

On neighborhood and degree based Symmetric Division deg index for some Silicate and Oxide Networks

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Abstract

In computational chemistry, numbers programming certain structural skin appearance of natural molecules and derivative from the parallel molecular graph are called the graph invariants or more frequently topological indices. Topological indices are numeric quantity that are derived from a molecular graph by mathematical calculations. In QSAR and QSPR study, topological indices are utilized to guess the bioactivity of chemical compounds. The Symmetric Division deg (SDD) is good estimate of total surface area for polychlorobiphenyls. In this paper we process the Symmetric Division deg index for Silicate, Oxide and Copper(II) Oxide network. We compare above network of Symmetric division deg index based on degree and neighbourhood.

On neighborhood and degree based Symmetric Division deg index for some Silicate and Oxide Networks

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In computational chemistry, numbers programming certain structural skin appearance of natural molecules and derivatives from the parallel molecular graph are called the graph invariants or more frequently topological indices. Topological indices are numeric quantities that are derived from a molecular graph by mathematical calculations. In QSAR and QSPR studies, topological indices are utilized to guess the bioactivity of chemical compounds. The Symmetric Division deg (SDD) is a good estimate of the total surface area for polychlorobiphenyls. In this paper, we explore the Symmetric Division deg index for Silicate, Oxide, and Copper(II) Oxide networks. We compute the degree and neighborhood based Symmetric division deg index for some network structures. Further, we compare those indices graphically.

KEYWORDS

Symmetric division deg index 1, Silicate network 2, Oxide network 3, Copper (II) Oxide network 4

1 | INTRODUCTION

A graph G is an ordered pair of sets $V(G)$ and $E(G)$, with the items $uv \in E(G)$ being a sub-collection of Vs' unordered pairs of elements (G). The members of $V(G)$ are referred to as vertices, while the elements of $E(G)$ are referred to as edges. If $e = pq$ is an edge, we say that the vertices p and q are adjacent, and that p, q are the two end points (or ends) of e . G has an order of n and a dimension of m if it has n vertices and m edges. An n -vertex graph is a graph

of order n . Chemical Graph theory is a branch of mathematical chemistry. To understand the physical characteristics of these chemical substances, graph theory is employed mathematically to represent molecules. This theory had an important effect on development of Chemical science. Quantitative structure property relations and Quantitative structure activity relations of the chemical structure require objective expressions for the topological property of these structures. Quantitative structure activity relations models mainly focus [1, 2] in reproduction system in biological field, chemical sciences, and control system engineering. One of the primary chemistry applications in quantitative structure activity relations is forecasting melting points. Mathematically, topological indices converts a structure as a graph and gives a numerical value for that graph. The idea of topological indices and structure based properties are developed by several authors in [3, 1, 4, 5, 6]. Several years ago, Vukicevic and Gasperov considered a new class of molecular descriptors, consisting of one hundred and forty eight descriptors, namely discrete Adriatic indices for improving the various QSPR/QSAR (quantitative structure property/activity relationships) studies and they found that only a few descriptors from this class are useful.[7, 8, 9, 10, 11, 12]

Besides indices, vertex-based indices are widely used in graph invariants [13]. The Symmetric division deg (SDD) index is one of the most useful discrete Adriatic indices, which is defined as [14, 15, 16]

$$SDD(G) = \sum_{p \sim q} \left(\frac{\max(d_p, d_q)}{\min(d_p, d_q)} + \frac{\min(d_p, d_q)}{\max(d_p, d_q)} \right)$$

The neighborhood of Symmetric division deg index is named as Fifth Neighborhood division index (ND_5) is defined as [17]

$$ND_5(G) = \sum_{p \sim q} \left(\frac{\delta_N(p^2) + \delta_N(q^2)}{\delta_N(p)\delta_N(q)} \right)$$

where $\delta_N(p) = \sum_{p' \in N(p)} d(p')$, $N(p) = \{p' \ni pp' \in E(G)\}$

In this paper, In section 2 we explore the SDD based on degree and neighborhood for the Silicate, Oxide and Copper (II) Oxide network structures. In section 3, we give the comparison and conclusion. In particular, we identify the significant difference between the network structures by means of the SDD index based on degree and neighborhood.

2 | MAIN RESULT

In mineral chemistry, metal oxides or metal carbonates are combined with sand to form silicates. Silicate is the largest, most vibrant, and hardest mineral over long distances on the earth. These silicates are used in three-dimensional metal cathode structures, reticular chemistry, and ultrahigh proton conductivity [18, 19, 20, 21]. Essential semantic component of Silicate (SiO_4) is tetrahedron. In graph theory, the silicate is drawn such that the oxygen nodes (blue vertices) and the middle vertex are perpendicular to the silicon node (red vertices) show in Figure1. The various silicate structures are obtained by arranging these tetrahedra. The structure of the Oxide network [22, 23, 24, 25] can be addressed by a mathematical graph and random. The Oxide network is obtained by removing all silicon ions from the Silicate network shown in Figure2.

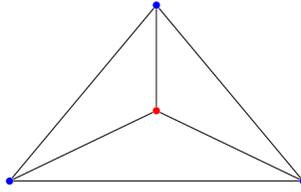


FIGURE 1 Graph Representation of SiO_4

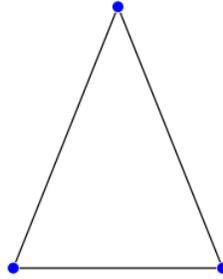


FIGURE 2 Graph Representation of Oxide

2.1 | Silicate network of SDD index based on degree and neighborhood

We have explored some of the structures of the Silicate network under this heading. They are Cyclic Silicate (CS), Double chain Silicate (DC), Rhombus Silicate ($RHSL$) and Regular Triangulate silicate ($RTSL$). Their structures are shown as Figure3, Figure5, Figure7 and Figure9 respectively. Cyclic Silicate networks are acquired organizing x unit Silicates in a cyclic combination by mixing oxygen molecules. The cardinality of vertex set (nodes) and edge set of Cyclic Silicate networks are $|V(CS_x)| = 3x$ and $|E(CS_x)| = 6x$ for $x \geq 3$. The graph representation of Cyclic Silicate based on degree and neighborhood of SDD index is shown in Figure4.

Theorem 1 Let G be a x dimension of Cyclic Silicate (CS_x), $SDD(G) = 14x$ for $x \geq 3$

Proof Consider the graph G is a Cyclic Silicate of x dimension (CS_x). The partitions of vertex set and edge set of CS_x with respect to degree of end vertices. There are two types of vertex set in CS_x . The cardinality of the V_1 vertex set is $2x$ of degree 3 and the cardinality of the V_2 vertex set is x of degree 6. So, $|V(G)| = |V_1| + |V_2| = 3x$. There are three partition of edges in G based on degree of end vertices. We have $E_1(3, 3) = \{pq \in E(G) \mid d_p = 3, d_q = 3\}$, $E_2(3, 6) = \{pq \in E(G) \mid d_p = 3, d_q = 6\}$ and $E_3(6, 6) = \{pq \in E(G) \mid d_p = 6, d_q = 6\}$ where $|E_1| = x$, $|E_2| = 4x$ and $|E_3| = x$. As

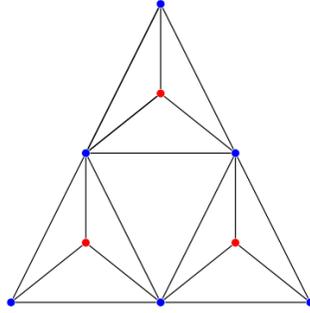


FIGURE 3 Cyclic Silicate

a consequence $|E(G)| = |E_1| + |E_2| + |E_3| = 6x$.

