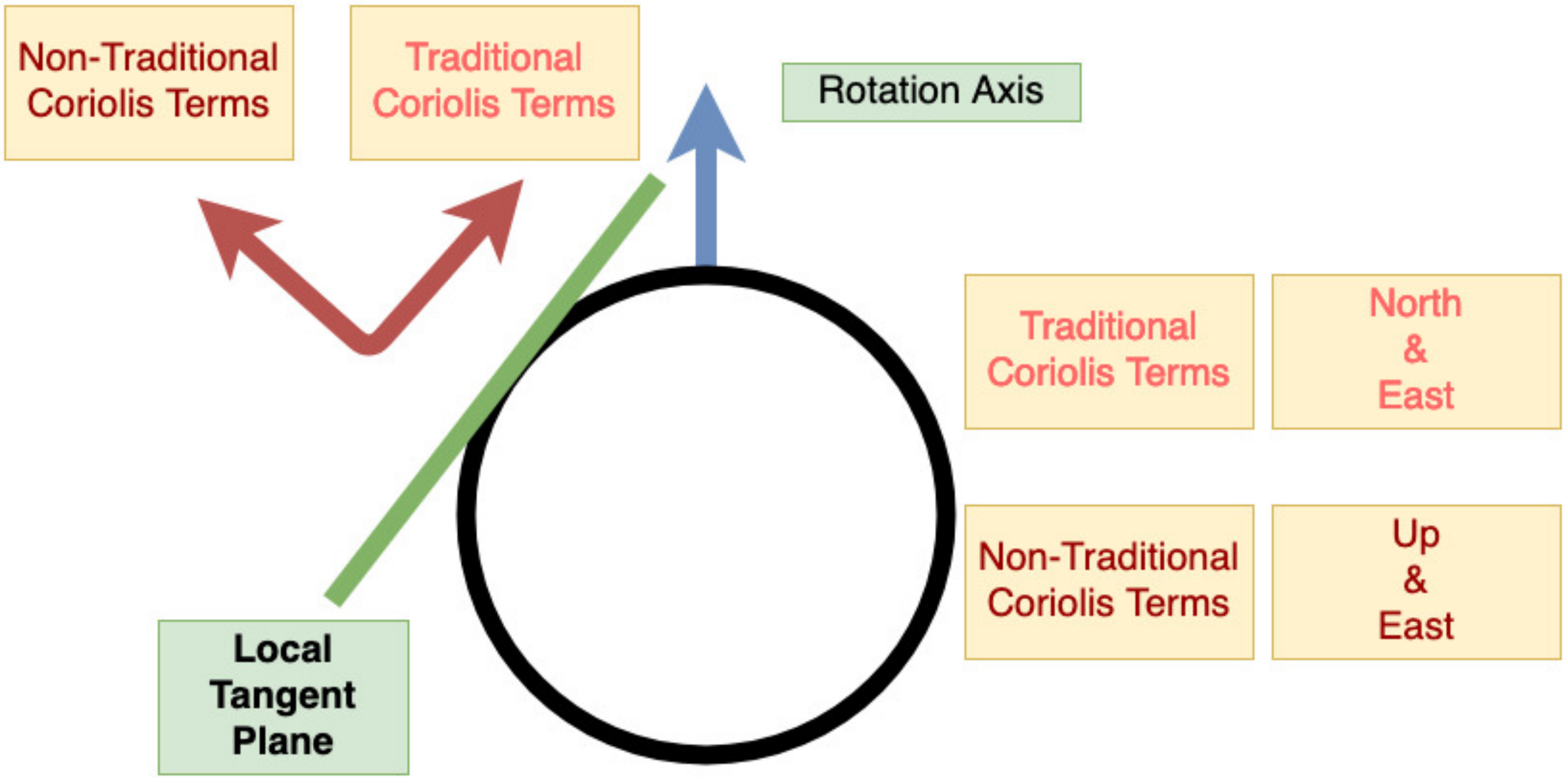
The Coriolis Force

The traditional simplification and the non-traditional terms



The Coriolis Force

The traditional simplification and the non-traditional terms

- The non-traditional terms are underlined
- The traditional terms vanish at the equator
- The non-traditional terms are negligible (and rightfully neglected) for flows which are horizontally much larger than the height of the troposphere
- The non-traditional terms, NCT, are significant for convective scale flows at the equator.
- Igel & B., JAS, 77, 2020

$$\frac{Du}{Dt} + \frac{\partial p}{\partial x} = 2\Omega_0 \sin(\phi)v - \underline{2\Omega_0 \cos(\phi)w}\,,$$

$$\frac{Dv}{Dt} + \frac{\partial p}{\partial y} = -2\Omega_0 \sin(\phi)u\,,$$

$$\frac{Dw}{Dt} + \frac{\partial p}{\partial z} = \underline{2\Omega_0 \cos(\phi)u} + B\,.$$

The Net Coriolis Force

Using the Leray projection to determine the pressure

- The net Coriolis force is the divergence-free force resulting from the pressure gradient in the Leray projection

$$\vec{F} = -\nabla p - 2\vec{\Omega} \times \vec{u}, \qquad \nabla \cdot \vec{F} = 0 \qquad \vec{F} \cdot \hat{n} = 0 \quad \text{on rigid boundaries,}$$

