Internal Bernstein Functions and Lévy-Laplace exponents

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Abstract

Bertoin, Roynette et Yor (missing citation) described new connections between the class $\Bd$ of L\'evy-Laplace exponents Ψ (also called the class (sub)critical branching mechanism) and the class of Bernstein functions ($\BF$) which are internal, i.e. those Bernstein functions ϕ s.t. $\Psi \circ \phi$ remains a Bernstein function for every Ψ. We complete their work and illustrate how the class f internal function is rich from the stochastic point of view. It is well known that every $\phi \in \BF$ corresponds univocally to: (i) a subordinator ${(X_t)}_{t\geq 0}$ (or equivalently to transition semigroups ${\big(\pr(X_t\in dx)\big)}_{t\geq 0}$; (ii) a L\'evy measure μ (which controls the jumps of the subordinator). It is also known that, on $\oi$, the measure $\pr(X_t \in dx)/t$ converges vaguely to $\dd \delta_0(dx)+ \mu(dx)$ as $t\to 0$, where $\dd$ is the drift term, but rare are the situations where we can compare the transition semigroups with the L\'evy measure. Our extensive investigations on the composition of L\'evy-Laplace exponents Ψ with Bernstein functions show, for instance, this remarkable facts: ϕ is internal is equivalent to: (a) $\phi^2 \in \BF$ or to (b) $t\mu(dx) - \pr(X_t\in dx)$ is a positive measure on $\oi$. We also provide conditions on μ insuring that ϕ is internal. We also show L\'evy-Laplace exponents are closely connected to the class of Thorin Bernstein function and provide conditions on μ insuring that ϕ is internal.

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References