$$\begin{aligned} SDD(G) &= \sum_{p \sim q} \left(\frac{\max(d_p, d_q)}{\min(d_p, d_q)} + \frac{\min(d_p, d_q)}{\max(d_p, d_q)} \right) \\ &= 2x + 10x + 2x \\ &= 14x \end{aligned}$$

Theorem 2 Let G be a Cyclic Silicate (CS_x), $ND_5(G) = \frac{129x}{10}$ for $x \geq 3$

Proof Let us consider the graph G has Cyclic Silicate of x -dimension (CS_x). There are two types of vertex sets of CS_x . The cardinality of the vertex set is $3x$ of degree 3 and 6. There are three partitions of edges in G based on degree sum of the neighborhood of end vertices. We have, $E_1(3, 3) = \{pq \in E(G) \mid \delta_p = 15, \delta_q = 15 \text{ where, } d_p = 3, d_q = 3\}$, $E_2(3, 6) = \{pq \in E(G) \mid \delta_p = 15, \delta_q = 24 \text{ where, } d_p = 3, d_q = 6\}$ and $E_3(6, 6) = \{pq \in E(G) \mid \delta_p = 24, \delta_q = 24 \text{ where, } d_p = 6, d_q = 6\}$. Where $|E(G)| = |E_1| + |E_2| + |E_3| = x + 4x + x = 6x$

$$\begin{aligned} ND_5 &= \sum_{p \sim q} \frac{\delta_N(p^2) + \delta_N(q^2)}{\delta_N(p)\delta_N(q)} \\ &= \frac{15^2 + 15^2}{15 \times 15} x + \frac{15^2 + 24^2}{15 \times 24} 4x + \frac{24^2 + 24^2}{24 \times 24} x \\ &= 2x + \frac{89}{40} 4x + 2x \\ &= \frac{20x + 89x + 20x}{10} \\ &= \frac{129x}{10} \end{aligned}$$

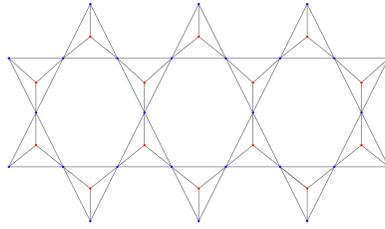


FIGURE 4 Double chain Silicate

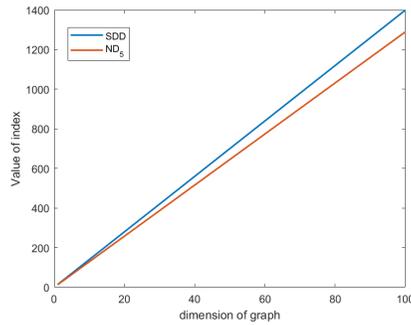


FIGURE 5 SDD and ND_5 of Cyclic Silicate

2.2 | Double Chain Silicate

Let DC_x be a x dimensional Double Chain silicate. DC_x is a combination of two Chain Silicates with dimension $2x + 1$. Number of vertices(nodes) and edges respectively $|V(DC_x)| = 11x + 7$ and $|E(DC_x)| = 12(2x + 1)$.

Theorem 3 Let G be a Double Chain Silicate of graph (DC_x) , $SDD(G) = 55x + 29$ for $x \geq 3$

Proof Let us consider the graph G has a Double Chain Silicate of x -dimension (DC_x) . The partitions of vertex set and edge set of $RHSL_x$ with respect to degree of end vertices. There are two types of vertex set in DC_x . The cardinality of the V_1 and V_2 vertex sets are $6x + 6$, $5x + 1$ of degree 3 and 6 respectively. So, $|V(G)| = |V_1| + |V_2| = 11x + 7$. There are three types of edges in G based on the degree of end vertices. We have $E_1(3, 3) = \{pq \in E(G) \mid d_p = 3, d_q = 3\}$, $E_2(3, 6) = \{pq \in E(G) \mid d_p = 3, d_q = 6\}$ and $E_3(6, 6) = \{pq \in E(G) \mid d_p = 6, d_q = 6\}$, where $|E_1| = 2x + 4$, $|E_2| = 14x + 10$, $|E_3| = 8x - 2$. As a consequence $|E(G)| = |E_1| + |E_2| + |E_3| = 24x + 12$.

$$\begin{aligned}
 SDD(G) &= \sum_{p \sim q} \left(\frac{\max(d_p, d_q)}{\min(d_p, d_q)} + \frac{\min(d_p, d_q)}{\max(d_p, d_q)} \right) \\
 &= 2(2x + 4) + \left(\frac{15}{6} \right) (14x + 10) + 2(8x - 2) \\
 &= 55x + 29
 \end{aligned}$$

Theorem 4 Let G be a Double chain silicate (DC_x) , $ND_5(G) = \frac{14064x+6843}{270}$ for $x \geq 3$

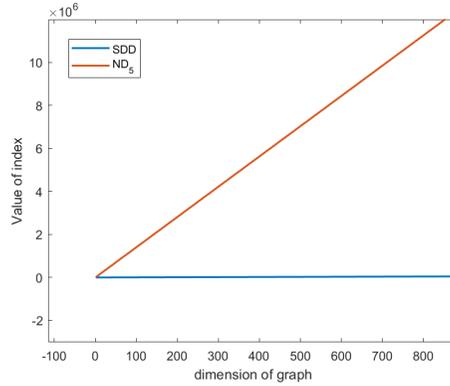


FIGURE 6 SDD and ND_5 of Double chain Silicate

Proof Let us consider the graph G has a Double chain Silicate of x -dimension (DC_x). There are two types of vertex set of DC_x . The cardinality of vertex set is $11x+7$ of degree 3 and 6. There are nine partition of edges in G based on degree sum of the neighborhood of end vertices. We have, $E_1(3,3) = \{pq \in E(G) \mid \delta_p = 15, \delta_q = 15 \text{ where, } d_p = 3, d_q = 3\}$, $E_2(3,6) = \{pq \in E(G) \mid \delta_p = 15, \delta_q = 24 \text{ where, } d_p = 3, d_q = 6\}$, $E_3(3,6) = \{pq \in E(G) \mid \delta_p = 15, \delta_q = 27 \text{ where, } d_p = 3, d_q = 6\}$, $E_4(3,6) = \{pq \in E(G) \mid \delta_p = 18, \delta_q = 27 \text{ where, } d_p = 3, d_q = 6\}$, $E_5(3,6) = \{pq \in E(G) \mid \delta_p = 18, \delta_q = 30 \text{ where, } d_p = 3, d_q = 6\}$, $E_6(6,6) = \{pq \in E(G) \mid \delta_p = 24, \delta_q = 24 \text{ where, } d_p = 6, d_q = 6\}$, $E_7(6,6) = \{pq \in E(G) \mid \delta_p = 24, \delta_q = 27 \text{ where, } d_p = 6, d_q = 6\}$, $E_8(6,6) = \{pq \in E(G) \mid \delta_p = 27, \delta_q = 27 \text{ where, } d_p = 6, d_q = 6\}$ and $E_9(6,6) = \{pq \in E(G) \mid \delta_p = 27, \delta_q = 30 \text{ where, } d_p = 6, d_q = 6\}$. Where