- Using the Leray projection we can compute the pressure and net force

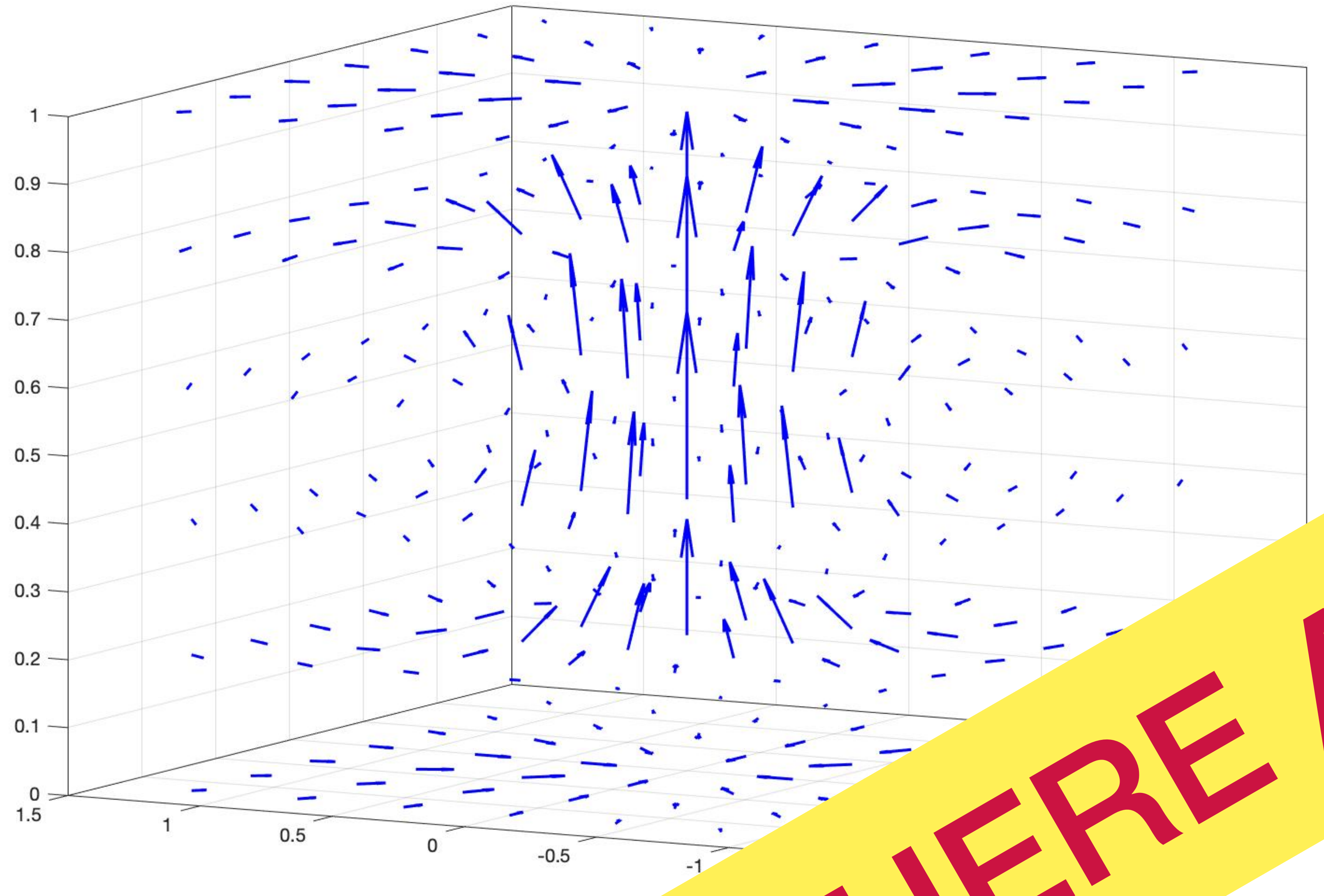
$$\nabla^2 p = 2\vec{\Omega} \cdot \vec{\omega},$$

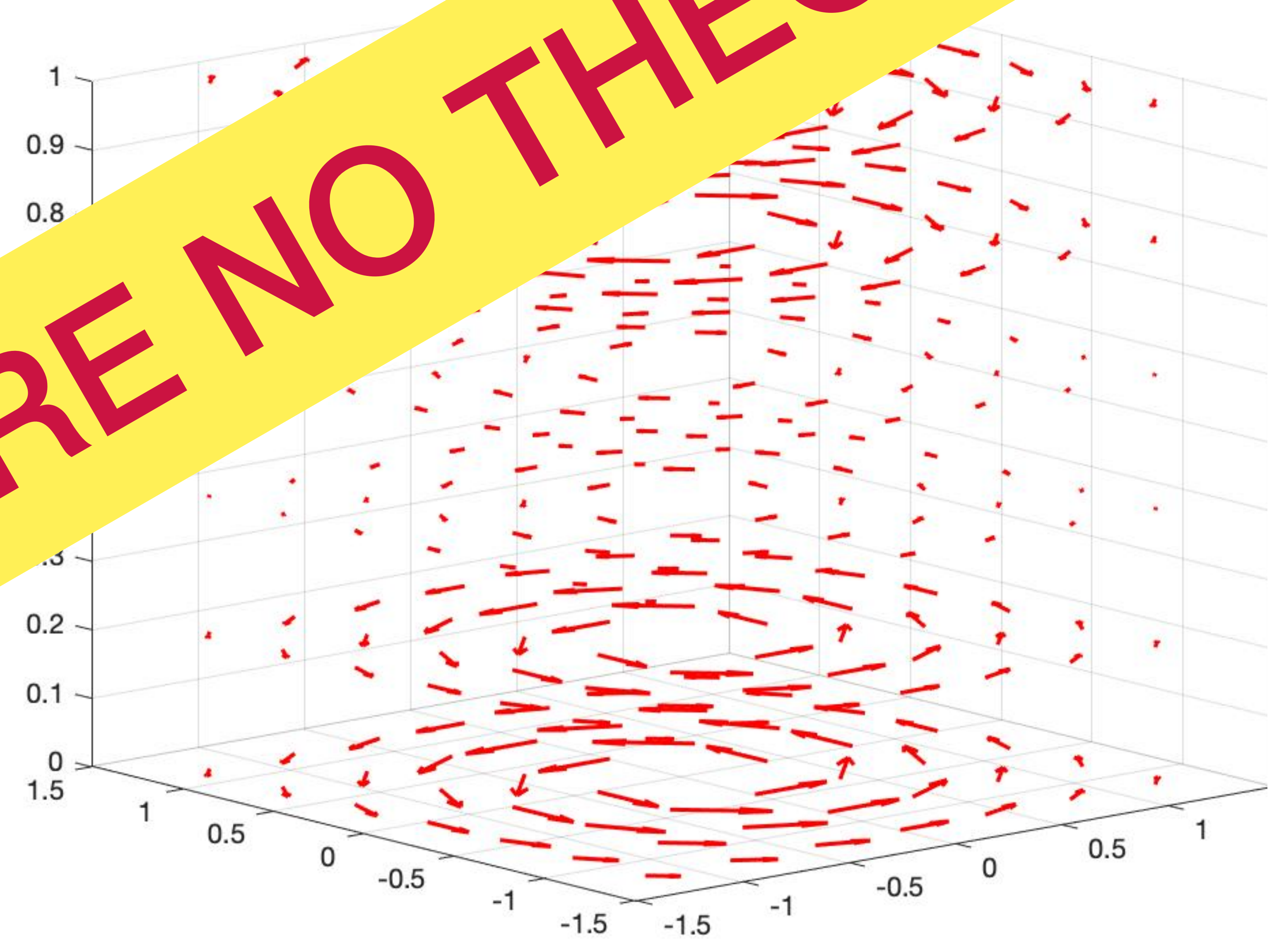
$$\hat{k} \cdot \nabla p = 2\left(\vec{u} \times \vec{\Omega}\right) \cdot \hat{k}, \quad \text{on} \quad z = 0.$$

