Lyapunov-Based PID controller design for Buck converter with Real-time Implementation

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Abstract

This paper presents a Lyapunov-based model reference controller for Buck converter operating under harmful disturbances. This strategy is an advanced version of the tuning method utilising Lyapunov stability function to reach a higher stability and a better disturbance rejection behavior in the practical applications. In addition, to reduce the computational burden and increase ease of implantation, Black-box technique is considered assuming no accurate mathematical model for the system. Lyapunov stability theory is used to enhance and tune the PID gain of this method with an adaptive mechanism while taking into account the real-time condition of a converter with regular changes. For this gain-based controller, an adaptive mechanism based on the Lyapunov concept is proposed, which can improve the stability and resilience of the system under various disturbances, particularly noise. Moreover, a Fuzzy-based PID controller and a PID technique with Particle swarm Optimization (PSO) algorithm are designed and compared with the presented Lyapunov-based technique. MATLAB\Simulink is utilized to examine the results of this improved controller. Additionally, this method produces better results in real-time contexts with faster dynamics and better frequency adaptation.

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Abstract: This paper presents a Lyapunov-based model reference controller for Buck converter operating under harmful disturbances. This strategy is an advanced version of the tuning method utilising Lyapunov stability function to reach a higher stability and a better disturbance rejection behavior in the practical applications. In addition, to reduce the computational burden and increase ease of implantation, Black-box technique is considered assuming no accurate mathematical model for the system. Lyapunov stability theory is used to enhance and tune the PID gain of this method with an adaptive mechanism while taking into account the real-time condition of a converter with regular changes. For this gain-based controller, an adaptive mechanism based on the Lyapunov concept is proposed, which can improve the stability and resilience of the system under various disturbances, particularly noise. Moreover, a Fuzzy-based PID controller and a PID technique with Particle swarm Optimization (PSO) algorithm are designed and compared with the presented Lyapunov-based technique. MATLAB\Simulink is utilized to examine the results of this improved controller. Additionally, this method produces better results in real-time contexts with faster dynamics and better frequency adaptation.

Keywords: Lyapunov-based strategy, PID controller, Buck converter, Disturbance

1 Introduction

In order to meet the growing need for renewable energy sources, DC/DC Power converters are now a crucial component. Here, a step-down-capable Buck converter has been chosen as the topology and is utilised in a variety of processes, including the motor and battery charging processes. Additionally, it benefits from fewer electrical components, greater efficiency, and a lighter, more affordable structure. Utilising various Converters primarily serves to regulate output voltage and ensure a strong dynamic performance [1, 2]. Nonetheless, it exhibits a discontinuous behavior under high-frequency switching conditions, and in real-time usage, some fractionality and formerly unheard-of nonlinearity can be seen [3–5]. Different feedback controllers that can enhance the converters' performance in various settings and control the output voltage have been presented. Additionally, the open-loop bandwidth is physically constrained due to the difficult nature of dynamics, which limits the advantages of the designed control method. This is demonstrated by the tracking control issue, where the control schemes can precisely follow the reference value. Moreover, the internal stability problem refers to the possibility of unstable system states.