$$\begin{aligned} |E(G)| &= |E_1| + |E_2| + |E_3| + |E_4| + |E_5| + |E_6| + |E_7| + |E_8| + |E_9| \\ &= (2x+4) + 24 + (8x-8) + (4x-4) + (2x-2) + 4 + 4(4x-4) + (4x-6) \\ &= 24x + 36 - 24 \end{aligned}$$

$$\begin{aligned} ND_5 &= \sum_{p \sim q} \frac{\delta_N(p^2) + \delta_N(q^2)}{\delta_N(p)\delta_N(q)} \\ &= \frac{15^2 + 15^2}{15 \times 15} (2x+4) + \frac{15^2 + 24^2}{15 \times 24} (24) + \frac{15^2 + 27^2}{15 \times 27} (8x-8) \\ &\quad + \frac{18^2 + 27^2}{18 \times 27} (4x-4) + \frac{18^2 + 30^2}{18 \times 30} (2x-2) + \frac{24^2 + 24^2}{24 \times 24} (4) \\ &\quad + \frac{24^2 + 27^2}{24 \times 27} (4) + \frac{27^2 + 27^2}{27 \times 27} (4x-4) + \frac{27^2 + 30^2}{27 \times 30} (4x-6) \\ &= 2(2x+4) + 8 + \frac{1}{27} \left(\frac{7428x - 5253}{10} \right) + \frac{1}{45} (566x - 747) + \frac{267}{5} \\ &= 4x + 8 + 8x + \frac{267}{5} + \frac{7428x}{270} - \frac{5253}{270} + \frac{566x}{45} - \frac{747}{45} \\ &= \frac{14064x + 6834}{270} \end{aligned}$$

Figure6 depicts the graph of Double Chain silicate depending on the degree and neighborhood of the SDD index.

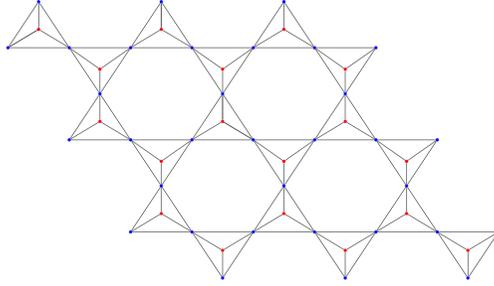


FIGURE 7 Rhombus Silicate

2.3 | Rhombus Silicate

An x dimensional Rhombus silicate denoted by $(RHSL_x)$. Number of vertices (nodes) and edges respectively $|V(RHSL_x)| = 5x^2 + 2x$ and $|E(RHSL_x)| = 12x^2$.

Theorem 5 Let G be a Rhombus Silicate of graph $(RHSL_x)$, $SDD(G) = 27x^2 + 2x - 2$ for $x \geq 2$

Proof Let us consider the graph G has Rhombus Silicate of x -dimension $(RHSL_x)$. The partitions of vertex set and edge set of $RHSL_x$ with respect to the degree of end vertices. There are two types of vertex set in $RHSL_x$. The cardinality of the V_1 and V_2 vertex sets are $2x(x+2)$, $x(3x-2)$ of degree 3 and 6 respectively. So, $|V(G)| = |V_1| + |V_2| = 5x^2 + 2x$. There are three types of edges in G based on the degree of end vertices. We have, $E_1(3,3) = \{pq \in E(G) \mid d_p = 3, d_q = 3\}$, $E_2(3,6) = \{pq \in E(G) \mid d_p = 3, d_q = 6\}$ and $E_3(6,6) = \{pq \in E(G) \mid d_p = 6, d_q = 6\}$. Where $|E_1| = 4x + 2$, $|E_2| = 6x^2 + 4x - 4$ and $|E_3| = 6x^2 - 8x + 2$. As a consequence $|E(G)| = |E_1| + |E_2| + |E_3| = 12x^2$.

$$\begin{aligned}
 SDD(G) &= \sum_{p \sim q} \left(\frac{\max(d_p, d_q)}{\min(d_p, d_q)} + \frac{\min(d_p, d_q)}{\max(d_p, d_q)} \right) \\
 &= (4x + 2)(2) + (6x^2 + 4x - 4) \left(\frac{15}{6} \right) + (6x^2 - 8x + 2)(2) \\
 &= 8x + 4 + 15x^2 + 10x - 10 + 12x^2 - 16x + 4 \\
 &= 27x^2 + 2x - 2
 \end{aligned}$$

Figure8 illustrates the graph representation of Rhombus Silicate depending on the degree and neighborhood of the SDD index.

Theorem 6 Let G be a Rhombus Silicate $(RHSL_x)$, $ND_5(G) = \frac{2304x^2 + 160x - 123}{90}$ for $x \geq 3$.

Proof Let us consider the graph G has Rhombus Silicate of x -dimension $(RHSL_x)$. There are two types of the vertex set of $RHSL_x$. The cardinality of the vertex set is $5x^2 + 2x$ of degree 3 and 6. There are twelve partitions of edges in G based on degree sum of the neighborhood of end vertices. We have $E_1(3,3) = \{pq \in E(G) \mid \delta_p = 12, \delta_q = 12 \text{ where, } d_p = 3, d_q = 3\}$, $E_2(3,3) = \{pq \in E(G) \mid \delta_p = 15, \delta_q = 15 \text{ where, } d_p = 3, d_q = 3\}$, $E_3(3,6) = \{pq \in E(G) \mid \delta_p = 12, \delta_q = 24 \text{ where, } d_p = 3, d_q = 6\}$, $E_4(3,6) = \{pq \in E(G) \mid \delta_p = 15, \delta_q = 24 \text{ where, } d_p = 3, d_q = 6\}$, $E_5(3,6) = \{pq \in E(G) \mid \delta_p = 15, \delta_q = 27 \text{ where, } d_p = 3, d_q = 6\}$, $E_6(3,6) = \{pq \in E(G) \mid \delta_p = 18, \delta_q =$

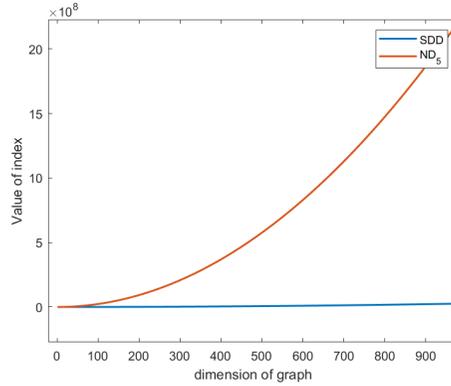


FIGURE 8 SDD and ND_5 of Rhombus Silicate

24 where, $d_p = 3, d_q = 6$, $E_7(3, 6) = \{pq \in E(G) \mid \delta_p = 18, \delta_q = 27\}$ where, $d_p = 6, d_q = 6$, $E_8(3, 6) = \{pq \in E(G) \mid \delta_p = 18, \delta_q = 30\}$ where, $d_p = 6, d_q = 6$, $E_9(6, 6) = \{pq \in E(G) \mid \delta_p = 24, \delta_q = 27\}$ where, $d_p = 6, d_q = 6$, $E_{10}(6, 6) = \{pq \in E(G) \mid \delta_p = 27, \delta_q = 30\}$ where, $d_p = 6, d_q = 6$ $E_{11}(6, 6) = \{pq \in E(G) \mid \delta_p = 27, \delta_q = 27\}$ where, $d_p = 6, d_q = 6$ and $E_{12}(6, 6) = \{pq \in E(G) \mid \delta_p = 30, \delta_q = 30\}$ where, $d_p = 6, d_q = 6$. Where