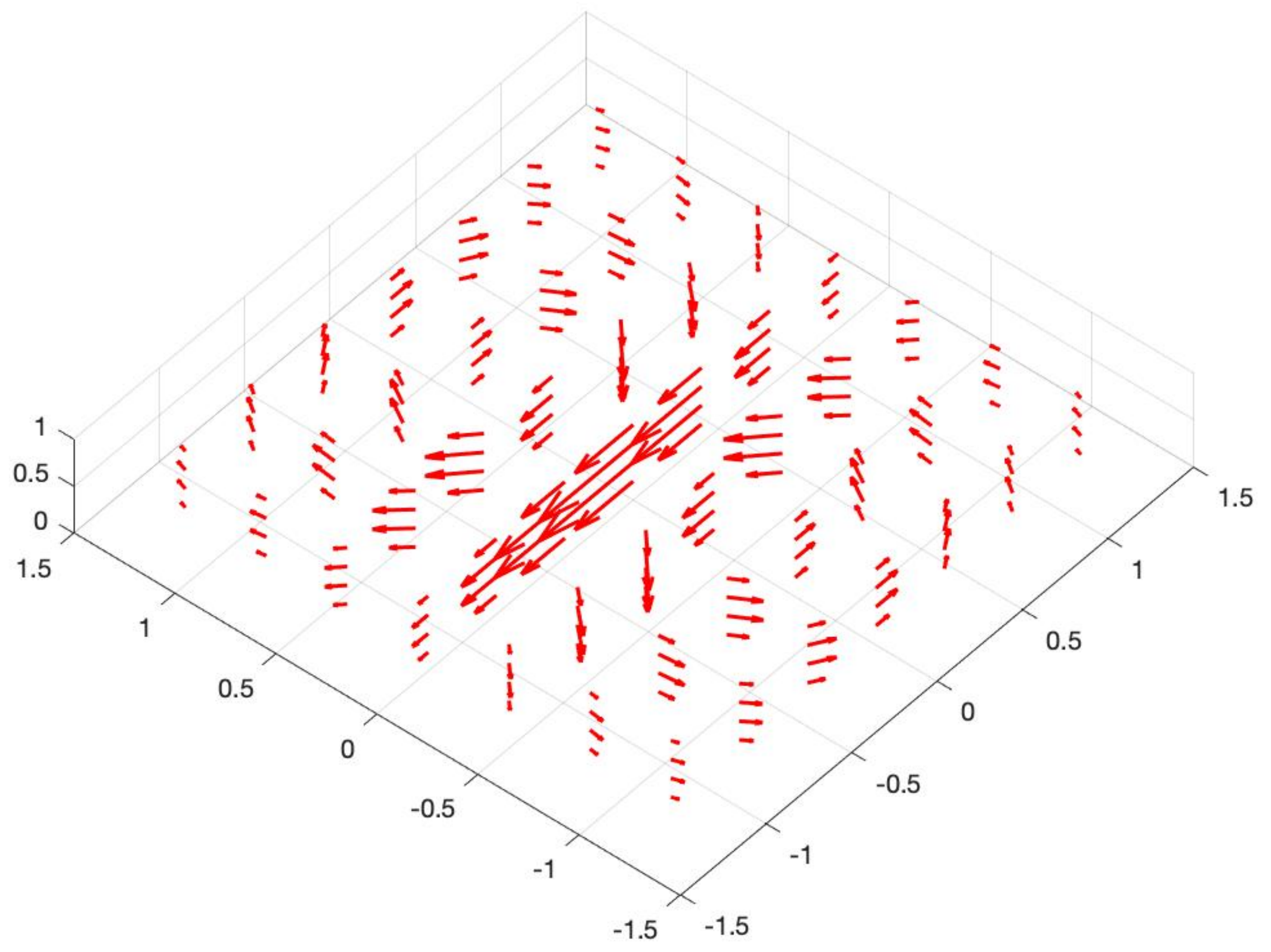
Theorem: The vertical component of the net Co… …orce is identically zero for Poloidal Fl…

Consequence: The pressure is computed a… …e vertical integral

THERE ARE NO THEOREMS AT AGU!!







Theorem: The vertical component of the net Coriolis Force is identically zero for Poloidal Flows

