A blow-up result for the wave equation with localized initial data: the scale-invariant damping and mass term with combined nonlinearities

Makram Hamouda[1] and M.Ali Hamza[1]

[1]Imam Abdulrahman Bin Faisal University

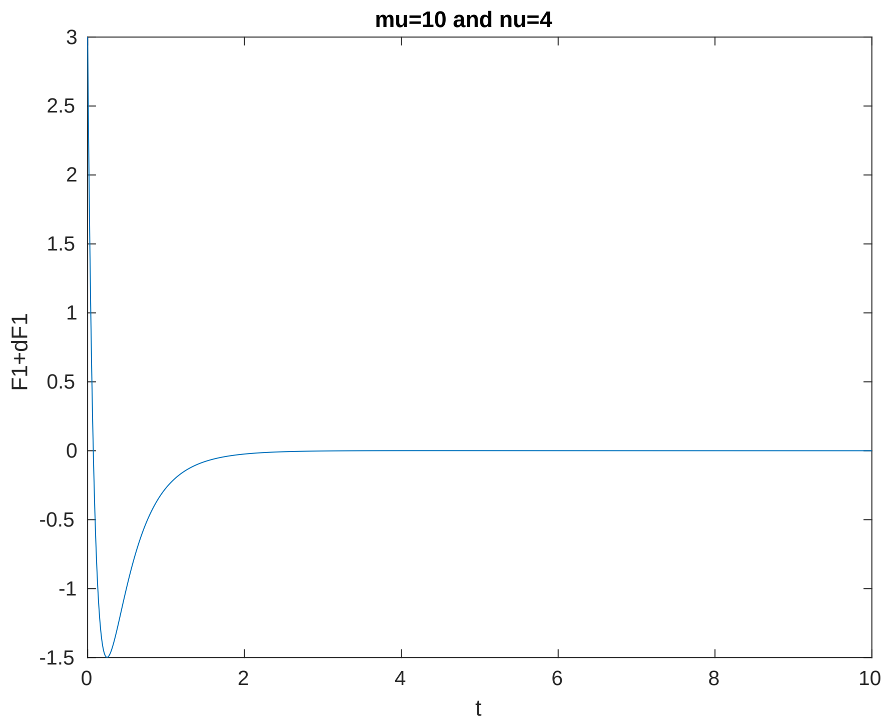
October 29, 2020

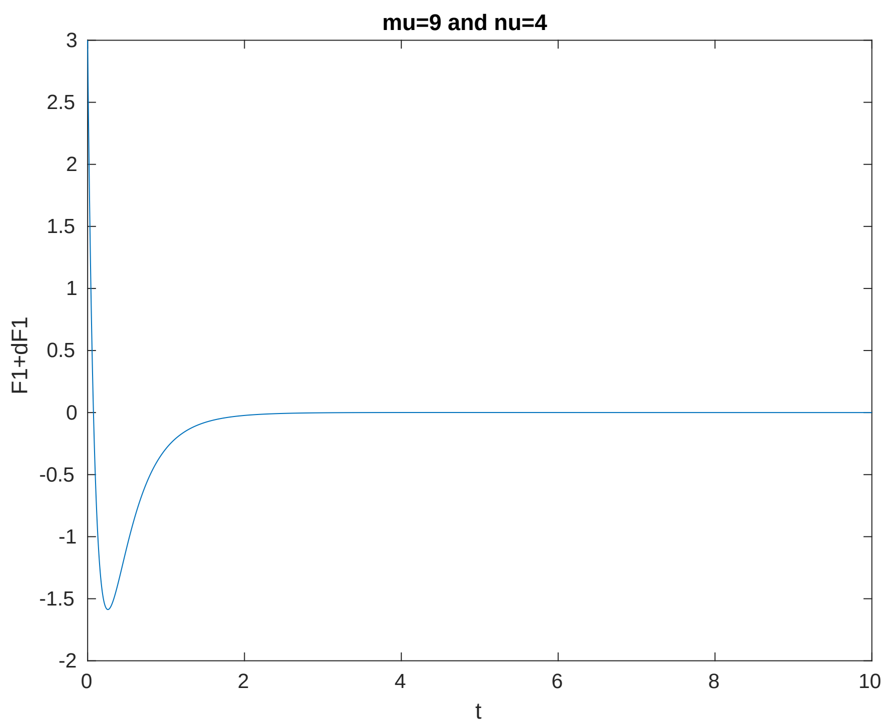
Abstract

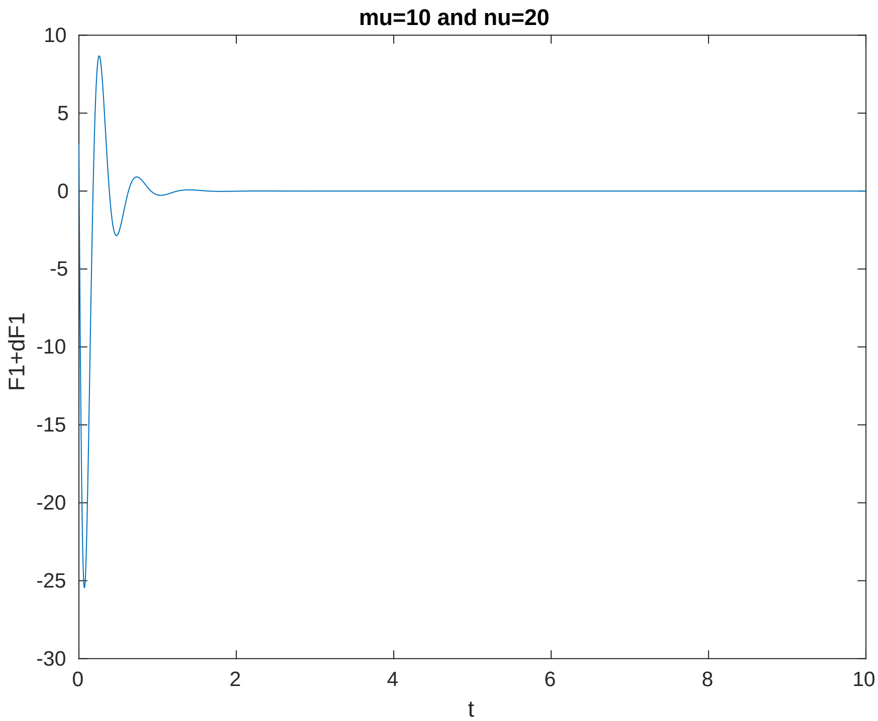
We are interested in this article in studying the damped wave equation with localized initial data, in the \textit{scale-invariant case} with mass term and two combined nonlinearities. More precisely, we consider the following equation: \begin{displaymath} \d (E) \hspace{1cm} u_{tt}-\Delta u+\frac{\mu}{1+t}u_t+\frac{\nu^2}{(1+t)^2}u=|u_t|^p+|u|^q, \quad \mbox{in}\ \R^N\times[0,\infty), \end{displaymath} with small initial data. Under some assumptions on the mass and damping coefficients, ν and $\mu>0$, respectively, we show that blow-up region and the lifespan bound of the solution of (E) remain the same as the ones obtained in (missing citation) in the case of a mass-free wave equation, {\it i.e.