Regarding its simplicity, low cost, and adequate efficiency, the Proportional-Integrated-Derivative (PID) scheme is the most widely utilized for power converters [6, 7], whereas providing a pure regulation and tracking of the reference signal was the goal, various techniques have been observed to perform as intended. These approaches are realistically unattractive because to their weakness against parametric uncertainty and a variety of disturbances. The stability of the method must also be guaranteed under parametric fluctuations, and the control parameters must once again be optimized while taking disturbances into account [8, 9]. It is also a difficult problem to ensure the technique's stability under parametric variations and to optimise the control settings. To keep the simplicity of these controllers and improve their dynamics, adaptive mechanism have been used for PID-Based techniques for higher efficiency. Some of the main techniques used to optimize the PID-based controllers are as follows: Sliding-Mode PID scheme [10, 11], Fuzzy-PID technique [12–14] , Fractional-order PID controllers [15, 16] , and Neural PID techniques [17, 18]. All of these methods exhibit improved control, greater robustness against disruptions and uncertainty, and better acclimatisation to the unpleasant conditions. However, given that these techniques affect the system, there were still some difficult problems. Sliding Mode Control is an advantageous substitute with improved frequency matching behavior and well-behaved dynamics in practical applications, but it has chattering problem and risky overshoot. Meanwhile, both Fuzzy and Neural-based strategies apply a high computational burden and higher complexity on the controller that can negatively effect the performance of the structure, particularly in industrial environments. Also, FO-PID controllers benefited from higher robustness and simpler technique, but they need more advanced optimization algorithms to reach more suitable gains that can be time-consuming; also, they are not suitable in noisy situations. Lyapunov theory is recommended to optimise the traditional PID controller in order to address these problems. the methodical, sequential, and recursive Easily implementable Control Lyapunov-Function (CLF)-based control scheme framework that ensures asymptotic error convergence in each of the subsystems [19, 20]. More interestingly, it is a good control theory since it can successfully reject both matched and mismatched uncertainty with linear parameters. Furthermore, in order to increase the efficiency of the controller with less complexity, CLF has been applied to a variety of power converter topologies in both non-adaptive and adaptive schemes. [21]. These studies exhibit less tracking error and transitory overshoot with quick responses, but they are more noise-dependent. The strength of the suggested technique is its increased robustness against interruptions in dealing with these problems.

The new aspect of the approach described in this research is creating a reliable Lyapunove-based PID controller for the Buck converter. This controller benefited from an online adaptive tuning structure that was optimized using the Lyapunov concept. Additionally, the contributions made by this technique are:

- Better regulation and lower sensitivity to error and disturbance is reported.
- Adaptive mechanism is designed using Lyapunov stability concept for more challenging conditions.
- Lower computational burden and complexity is provided with robust structure.
- This control technique is assured to be robust in a variety of difficult situations, notably those involving supply voltage and parametric variation.

2 Mathematical Model of Buck Converter

Fig .1 illustrates the general structure of the Buck converter used in step-down modes. Additionally, Table 1 lists the elements of it along with their definitions and values [22].
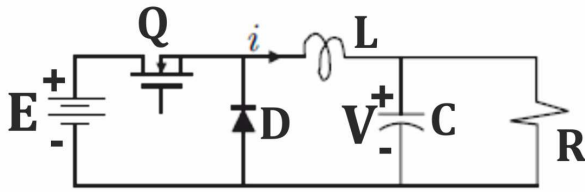
Fig. 1: Buck converter

and i_L are state variables used to model the system based on the switching function of u and Eq.1 demonstrates the switching equations considering Fig.2.

$$L \ (d_{il})/d_t = E - (1-u)V_c$$
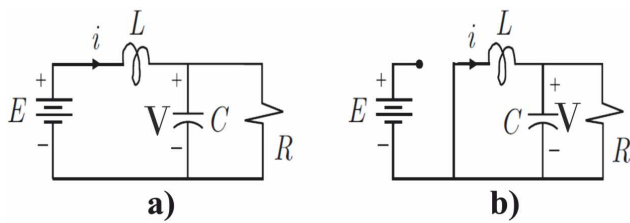$$C(d_{Vc})/d_t = (1-u)(d_{il})/d_t - V_c/R \tag{1}$$

Fig. 2: Modes of operation for Buck converters.