Consequence: The pressure is computed as a simple vertical integral

$$p = -2\Omega_0 \cos(\lambda) \int_z^\infty u(x, y, z')\, dz';$$

$$\Phi = \int_z^\infty \phi(x, y, z')\, dz'.$$

$$\vec{F}_{\mathrm{TCT}} = -2\Omega_0 \sin(\lambda)\, U\, \hat{\theta}$$

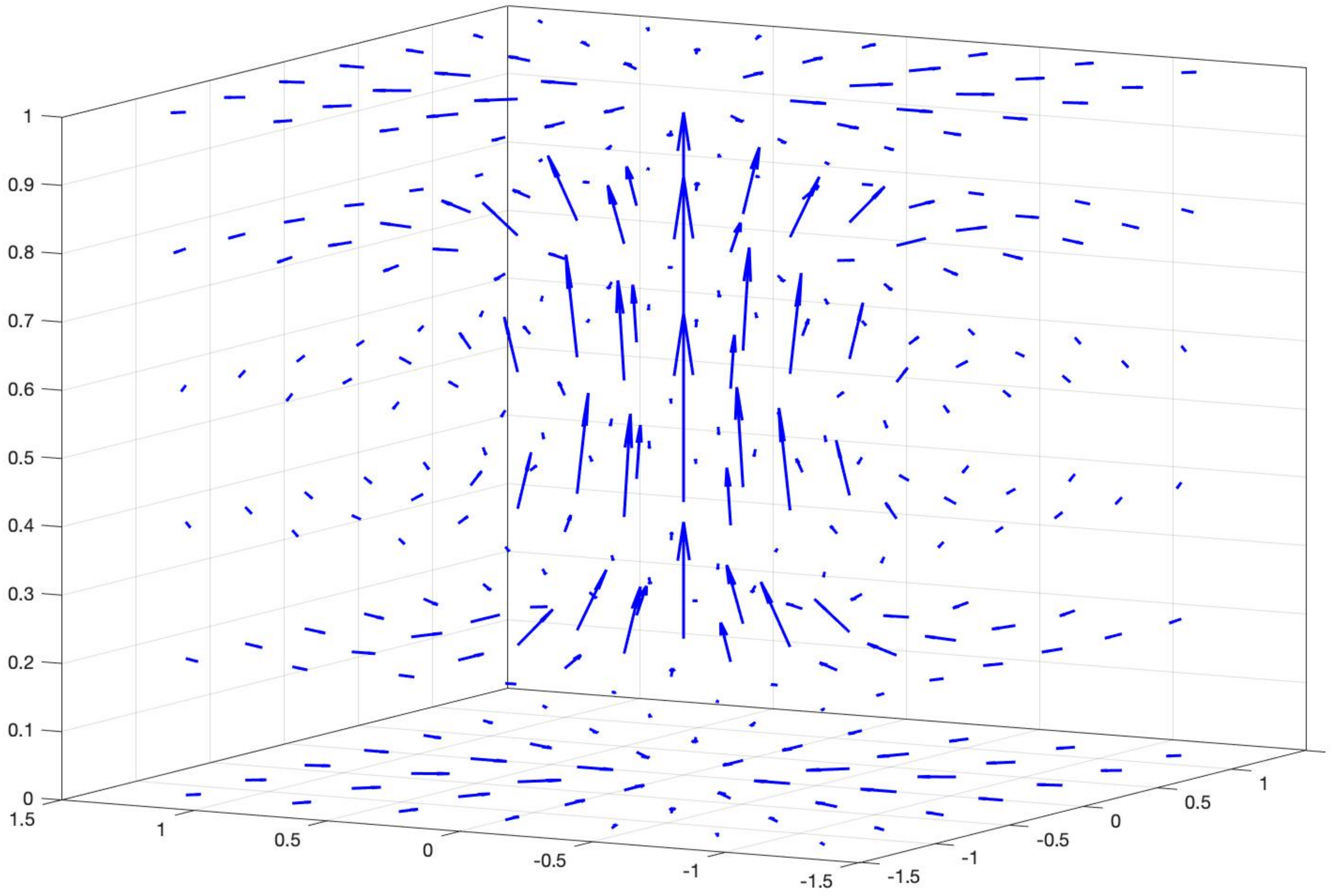
$$\vec{F}_{\mathrm{NCT}} = 2\Omega_0 \cos(\lambda) \left[\Phi_{yy}\, \hat{i} - \Phi_{xy}\, \hat{j}\right]$$