} (E) with $\nu=0$. Furthermore, using in part the computations done for (E), we enhance the result in (missing citation) on the Glassey conjecture for the solution of (E) with omitting the nonlinear term $|u|^q$. Indeed, the blow-up region is extended from $p \in (1, p_G(N+\sigma)]$, where σ is given by \eqref{sigma} below, to $p \in (1, p_G(N+\mu)]$ yielding, hence, a better estimate of the lifespan when $(\mu-1)^2-4\nu^2<1$. Otherwise, the two results coincide. Finally, we may conclude that the mass term {\it has no influence} on the dynamics of (E) (resp. (E) without the nonlinear term $|u|^q$), and the conjecture we made in (missing citation) on the threshold between the blow-up and the global existence regions obtained holds true here.

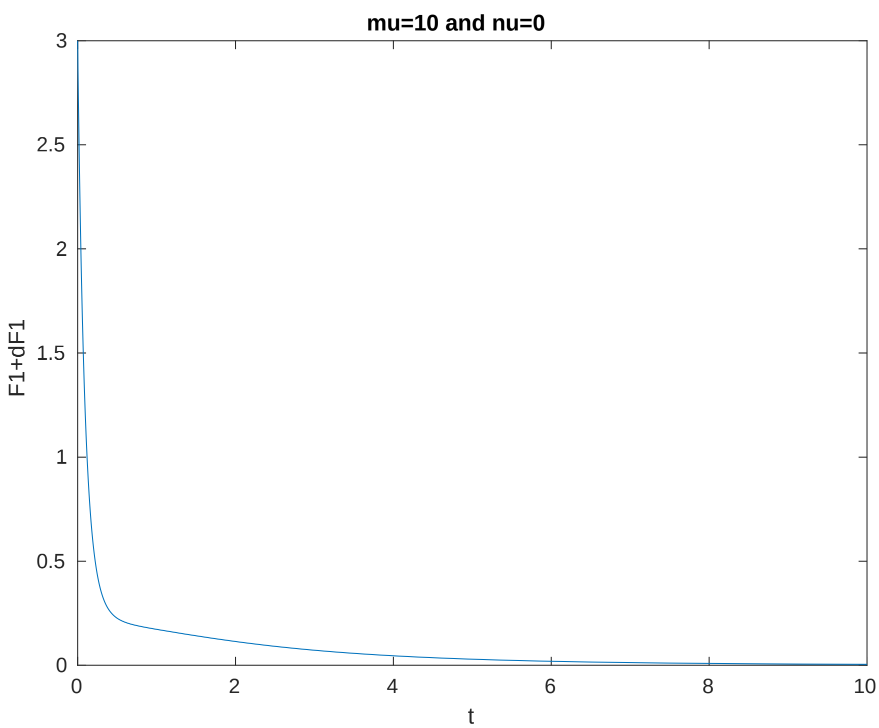
Hosted file

`Blow-up-Damped wave-mass-MMAS.pdf` available at https://authorea.com/users/44799/articles/489611-a-blow-up-result-for-the-wave-equation-with-localized-initial-data-the-scale-invariant-damping-and-mass-term-with-combined-nonlinearities


mu=10 and nu=4
F1+dF1
t
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
2
4
6
8
10


mu=9 and nu=4
F1+dF1
t
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
2
4
6
8
10


mu=10 and nu=20
F1+dF1
t
10
5
0
-5
-10
-15
-20
-25
-30
0
2
4
6
8
10


mu=10 and nu=0
F1+dF1
t
3
2.5
2
1.5
1
0.5
0
0
2
4
6
8
10

References