The switching equations of (1), regarding transfer functions , lead to the state-space matrices of the system (Eq. 2). Table 1 includes a list of the components' quantities.

$$A = \begin{bmatrix} 0 & \dfrac{-1}{L} \\ \dfrac{1}{C} & \dfrac{-1}{RC} \end{bmatrix} \quad , \quad B = \begin{bmatrix} \dfrac{E}{L} \\ 0 \end{bmatrix} \quad , \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix} \tag{2}$$

$$G(s) = \frac{E/LC}{s^2 + (1/RC)s + 1/LC}$$

Table 1 List of components

Components	Definitions	Amounts
E	Supply Voltage	10 - 20V
V_{out}	Output voltage	5 - 18V
P_{out}	Output power	10 - 20W
R	Resistor (Load)	10Ω
C	Capacitor	$100\mu F$
L	Inductor	$2\mu H$

3 Design Of The Control Strategies

3.1 Model Reference adaptive control (MRAC)

Typically, an MRAC scheme shows the ideal plant's behavior regarding reference model. By using Error-driven feedback from the difference between reference model output and plant output, the controller's parameters are discovered [23, 24]. The description of a Lyapunov-based MRAC that locates the best-adjusted parameters for the PID controller is covered in the next section.

3.1.1 Lyapunov-based PID Structure: According to the Buck converter system, a second order reference model is required, and the gains of the forward and feedback channels are assisted by the Lyapunov stability theory to produce an adaptive adjustment [25]. The schematic block of this method is in Fig.3.
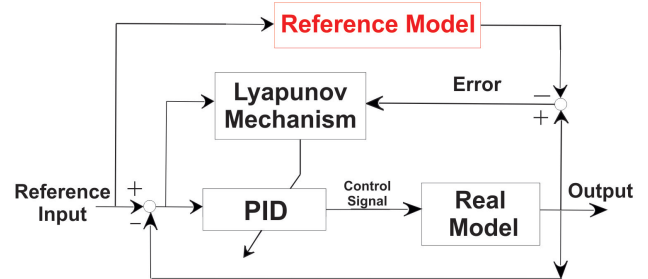
Fig. 3: Adaptation related to Lypunov-based PID strategy.

The plant equation is written here:

$$\dot{x} = Ax + Bu \quad , \quad x \in \mathfrak{R}^n \tag{3}$$

where, A and B are controllable polynomials. Equation (4) is illustrated as the reference model.

$$\dot{x}_m = A_m x_m + B_m u_c \quad , \quad x \in \mathfrak{R}^n \tag{4}$$

After assigning real model and reference model, the control law is shown as follows

$$u = -K^* x + L^* u_c \tag{5}$$

Furthermore, Eq.6 depicts the closed-loop system.

$$\dot{x} = (A - BK^*)x + BL^* u_c \tag{6}$$

Applying the model matching condition

$$A - BK^* = A_m, BL^* = B_m \tag{7}$$

The resulting control law reaches as

$$u = -K(t)x + u_c L(t) \tag{8}$$

In Eq.8, the coefficiencies of $L(t), K(t)$ are the estimated values of K^*, L^*. The tracking error is $e = x - x_m$ along with the approximated value of $\dot{e} = A_m e + B_m(-\tilde{K}x + \tilde{L}u_c)$. Next, the Lyapunov

function will be selected:

$$V(e, \widetilde{K}, \widetilde{L}) = e^T P e + tr(\widetilde{K}^T \Gamma^{-1} \widetilde{K} + \widetilde{L}^T \Gamma^{-1} \widetilde{L}) \qquad (9)$$

where, $\widetilde{L} \cong L - L^*, \widetilde{K} \cong K - K^*$.'P' satisfies the Lyapunov equation

$$\begin{aligned}
&PA_m + A_m^T P = -Q \\
&\dot{V}(e, \widetilde{K}, \widetilde{L}) = e^T(PA_m + A_m^T P)e + \\
&B_m(-\widetilde{K}x + \widetilde{L}u_c) + B_m(-\widetilde{K}x + \widetilde{L}u_c)^T Pe + \\
&2tr(\widetilde{K}^T \Gamma^{-1}\dot{\widetilde{K}} + \widetilde{L}^T \Gamma^{-1}\dot{\widetilde{L}})
\end{aligned} \qquad (10)$$

However, one can rewrite (10) as below

$$\begin{aligned}
&\dot{V}(e, \widetilde{K}, \widetilde{L}) = e^T(-Q)e + 2(-\widetilde{K}x + \widetilde{L}u_c)^T B_m^T Pe + \\
&2tr(\widetilde{K}^T \Gamma^{-1}\dot{\widetilde{K}} + \widetilde{L}^T \Gamma^{-1}\dot{\widetilde{L}})
\end{aligned} \qquad (11)$$