$$\begin{aligned}
 |E(G)| &= |E_1| + |E_2| + |E_3| + |E_4| + |E_5| + |E_6| + |E_7| + |E_8| + |E_9| + |E_{10}| + |E_{11}| + |E_{12}| \\
 &= 6 + 4(x-1) + 6 + 8 + 8(2x-3) + 2 + 4(2x-3) \\
 &\quad + 2(x-2)(3x-4) + 8 + 8(x-2) + 8(x-2) + 2 + 6(x-2)^2 \\
 &= 24x + 36 - 24
 \end{aligned}$$

$$\begin{aligned}
 ND_5(G) &= \sum_{p \sim q} \frac{\delta_N(p^2) + \delta_N(q^2)}{\delta_N(p)\delta_N(q)} \\
 &= \frac{12^2 + 12^2}{12 \times 12} (6) + \frac{15^2 + 15^2}{15 \times 15} 4(x-1) + \frac{12^2 + 24^2}{12 \times 24} (6) + \frac{15^2 + 24^2}{15 \times 24} (8) \\
 &\quad + \frac{15^2 + 27^2}{15 \times 27} 8(2x-3) + \frac{18^2 + 24^2}{18 \times 24} (2) + \frac{18^2 + 27^2}{18 \times 27} 4(2x-3) \\
 &\quad + \frac{18^2 + 30^2}{18 \times 30} 2(x-2)(3x-4) + \frac{24^2 + 27^2}{24 \times 27} (8) \\
 &\quad + \frac{27^2 + 30^2}{27 \times 30} 8(x-2) + \frac{27^2 + 27^2}{27 \times 27} 8(x-2) + 2 + \frac{30^2 + 30^2}{30 \times 30} 6(x-2)^2 \\
 &= 2(6) + 2(4(x-1)) + \frac{720}{288} (6) + \frac{801}{360} (8) + \frac{954}{405} 8(2x-3) \\
 &\quad + \frac{900}{432} (2) + \frac{1053}{486} 4(2x-3) + \frac{1224}{540} 2(x-2)(3x-4) \\
 &\quad + \frac{1305}{648} (8) + \frac{1629}{810} 8(x-2) + 2(8(x-2) + 2) + 2(6(x-2)^2) \\
 &= \frac{2304x^2 + 160x - 123}{90}
 \end{aligned}$$

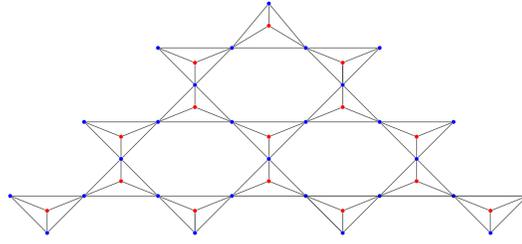


FIGURE 9 Regular Triangulate Silicate

2.4 | Regular Triangulate Silicate

The molecular structure of a x dimension Regular Triangulate Silicate network is denoted by $RTSL_x$. The cardinality of vertex and edge sets are respectively $\frac{1}{2}(5x^2 + 13x + 2)$, $6x^2 + 12x$. In Figure10 presents the graph representation of $RTSL_x$ depending on the degree and neighborhood of the SDD index.

Theorem 7 Let G be a Regular Triangulate Silicate of graph ($RTSL_x$), $SDD(G) = \frac{27x^2 + 57x - 2}{2}$ for $x \geq 2$.

Proof Let us consider the graph G has Regular Triangulate Silicate of x -dimension ($RTSL_x$). The partitions of vertex set and edge set of $RTSL_x$ with respect to degree of end vertices. There are two types of vertex set in $RTSL_x$. The cardinality of the V_1 vertex set is $\frac{5x^2 - 11x + 22}{2}$ of degree 6 and $|V_2| = 12x - 10$ of degree 3. So, $|V(G)| = |V_1| + |V_2| = \frac{5x^2 + 13x + 2}{2}$. There are three types of edges in G based on degree of end vertices. We have $E_1(3, 3) = \{pq \in E(G) \mid d_p = 3, d_q = 3\}$, $E_2(3, 6) = \{pq \in E(G) \mid d_p = 3, d_q = 6\}$ and $E_3(6, 6) = \{pq \in E(G) \mid d_p = 6, d_q = 6\}$. where $|E_1| = 3x + 4$, $|E_2| = 3x^2 - 2$ and $|E_3| = (3x^2 + 9x - 2)$. As a consequence $|E(G)| = 6x^2 + 12x$.

$$\begin{aligned} SDD(G) &= \sum_{p \sim q} \left(\frac{\max(d_p, d_q)}{\min(d_p, d_q)} + \frac{\min(d_p, d_q)}{\max(d_p, d_q)} \right) \\ &= \left(\frac{3^2 + 3^2}{3 \times 3} \right) 3x + 4 + \left(\frac{6^2 + 6^2}{6 \times 6} \right) 3x^2 - 2 + \left(\frac{3^2 + 6^2}{3 \times 6} \right) (3x^2 + 9x - 2) \\ &= 2(3x + 4) + 2(3x^2 - 2) + \frac{5}{2}(3x^2 + 9x - 2) \\ &= \frac{27x^2 + 57x - 2}{2} \end{aligned}$$

Theorem 8 Let G be a Regular Triangulate Silicate network ($RTSL_x$), $x \geq 3$, $ND_5(G) = \frac{2304x^2 + 3768x + 4229}{180}$

Proof Let us consider the graph G has Regular Triangulate Silicate of x -dimension ($RHSL_x$). There are two types of vertex set of $RTSL_x$. The cardinality of vertex set is $\frac{5x^2 + 13x + 2}{2}$ of degree 3 and 6. There are thirteen partitions of edges in G based on degree sum of the neighborhood of end vertices. We have, $E_1(3, 3) = \{pq \in E(G) \mid \delta_p = 12, \delta_q = 12 \text{ where, } d_p = 3, d_q = 3\}$, $E_2(3, 3) = \{pq \in E(G) \mid \delta_p = 15, \delta_q = 15 \text{ where, } d_p = 3, d_q = 3\}$, $E_3(3, 6) = \{pq \in E(G) \mid \delta_p = 12, \delta_q = 24 \text{ where, } d_p = 3, d_q = 6\}$, $E_4(3, 6) = \{pq \in E(G) \mid \delta_p = 15, \delta_q = 24 \text{ where, } d_p = 3, d_q = 6\}$, $E_5(3, 6) = \{pq \in E(G) \mid \delta_p = 15, \delta_q = 27 \text{ where, } d_p = 3, d_q = 6\}$, $E_6(3, 6) = \{pq \in E(G) \mid \delta_p = 18, \delta_q = 24 \text{ where, } d_p = 3, d_q = 6\}$, $E_7(3, 6) = \{pq \in E(G) \mid \delta_p = 18, \delta_q = 27 \text{ where, } d_p = 6, d_q = 6\}$, $E_8(3, 6) = \{pq \in E(G) \mid \delta_p = 18, \delta_q = 30 \text{ where, } d_p = 6, d_q = 6\}$, $E_9(6, 6) = \{pq \in E(G) \mid \delta_p = 24, \delta_q = 24 \text{ where, } d_p = 6, d_q = 6\}$,