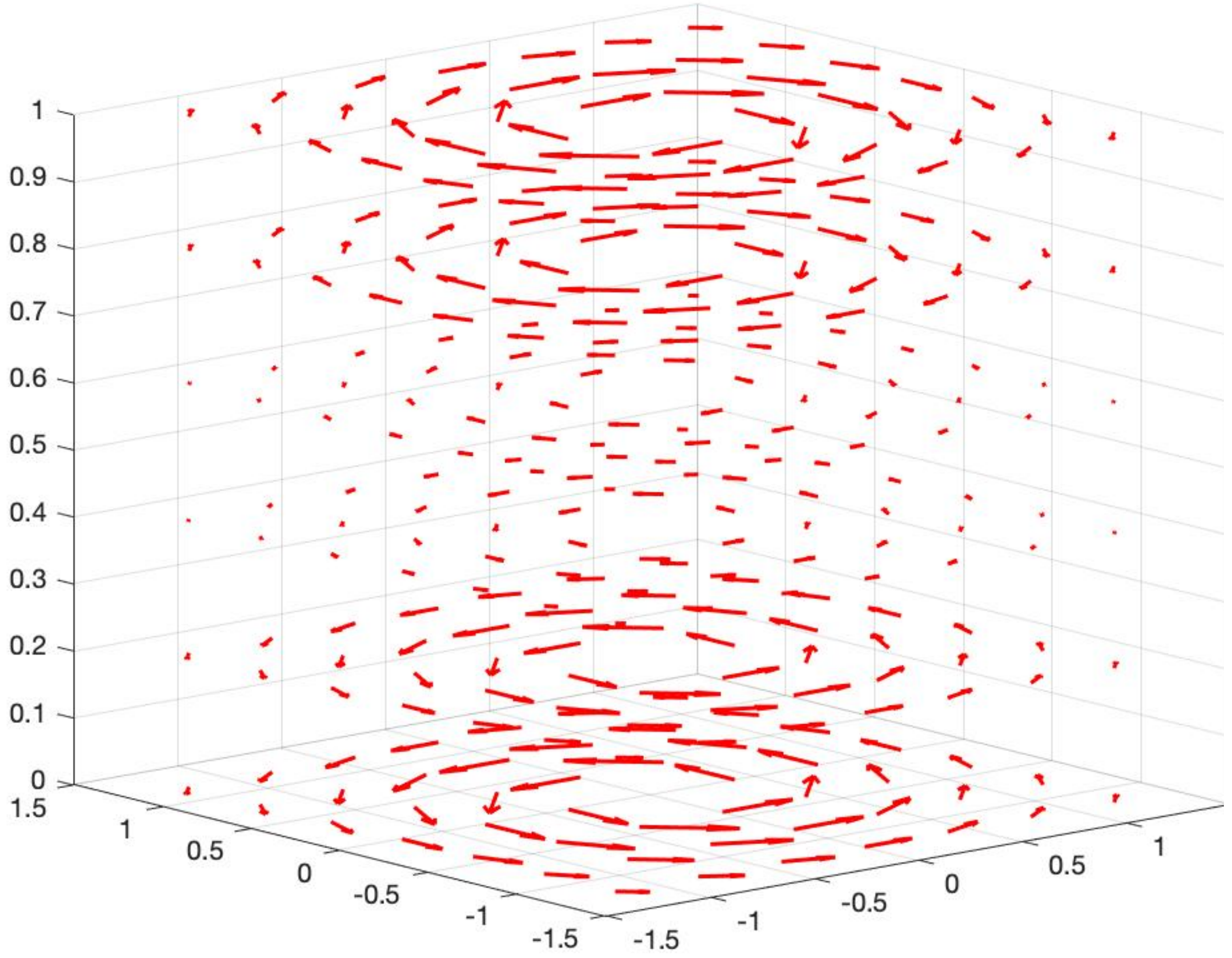
Tropical Cyclogenesis

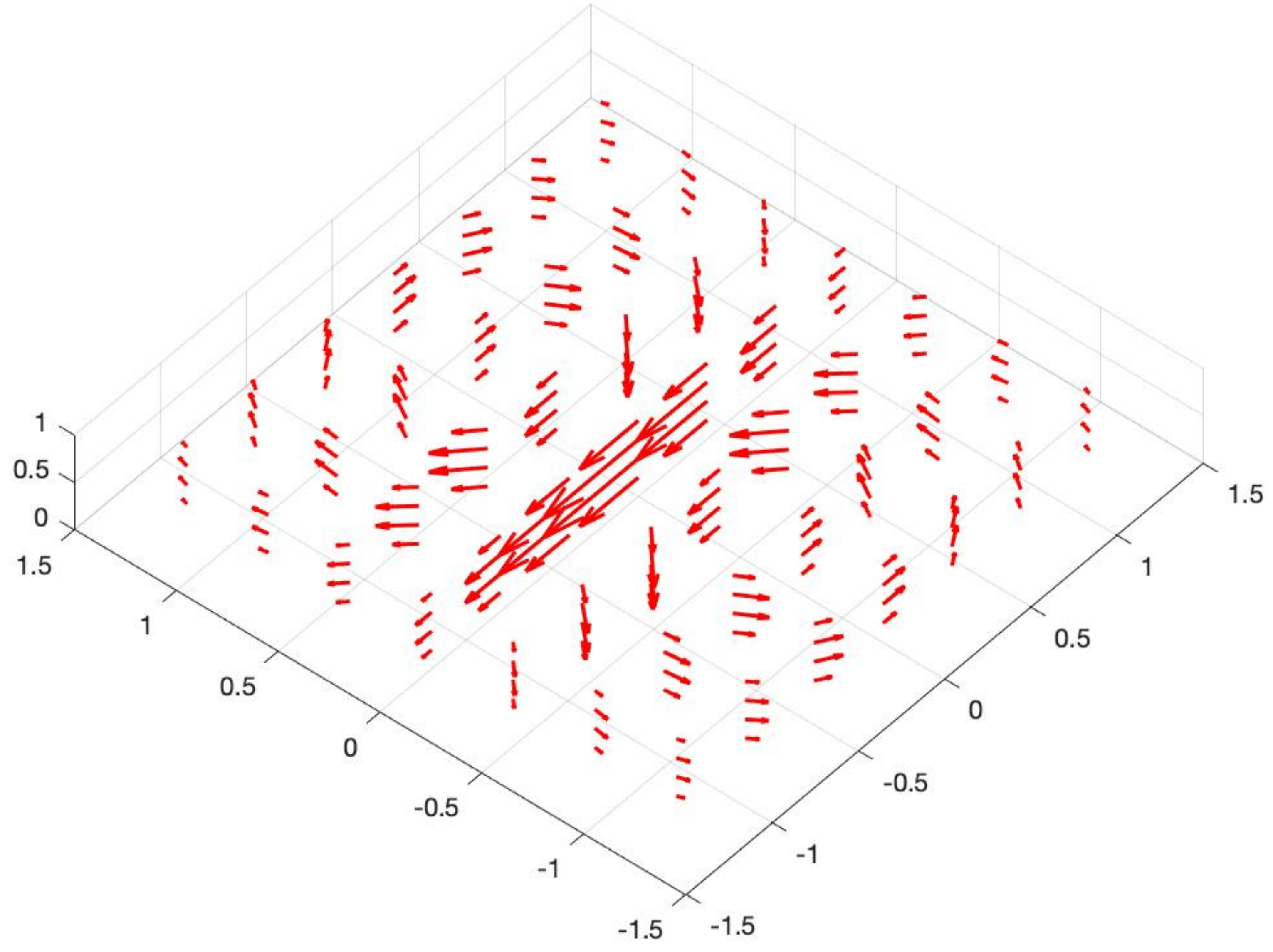
MJO-genesis?

Theorem: The vertical component of the net Coriolis Force is identically zero for Poloidal Flows