To make $\dot{V} = -e^T Q e$ (negative definite)

$$\dot{\widetilde{K}} = \dot{K} = \Gamma x^T B_m^T Pe \quad , \quad \dot{\widetilde{K}} = \dot{L} = -\Gamma u_c^T B_m^T Pe \qquad (12)$$

Variable x_p is actually the plant state x and Γ is the adaptation gain matrix, which is positive definite.

3.2 Comparison With Fuzzy-PID Controller

This fuzzy-based PID strategy includes three gains that are optimized
with fuzzy logic: differential, proportional, and integral gains. The principle of this platform is based on a fuzzy variable. The load voltage error on the topology is addressed as the fuzzy variable that needs to be defined by linguistic variables. Fuzzy output is the result of the implication and aggregation step validated by the union of all the individual rules. The centre of area (COA) is brought back to being beneath the curve using the centroid calculation, which is the defuzzification technique. Additionally, the COA method has been applied to achieve this goal as follows:

$$Output = \frac{\displaystyle\sum_{i=1}^{n} Z_i \mu_{out}(Z_i)}{\displaystyle\sum_{i=1}^{n} \mu_{out}(Z_i)} \qquad (13)$$

where $\mu_{out}(Z_i); i = 1, 2, ..., n$ are the sampled values of the aggregated output membership function and 'Output' is the crisp value that describes the duty command. Figs.4 and 5 show the general flow chart and fuzzy memberships in this technique.

3.3 PSO Algorithm

Nowadays, optimising algorithms have been utilised to get the controllers' best gains. The Particle Swarm Optimisation (PSO) approach is a well-known evolutionary computing technology with an effective algorithm. It is a manipulating programmer that makes use of evolutionary operatives and was created based on the behavior of swarms. For the d-dimensional search space, a particles flock is recommended as a solution to the d-variable optimization problem. With prior knowledge of the particles, the best gains may then be controlled. The scheme of determining gains using this method is shown in Fig. 9 [19].

4 Result of Simulations

The results of the Lyapunov-PID controller in various situations depending on the circuit values specified in Table 1 are discussed
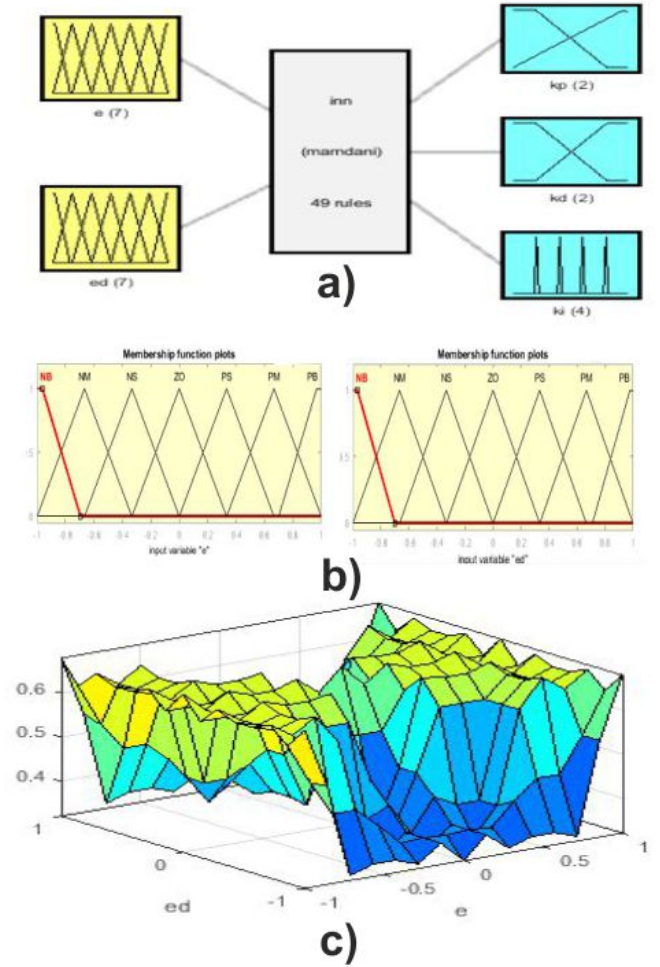