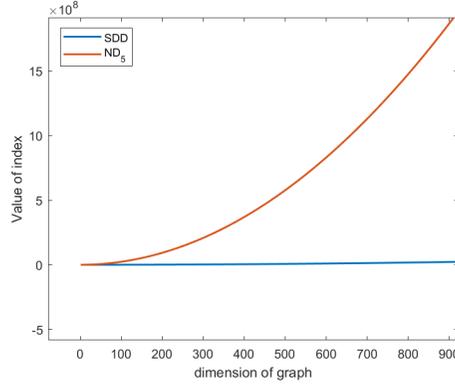


FIGURE 10 SDD and ND_5 of Regular Triangulate Silicate

$E_{10}(6, 6) = \{pq \in E(G) \mid \delta_p = 24, \delta_q = 27 \text{ where, } d_p = 6, d_q = 6\}$, $E_{11}(6, 6) = \{pq \in E(G) \mid \delta_p = 27, \delta_q = 27 \text{ where, } d_p = 6, d_q = 6\}$, $E_{12}(6, 6) = \{pq \in E(G) \mid \delta_p = 27, \delta_q = 30 \text{ where, } d_p = 6, d_q = 6\}$ and $E_{13}(6, 6) = \{pq \in E(G) \mid \delta_p = 30, \delta_q = 30 \text{ where, } d_p = 6, d_q = 6\}$. Where

$$\begin{aligned}
 |E(G)| &= |E_1| + |E_2| + |E_3| + |E_4| + |E_5| + |E_6| + |E_7| + |E_8| + |E_9| + |E_{10}| + |E_{11}| + |E_{12}| \\
 &= 6 + 6 + (3x - 2) + 8 + (12x - 16) + 2 + (6x - 8) \\
 &+ (3x^2 - 9x + 6) + 1 + 6 + (3x + 3) + (6x - 12) + (3x^2 - 9x) \\
 &= 6x^2 + 12x
 \end{aligned}$$

$$\begin{aligned}
 ND_5(G) &= \sum_{p \sim q} \frac{\delta_N(p^2) + \delta_N(q^2)}{\delta_N(p)\delta_N(q)} \\
 &= \frac{12^2 + 12^2}{12 \times 12} (6) + \frac{12^2 + 24^2}{12 \times 24} (6) + \frac{15^2 + 15^2}{15 \times 15} (3x - 2) + \frac{15^2 + 24^2}{15 \times 24} (8) \\
 &+ \frac{15^2 + 27^2}{15 \times 27} (12x - 16) + \frac{18^2 + 24^2}{18 \times 24} (2) + \frac{18^2 + 27^2}{18 \times 27} (6x - 8) \\
 &+ \frac{18^2 + 30^2}{18 \times 30} (3x^2 - 9x + 6) + \frac{24^2 + 24^2}{24 \times 24} (1) + \frac{24^2 + 27^2}{24 \times 27} (6) \\
 &+ \frac{27^2 + 27^2}{27 \times 27} (3x + 3) + \frac{27^2 + 30^2}{27 \times 30} (6x - 12) + \frac{30^2 + 30^2}{30 \times 30} (3x^2 - 9x) \\
 &= 2(6) + 15 + 2(3x - 2) + \frac{801}{45} + \frac{954}{405} (12x - 16) + \frac{900}{216} \\
 &+ \frac{1053}{486} (6x - 8) + \frac{1224}{540} (3x^2 - 9x + 6) + 2 \\
 &+ \frac{1305}{108} + 2(3x + 3) + \frac{1629}{810} (6x - 12) + 2(3x^2 - 9x) \\
 &= \frac{2304x^2 + 3768x + 4229}{180}
 \end{aligned}$$

2.5 | Oxide Network of SDD index based on degree and neighborhood

In this section, we identify the SDD index based on for some of the structures of the Oxide networks. The OX_x be an x -dimension of the Oxide network. The number of vertices(nodes) and edges $9x^2 + 3x$ and $18x^2$ respectively. Figure11 displays a graph of OX_x depending on SDD and ND_5 .

Theorem 9 Let G be a Oxide Network graph (OX_x), $x \geq 3$, $SDD(G) = 6x + 36x^2$.

Proof Let us consider the graph G has Oxide network of x -dimension (OX_x). The partitions of vertex set and edge set of OX_x with respect to degree of end vertices. There are two types of vertex set in OX_x . The cardinality of the V_1 vertex set is $6x$ of degree 2 and the cardinality of the V_2 vertex set is $9x^2 - 3x$ of degree 4. So, $|V(G)| = |V_1| + |V_2| = 9x^2 + 3x$. There are three types of edges in G based on degree of end vertices. We have $E_1(2, 2) = \{pq \in E(G) \mid d_p = 2, d_q = 2\}$, $E_2(4, 4) = \{pq \in E(G) \mid d_p = 4, d_q = 4\}$ where

$$\begin{aligned} |E(G)| &= |E_1| + |E_2| \\ &= 12x + 18x^2 - 12x \\ &= 18x^2 \end{aligned}$$

$$\begin{aligned} SDD(G) &= \sum_{p \sim q} \left(\frac{\max(d_p, d_q)}{\min(d_p, d_q)} + \frac{\min(d_p, d_q)}{\max(d_p, d_q)} \right) \\ &= \left(\frac{10}{4} \right) (12x) + (2)(6(3x^2 - 2x)) \\ &= 30x + 2(18x^2 - 12x) \\ &= 6x + 36x^2 \end{aligned}$$

Theorem 10 Let G be a Oxide network (OX_x), $x \geq 3$, $ND_5(G) = \frac{1008x^2 + 89x}{28}$.

Proof Let us consider the graph G has Oxide network x -dimension (OX_x). There are two types of vertex set of OX_x . The cardinality of vertex set is $9x^2 + 3x$. There are six types of edges in G based on degree sum of the neighborhood of end vertices. We have $E_1(2, 2) = \{pq \in E(G) \mid \delta_p = 8, \delta_q = 12 \text{ where, } d_p = 2, d_q = 2\}$, $E_2(2, 4) = \{pq \in E(G) \mid \delta_p = 8, \delta_q = 14 \text{ where, } d_p = 2, d_q = 4\}$, $E_3(4, 4) = \{pq \in E(G) \mid \delta_p = 12, \delta_q = 14 \text{ where, } d_p = 4, d_q = 4\}$, $E_4(4, 4) = \{pq \in E(G) \mid \delta_p = 14, \delta_q = 14 \text{ where, } d_p = 4, d_q = 4\}$, $E_5(4, 4) = \{pq \in E(G) \mid \delta_p = 14, \delta_q = 16 \text{ where, } d_p = 4, d_q = 4\}$ and $E_6(4, 4) = \{pq \in E(G) \mid \delta_p = 16, \delta_q = 16 \text{ where, } d_p = 4, d_q = 4\}$. Where

$$\begin{aligned} |E(G)| &= |E_1| + |E_2| + |E_3| + |E_4| + |E_5| + |E_6| \\ &= 6x + 6x + 6x + 3x + 6x + (18x^2 - 27x) \\ &= 18x^2. \end{aligned}$$