Consequence: The pressure is computed as a simple vertical integral

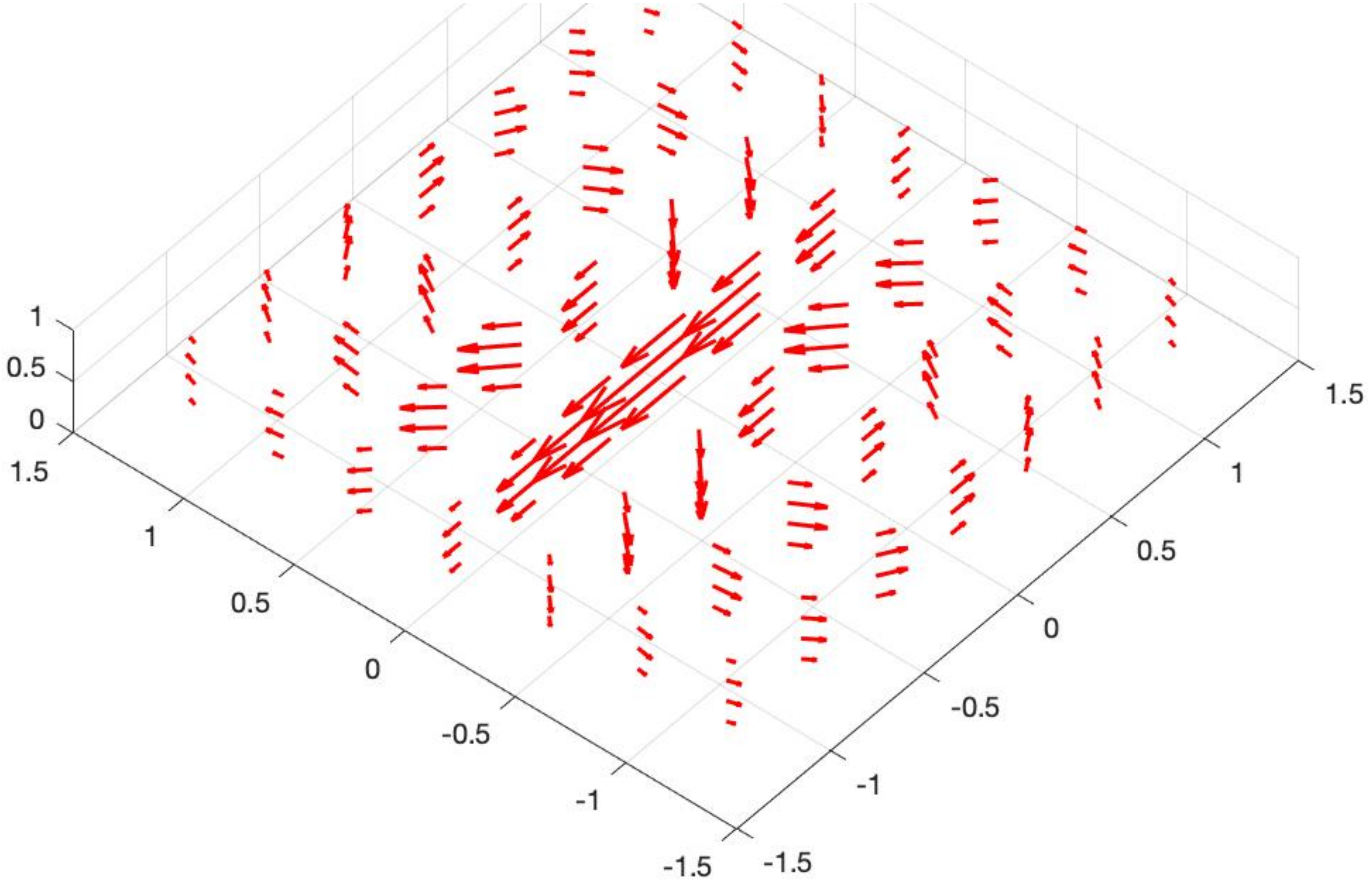


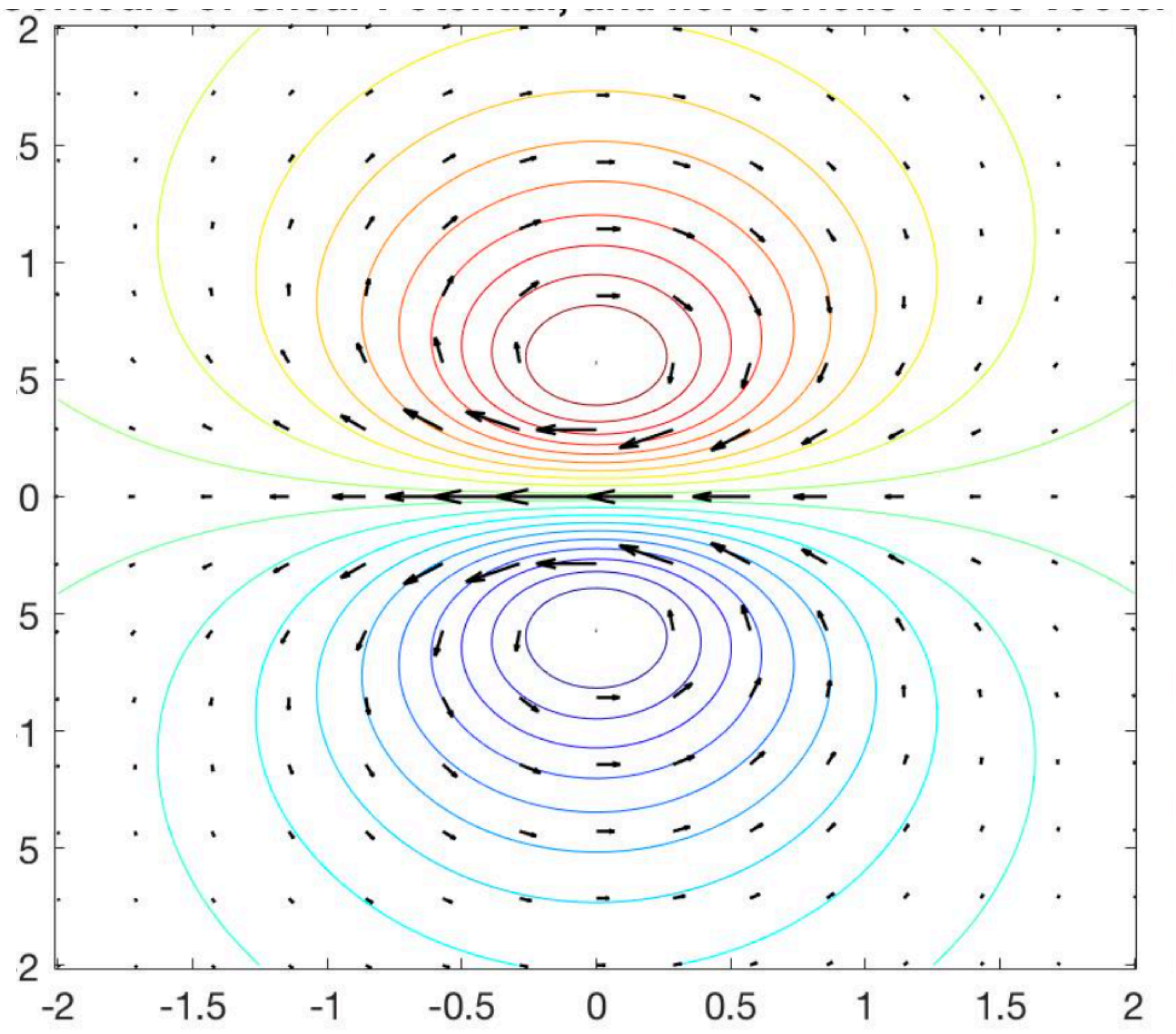




The Net NCT for Poloidal flows

Westward in the up flow, eastward in the down flow



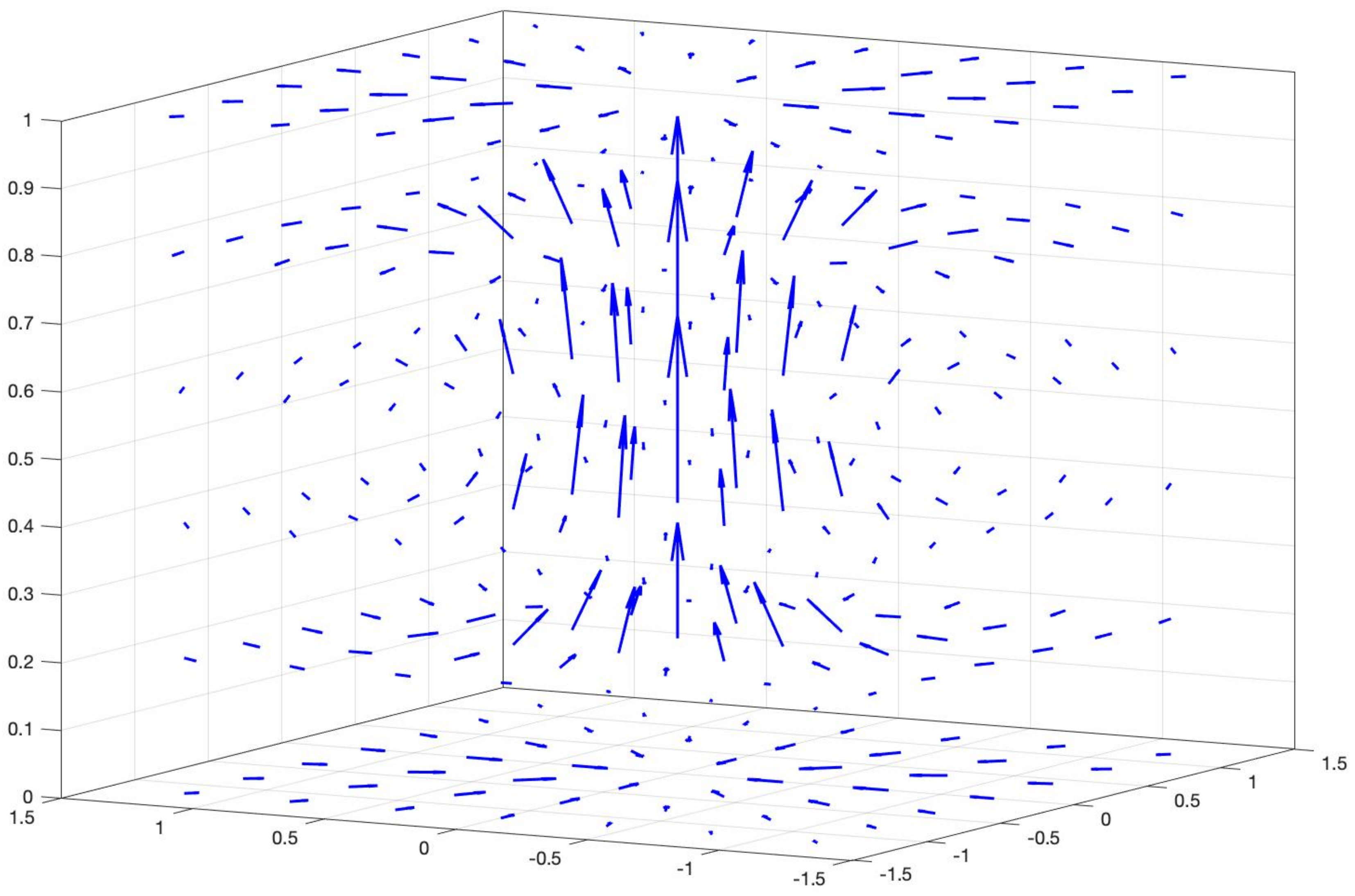


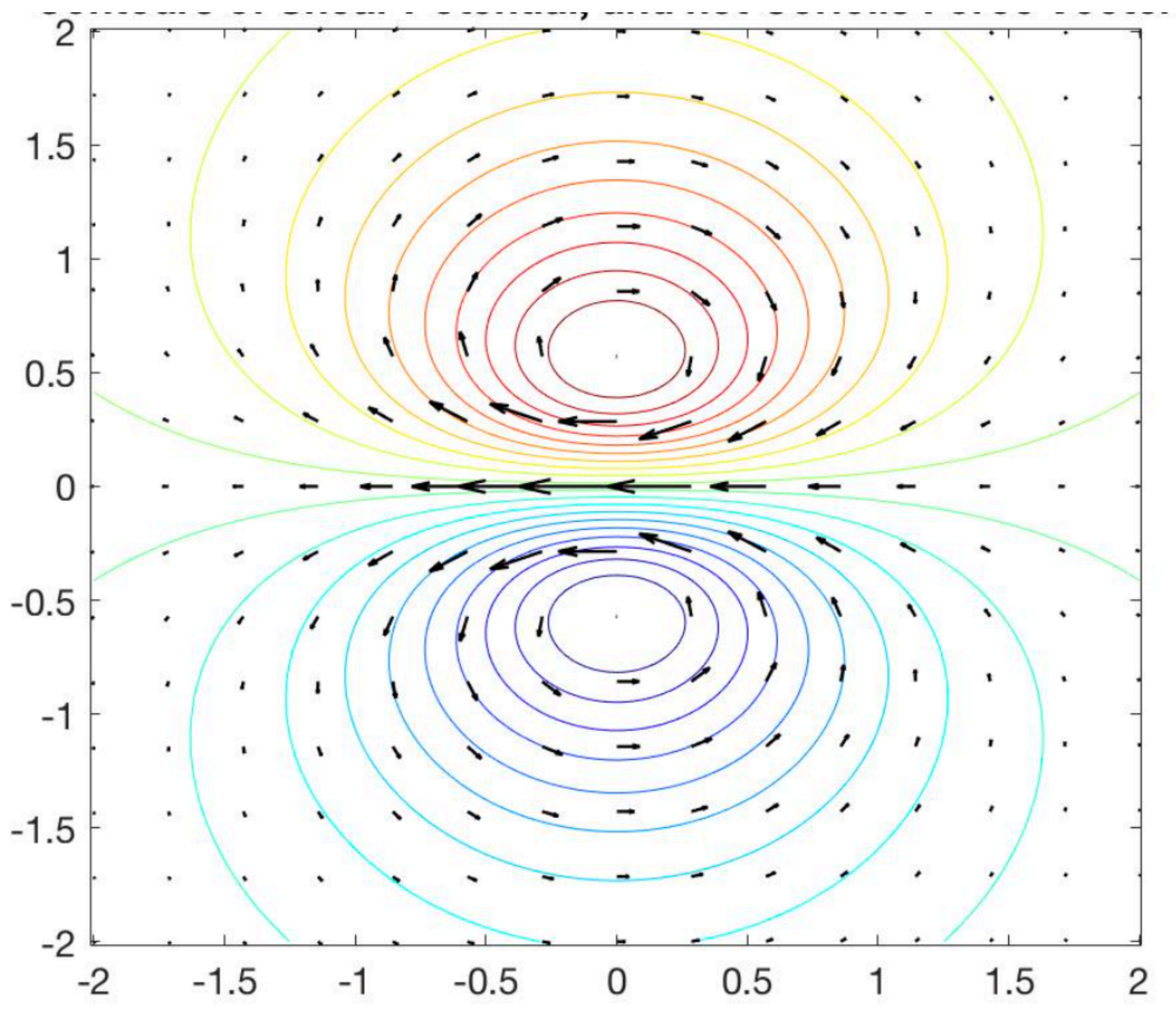