Fig. 4: Process of Fuzzy-based PID scheme; a) schematic diagram, b) membership functions, and c) approximated surface of the technique.
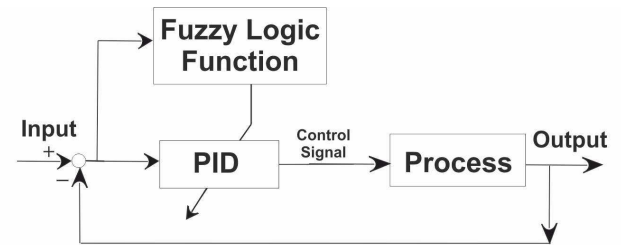

Fig. 5: Procedure of Fuzzy-based PID control strategy [19] .
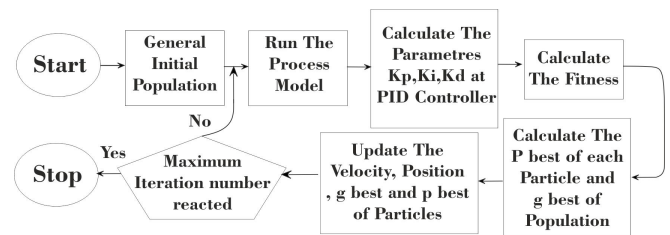

Fig. 6: Flow chart of PSO [6] .

in this section. In order to examine the regulation in both modes, the controllers' functionality will first be checked without any interruptions. The tracking signals of different control strategies for the Buck converter with a resistance load are shown in Fig. 7. ($R = 50\Omega$).
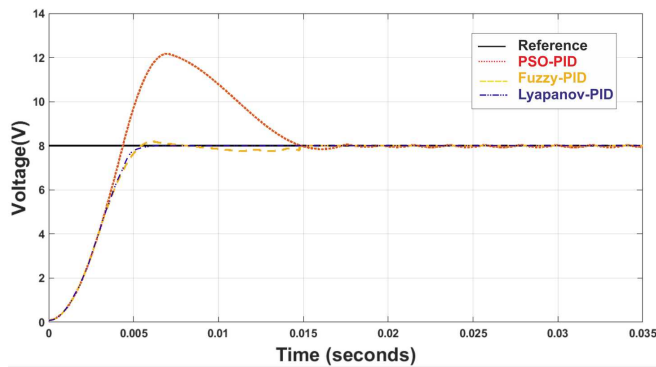

Fig. 7: Tracking responses of the controllers to the reference signal.

Different circumstances call for various output voltage levels at the load; as a result, reference voltage fluctuations must be used to assess the controllers' performance. In Fig. 8, the workings of the converter are tested with two adverse and favourable changes. First, a decrease in voltage from 8 to 5 volts has been applied, followed by an increase in voltage from 5 to 9 volts. It should be highlighted that because the controllers' levels are difficult to correct for, these reference changes are highly difficult alterations.
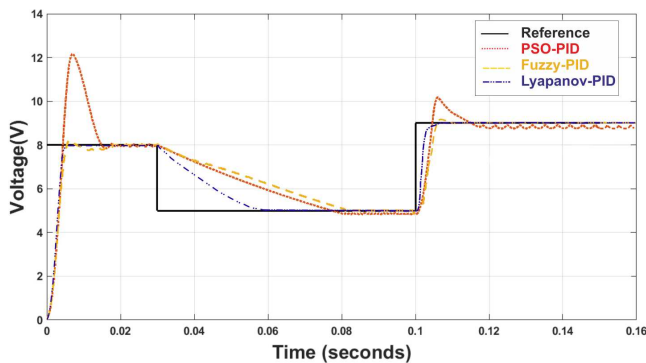

Fig. 8: Tracking performance of the controllers in reference voltage variations.