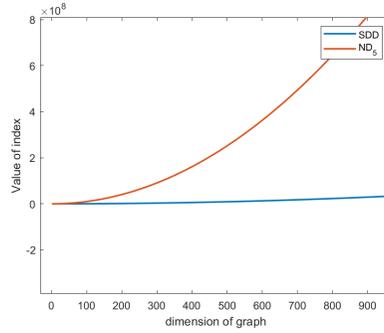


FIGURE 11 SDD and ND_5 of Oxide

$$\begin{aligned}
 ND_5(G) &= \sum_{p \sim q} \frac{\delta_N(p^2) + \delta_N(q^2)}{\delta_N(p)\delta_N(q)} \\
 &= \frac{8^2 + 12^2}{8 \times 12} (6x) + \frac{8^2 + 14^2}{8 \times 14} (6x) \\
 &\quad + \frac{12^2 + 14^2}{12 \times 14} (6x) + \frac{14^2 + 14^2}{14 \times 14} (3x) \\
 &\quad + \frac{14^2 + 16^2}{14 \times 16} (6x) + \frac{16^2 + 16^2}{16 \times 16} (18x^2 - 27x) \\
 &= \frac{208}{96} (6x) + \frac{260}{112} (6x) + \frac{340}{168} (6x) + 2(3x) + \frac{452}{224} (6x) + 2(18x^2 - 27x) \\
 &= \frac{1008x^2 + 89x}{28}
 \end{aligned}$$

2.6 | Rhombus Oxide network

An x dimensional Rhombus Oxide Network denoted by $(RHOX_x)$. Number of vertices (nodes) and edges respectively $|V(RHOX_x)| = 3x^2 + 2x$ and $|E(RHOX_x)| = 6x^2$. Figure12 shows a graph representation of $RHOX_x$ based on the neighborhood and degree of the SDD index.

Theorem 11 Let G be a Rhombus Oxide of graph $(RHOX_x)$, $x \geq 2$, $SDD(G) = 12x^2 + 4x - 2$.

Proof Let us consider the graph G has Rhombus Oxide network of x -dimension $(RHOX_x)$. The partitions of vertex set and edge set of $RHOX_x$ with respect to degree of end vertices. There are two types of vertex set in $RHOX_x$. The cardinality of the V_1 vertex set is $4x$ of degree 2 and the cardinality of the V_2 vertex set is $3x^2 - 2x$ of degree 4. So, $|V(G)| = |V_1| + |V_2| = 3x^2 + 2x$. There are three types of edges in G based on the degree of end vertices. We have, $E_1(2, 2) = \{pq \in E(G) \mid d_p = 2, d_q = 2\}$, $E_2(2, 4) = \{pq \in E(G) \mid d_p = 2, d_q = 4\}$ and $E_3(4, 4) = \{pq \in E(G) \mid d_p = 4, d_q = 4\}$. where

$$\begin{aligned}
 |E(G)| &= |E_1| + |E_2| + |E_3| \\
 &= 2 + 4(2x - 1) + 6x^2 - 8x + 2 \\
 &= 6x^2.
 \end{aligned}$$

$$\begin{aligned}
SDD(G) &= \sum_{p \sim q} \left(\frac{\max(d_p, d_q)}{\min(d_p, d_q)} + \frac{\min(d_p, d_q)}{\max(d_p, d_q)} \right) \\
&= (2)(2) + 4(2x - 1) \left(\frac{10}{4} \right) + (6x^2 - 8x + 2)(2) \\
&= 12x^2 + 4x - 2.
\end{aligned}$$

Theorem 12 Let G be a Rhombus Oxide of graph (OX_x) , $x \geq 3$, $ND_5(G) = \frac{84x^2 + 19x - 9}{7}$.

Proof Let us consider the graph G has Oxide network x -dimension $(RHOX_x)$. There are two types of the vertex set of $RHOX_x$. The cardinality of vertex set is $3x^2 + 2x$. There are eight types of edges in G based on degree sum of the neighborhood of end vertices. We have, $E_1(2, 2) = \{pq \in E(G) \mid \delta_p = 6, \delta_q = 6 \text{ where, } d_p = 2, d_q = 2\}$, $E_2(2, 4) = \{pq \in E(G) \mid \delta_p = 6, \delta_q = 12 \text{ where, } d_p = 2, d_q = 4\}$, $E_3(2, 4) = \{pq \in E(G) \mid \delta_p = 8, \delta_q = 12 \text{ where, } d_p = 2, d_q = 4\}$, $E_4(2, 4) = \{pq \in E(G) \mid \delta_p = 8, \delta_q = 14 \text{ where, } d_p = 2, d_q = 4\}$, $E_5(4, 4) = \{pq \in E(G) \mid \delta_p = 12, \delta_q = 14 \text{ where, } d_p = 4, d_q = 4\}$, $E_6(4, 4) = \{pq \in E(G) \mid \delta_p = 14, \delta_q = 14 \text{ where, } d_p = 4, d_q = 4\}$, $E_7(4, 4) = \{pq \in E(G) \mid \delta_p = 14, \delta_q = 16 \text{ where, } d_p = 4, d_q = 4\}$ and $E_8(4, 4) = \{pq \in E(G) \mid \delta_p = 16, \delta_q = 16 \text{ where, } d_p = 4, d_q = 4\}$. where

$$\begin{aligned}
|E(G)| &= |E_1| + |E_2| + |E_3| + |E_4| + |E_5| + |E_6| \\
&= 2 + 4 + 4 + 4(2x - 3) + 8 + 2(4x - 7) + 8(x - 2) + 6(x - 2)^2 \\
&= 6x^2
\end{aligned}$$

$$\begin{aligned}
ND_5(G) &= \sum_{p \sim q} \frac{\delta_N(p^2) + \delta_N(q^2)}{\delta_N(p)\delta_N(q)} \\
&= \frac{6^2 + 6^2}{6 \times 6} (2) + \frac{6^2 + 12^2}{6 \times 12} (4) + \frac{8^2 + 12^2}{8 \times 12} (4) \\
&\quad + \frac{8^2 + 14^2}{8 \times 14} 4(2x - 3) + \frac{12^2 + 14^2}{12 \times 14} (8) + \frac{14^2 + 14^2}{14 \times 14} 2(4x - 7) \\
&\quad + \frac{14^2 + 16^2}{14 \times 16} 8(x - 2) + \frac{16^2 + 16^2}{16 \times 16} 6(x - 2)^2 \\
&= 4 + 10 + \frac{26}{3} + \frac{65}{7} (2x - 3) + \frac{340}{21} + 4(4x - 7) + \frac{113}{7} (x - 2) + 12(x - 2)^2 \\
&= \frac{84x^2 + 19x - 9}{7}
\end{aligned}$$

2.7 | Regular Triangulate Oxide Network

Let $RTOX_n$ be the group of Regular Triangulate Oxide network for $x \geq 3$. The number of vertices(nodes) and edges $\frac{3x^2 + 9x + 2}{2}$ and $3x^2 + 6x$ respectively. Figure13 illustrates the graph representation of Regular Triangulate Oxide depending on the degree and neighborhood of the SDD index.

Theorem 13 Let G be a Regular Triangulate Oxide Network of graph $(RTOX_x)$, $x \geq 3$, $SDD(G) = 6x^2 + 15x$.