Vertical Upscale Flux of Zonal Momentum

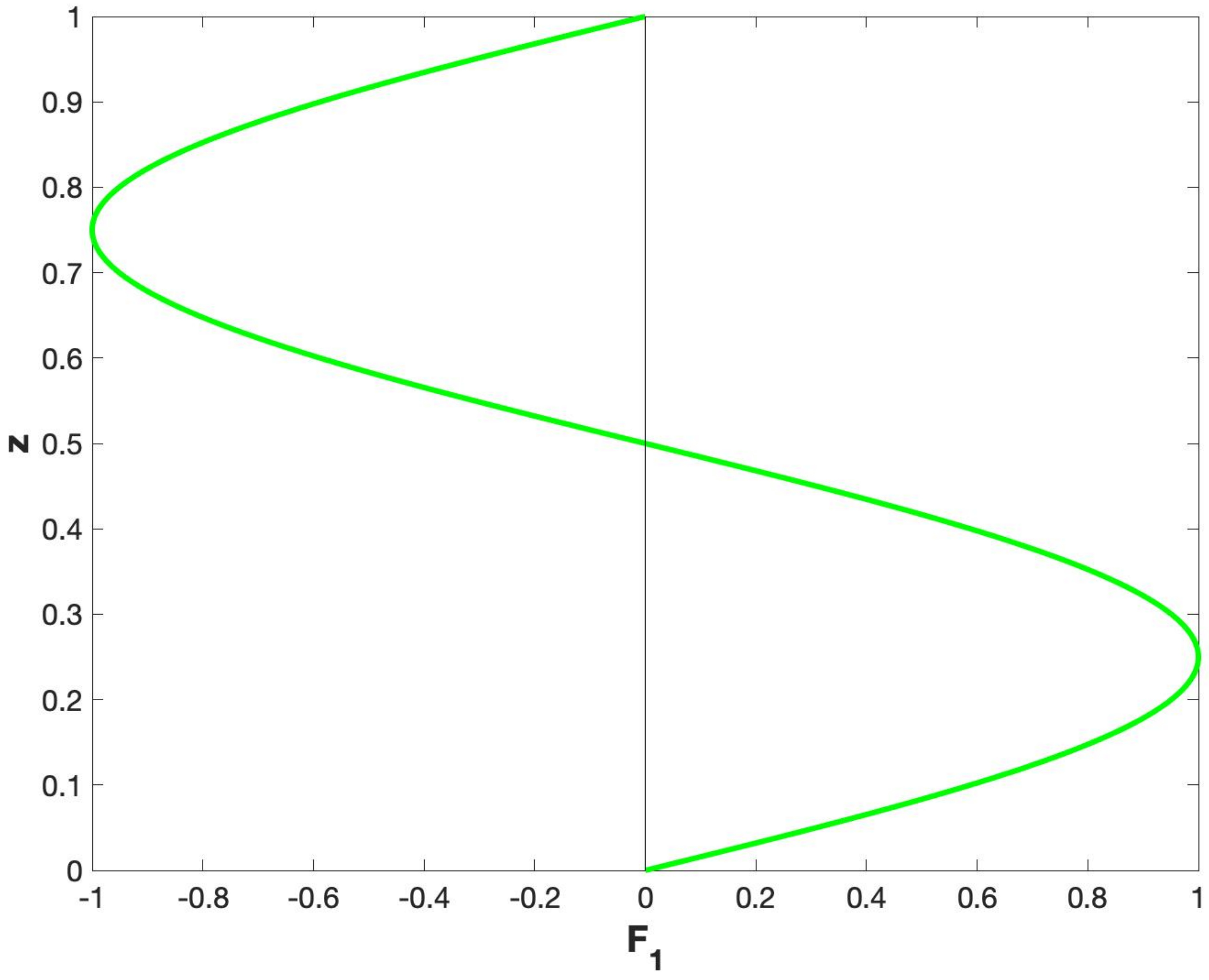
Lower tropospheric westerlies, upper tropospheric easterlies

$$\vec{F} = -(\overline{uw})_z \hat{i}$$

$$\overline{F}_1 = \frac{f}{H}\left(\frac{\Omega}{D}\right)\left(\frac{S_0^2}{\Gamma^2}\right)\sin\left(\frac{2z}{H}\right)$$







The DoNUT circulation + Net Non-Traditional Coriolis force provide the vertical upscale fluxes of zonal momentum which were needed in the Multiscale MJO Models of Majda & B.


DONUT
Equatorial Non-Traditional Coriolis Force
IPESD + MJO
Large Scale Lower tropospheric Westerly Wind