An Engel Transform

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We can transform functions in the summands of an Engel expansion to sequence terms and or generating functions. A list gives

$$E\left[\frac{(-1)^n}{G(1+n)G(2+n)(2,2)_n}\right] \to A273935 = n!(n-1)!(2^n-1) \tag{1}$$

$$E\left[\frac{1}{G(1+n)G(2+n)}\right] \to A010790 = n!(n-1)! \tag{2}$$

$$E\left[\frac{1}{G(1+n)G(3+n)}\right] \to A129464 = -n(n+1)(n-1)!^2 \tag{3}$$

$$E\left[\frac{\sqrt[8]{e}2^{-n^2-n+\frac{1}{24}}\pi^{\frac{n}{2}+\frac{1}{4}}G(n+3)}{A^{3/2}G\left(n+\frac{3}{2}\right)}\right] = C(n) = \frac{(2n)!}{(n+1)!n!} \tag{4}$$

$$E\left[\frac{1}{\Gamma(n+1)}\right] \to 1,1,2,3,4,5,6,7,8,9,\cdots n \tag{5}$$

$$E\left[\frac{1}{\Gamma(n+1)^2}\right] \to 1,1,4,9,16,25,36,49,\cdots n^2 \tag{6}$$

$$E\left[\frac{1}{G(1+n)}\right] \to 1,1,1,2,6,24,120,720,\cdots n! \tag{7}$$

$$\tag{8}$$

this is generated by the product of the reciprocals of the sequence terms

$$\prod_{k=0}^{n}\frac{1}{a(k)} = E^{-1}(a(n)) \tag{9}$$