Proof Let us consider the graph G has Rhombus Oxide network of x -dimension $(RHOX_x)$. The partitions of vertex set and edge set of $RHOX_x$ with respect to degree of end vertices. There are two types of vertex set in $RHOX_x$. So

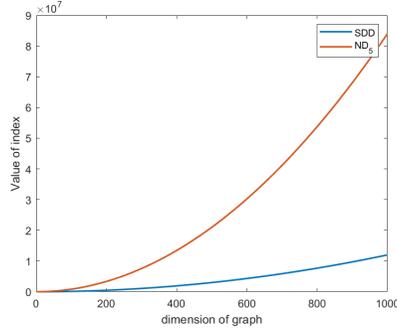


FIGURE 12 SDD and ND_5 of Rhombus Oxide

, $|V(G)| = |V_1| + |V_2| = 3x^2 + 6x$. There are three types of edges in G based on the degree of end vertices. We have $E_1(2,2) = \{pq \in E(G) \mid d_p = 2, d_q = 2\}$, $E_2(2,4) = \{pq \in E(G) \mid d_p = 2, d_q = 4\}$ and $E_3(4,4) = \{pq \in E(G) \mid d_p = 4, d_q = 4\}$. Where

$$\begin{aligned} |E(G)| &= |E_1| + |E_2| + |E_3| \\ &= 2 + 6x + (3x^2 - 2) \\ &= 3x^2 + 6x \end{aligned}$$

$$\begin{aligned} SDD(G) &= \sum_{p \sim q} \left(\frac{\max(d_p, d_q)}{\min(d_p, d_q)} + \frac{\min(d_p, d_q)}{\max(d_p, d_q)} \right) \\ &= 2(2) + \left(\frac{10}{4} \right) (6x) + 2(3x^2 - 2) \\ &= 4 + 15x + 6x^2 - 4 \\ &= 6x^2 + 15x \end{aligned}$$

Theorem 14 Let G be a Regular Triangulate Oxide Network of graph (OX_x) , $x \geq 3$, $ND_5(G) = \frac{504x^2 + 1179x + 2}{84}$

Proof Let us consider the graph G has Oxide network x -dimension $(RTOX_x)$. There are two types of vertex set of $RTOX_x$. The cardinality of vertex set is $3x^2 + 6x$. There are nine types of edges in G based on degree sum of the neighborhood of end vertices. We have, $E_1(2,2) = \{pq \in E(G) \mid \delta_p = 6, \delta_q = 6 \text{ where, } d_p = 2, d_q = 2\}$, $E_2(2,4) = \{pq \in E(G) \mid \delta_p = 6, \delta_q = 12 \text{ where, } d_p = 2, d_q = 4\}$, $E_3(2,4) = \{pq \in E(G) \mid \delta_p = 8, \delta_q = 12 \text{ where, } d_p = 4, d_q = 4\}$, $E_4(2,4) = \{pq \in E(G) \mid \delta_p = 8, \delta_q = 14 \text{ where, } d_p = 4, d_q = 4\}$, $E_5(4,4) = \{pq \in E(G) \mid \delta_p = 12, \delta_q = 12 \text{ where, } d_p = 4, d_q = 4\}$, $E_6(4,4) = \{pq \in E(G) \mid \delta_p = 12, \delta_q = 14 \text{ where, } d_p = 4, d_q = 4\}$, $E_7(4,4) = \{pq \in E(G) \mid \delta_p = 14, \delta_q = 14 \text{ where, } d_p = 4, d_q = 4\}$, $E_8(4,4) = \{pq \in E(G) \mid \delta_p = 14, \delta_q = 16 \text{ where, } d_p = 4, d_q = 4\}$ and

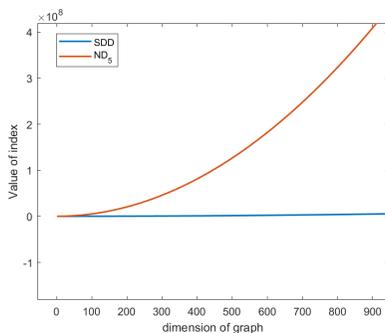


FIGURE 13 SDD and ND_5 of Regular Triangulate Oxide

$E_9(4,4) = \{pq \in E(G) \mid \delta_p = 16, \delta_q = 16 \text{ where, } d_p = 4, d_q = 4\}$. Where

$$\begin{aligned} |E(G)| &= |E_1| + |E_2| + |E_3| + |E_4| + |E_5| + |E_6| + |E_7| + |E_8| + |E_9| \\ &= 2 + 4 + 4 + (6x - 8) + 1 + 6 + (6x - 9) + (6x - 12) + (3x^2 - 12x + 12) \\ &= 3x^2 + 6x \end{aligned}$$

$$\begin{aligned} ND_5(G) &= \sum_{p \sim q} \frac{\delta_N(p^2) + \delta_N(q^2)}{\delta_N(p)\delta_N(q)} \\ &= \frac{6^2 + 6^2}{6 \times 6} (2) + \frac{6^2 + 12^2}{6 \times 12} (4) + \frac{8^2 + 12^2}{8 \times 12} (4) \\ &\quad + \frac{8^2 + 14^2}{8 \times 14} (6x - 8) + \frac{12^2 + 12^2}{12 \times 12} (1) \\ &\quad + \frac{12^2 + 14^2}{12 \times 14} (6) + \frac{14^2 + 14^2}{14 \times 14} (6x - 9) \\ &\quad + \frac{14^2 + 16^2}{14 \times 16} (6x - 12) + \frac{16^2 + 16^2}{16 \times 16} (3x^2 - 12x + 12) \\ &= \frac{504x^2 + 1179x + 2}{84} \end{aligned}$$

2.8 | The degree and neighborhood version of Symmetric division deg index of Copper (II) Oxide Network

In this section [26], we acquire the Symmetric division deg index for Copper (II) oxide. The octagons are connected to one another in columns and rows, in the CuO structure. The association between two octagons is accomplished by creating one C4 bond between two octagons. It has $4xy + 3y + x$ vertices (nodes) and $6xy + 2y$ edges, where x and y represent the number of octagons in rows and columns, respectively [27].

Theorem 15 Let G be a Copper (II) Oxide network of graph for $x, y > 2$, then $SDD(G) = \frac{38xy+x+16y-7}{3}$.

Proof There are four types of edges in G on the bases of different degree of end vertices. We have $E_1(2,2) = \{pq \in E(G) \mid d_p = 2, d_q = 2\}$, $E_2(2,4) = \{pq \in E(G) \mid d_p = 2, d_q = 4\}$, $E_3(3,4) = \{pq \in E(G) \mid d_p = 3, d_q = 4\}$ and

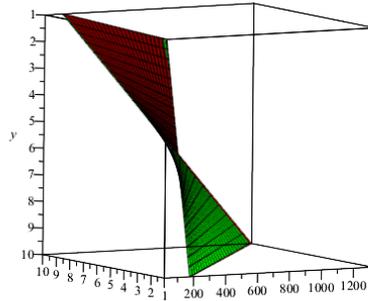


FIGURE 14 SDD and ND_5 of Copper(II) Oxide

$E_4(2,3) = \{pq \in E(G) \mid d_p = 2, d_q = 3\}$ where

$$\begin{aligned} |E(G)| &= |E_1| + |E_2| + |E_3| + |E_4| \\ &= 4(y+1) + 4(y-1) + \\ &4(xy - x - y + 1) + 2(xy + 2x - y - 2) \\ &= 6xy + 2y \end{aligned}$$

$$\begin{aligned} SDD(G) &= \sum_{p \sim q} \left(\frac{\max(d_p, d_q)}{\min(d_p, d_q)} + \frac{\min(d_p, d_q)}{\max(d_p, d_q)} \right) \\ &= 4(y+1)(2) + 4(y-1) \left(\frac{10}{4} \right) + 2(xy + 2x - y - 2) \left(\frac{13}{6} \right) + 4(xy - x - y + 1) \left(\frac{25}{12} \right) \\ &= \frac{38xy + x + 16y - 7}{3} \end{aligned}$$

Theorem 16 Let G be a Copper (II) Oxide network of graph for $x, y > 2$, then $ND_5(G) = \frac{380xy - 20x + 148y - 18}{30}$.

Proof There are nine types of edges in G and then the neighborhood on the bases of different degrees of end vertices. We have, $E_1(2,2) = \{pq \in E(G) \mid \delta_p = 4, \delta_q = 4 \text{ where, } d_p = 2, d_q = 2\}$, $E_2(2,2) = \{pq \in E(G) \mid \delta_p = 4, \delta_q = 5 \text{ where, } d_p = 2, d_q = 2\}$, $E_3(2,2) = \{pq \in E(G) \mid \delta_p = 4, \delta_q = 6 \text{ where, } d_p = 2, d_q = 2\}$, $E_4(2,3) = \{pq \in E(G) \mid \delta_p = 5, \delta_q = 6 \text{ where, } d_p = 2, d_q = 3\}$, $E_5(2,3) = \{pq \in E(G) \mid \delta_p = 6, \delta_q = 6 \text{ where, } d_p = 2, d_q = 3\}$, $E_6(2,3) = \{pq \in E(G) \mid \delta_p = 6, \delta_q = 10 \text{ where, } d_p = 2, d_q = 3\}$, $E_7(2,4) = \{pq \in E(G) \mid \delta_p = 6, \delta_q = 10 \text{ where, } d_p = 2, d_q = 4\}$, $E_8(3,4) = \{pq \in E(G) \mid \delta_p = 10, \delta_q = 12 \text{ where, } d_p = 3, d_q = 4\}$ and $E_9(3,4) = \{pq \in E(G) \mid \delta_p = 10, \delta_q = 10 \text{ where, } d_p = 3, d_q = 4\}$.

The following Figure14 shows SDD and ND_5 of Copper (II) Oxide. Green and red indicate SDD and ND_5 respectively, which shows there is no physical properties significant difference between Copper (II) Oxide. Where

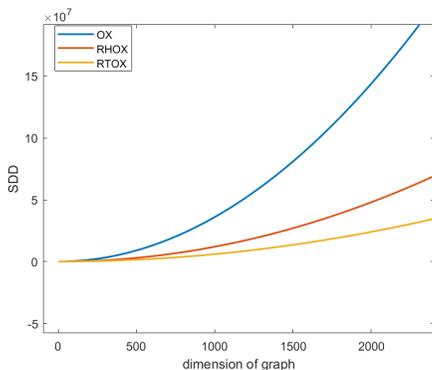
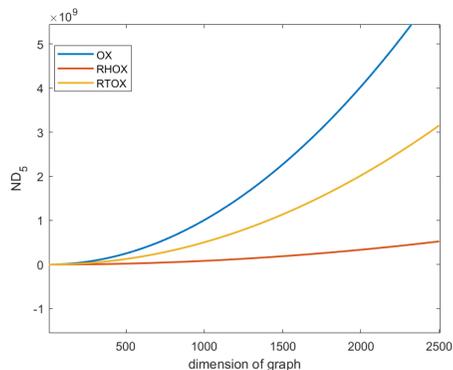


FIGURE 15 SDD

FIGURE 16 ND_5

$$\begin{aligned}
 |E(G)| &= |E_1| + |E_2| + |E_3| + |E_4| + |E_5| + |E_6| + |E_7| + |E_8| + |E_9| \\
 &= 4 + 4 + (4y - 4) + 4 + (6x - 10) + (2xy - 2x + 2y) + \\
 &\quad (2xy - 2x + 2y) + (4xy - 4x - 8y + 8) + (4x - 4) \\
 &= 6xy + 2y
 \end{aligned}$$

$$\begin{aligned}
 ND_5(G) &= \sum_{p=q} \frac{\delta_N(p^2) + \delta_N(q^2)}{\delta_N(p)\delta_N(q)} \\
 &= \frac{4^2 + 4^2}{4 \times 4} (4) + \frac{4^2 + 5^2}{4 \times 5} (4) + \frac{4^2 + 6^2}{4 \times 6} (4y - 4) \\
 &\quad + \frac{5^2 + 6^2}{5 \times 6} (4) + \frac{6^2 + 6^2}{6 \times 6} (6x - 10) + \frac{6^2 + 10^2}{6 \times 10} (2xy - 2x + 2y - 2) \\
 &\quad + \frac{10^2 + 10^2}{10 \times 10} (4x - 4) + \frac{10^2 + 12^2}{10 \times 12} (4xy - 4x - 8y + 8) \\
 &= \frac{380xy - 20x + 148y - 18}{30}
 \end{aligned}$$

3 | COMPARISON AND CONCLUSION

In Figure15,16,17 and 18 shows the graphs of degree and neighborhood of SDD index for some structures of Silicate network and Oxide network, where x - axis represents the dimension of a graph and y - axis represents the values of the SDD and ND_5 index respectively.

In Figure15 and 16 we have compared CS_x , DC_x , $RHSL_x$ and $RTSL_x$ based on the degree and neighborhood SDD index so the frequency curve for the structures $RHSL_x$ and $RTSL_x$ are increasing, so we conclude that these two silicate networks ($RHSL_x$, $RTSL_x$) obey with the physical properties (boiling points, melting points, molar value, etc.)

In Figure17 and 18 we have made a comparison between OX , $RTOX_x$ and $RHOX_x$ based on the degree and neighborhood SDD index. The frequency curve of OX is increasing slowly, which means that OX will obey the physical property like (boiling point, melting point, molar value, etc.). Prior to $RTOX_x$ and $RHOX_x$.

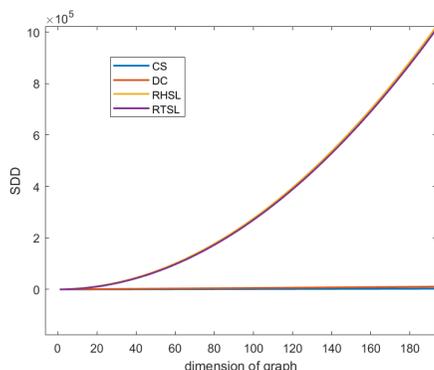


FIGURE 17 SDD

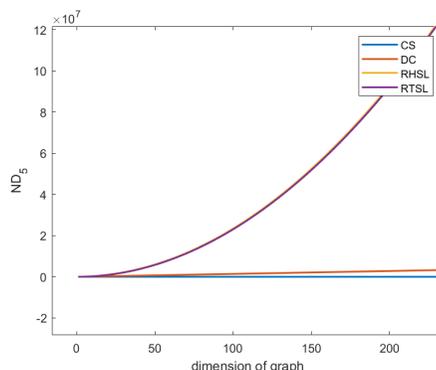


FIGURE 18 ND_5

From these, it is observed that when the dimension is maximum Silicate and Oxide will obey the physical properties. Also if the dimension curve is not increasing (linear) those particular Silicate and Oxide networks won't obey the physical properties of those networks.

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Conflict of Interest

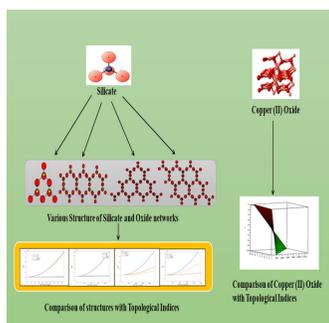
The authors declare that they have no conflict of interest.

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GRAPHICAL ABSTRACT



The physical properties of Silicate, Oxide, and Copper (II) Oxide networks are examined in this research utilizing topological indices.