Observing Fig.8, the Ly-PID scheme has significant performance with faster dynamics. The Fuzzy-based PID system shows more reliable behavior under the impact of this quick variation with better reactions, while the PSO-based PID technique can correct for the change over a longer period of time. The fundamental problem for controllers used with converters is parametric variation or parametric uncertainty, which can be caused by these structures' inadequate working conditions. Age and heat are two significant variables that might lead to deviation from the reference signal and for which the controllers must account. In Fig. 9, a sudden load change is induced to examine the impact of parametric variation on the controller's performance.

Figure 9 illustrates a sudden load variation with an amount of $R_{Load} = 50\Omega$ to 100Ω. When looking at Fig. 9, it is clear that the PSO-PID controllers is unable to account for this parametric fluctuation, but the Ly-PIDcontroller shows great performance against this disturbance since it can regularly tune the parameters. Additionally, the harmful nature of noise in industrial settings must be taken into account while building the controllers. The structure in Fig. 10 is subjected to noise in various ranges to test this disruption.
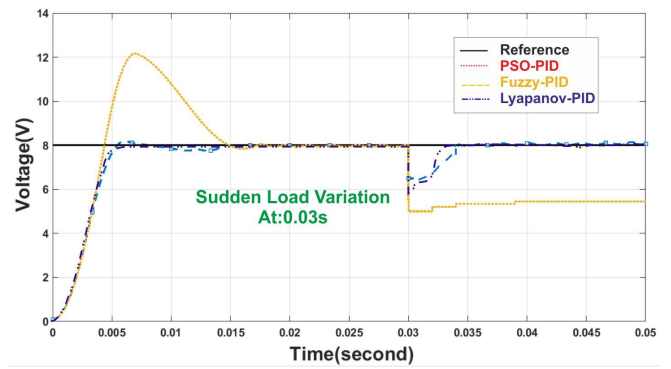

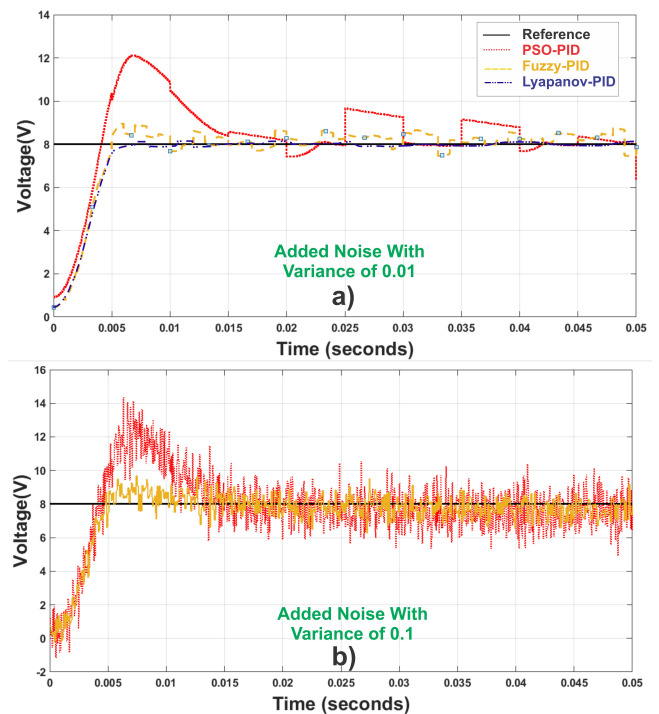**Fig. 9**: Tracking results of these controllers under sudden parametric variation.

Fig. 10: Tracking outcomes in noisy conditions, a) applied noise of 0.01 variance, b) applied noise of 0.1 variance.

Figure 10 shows how noise with variances of 0.01 and 0.1 affects the approaches. Both figures show that the PSO-PID controller is completely unable to deal with the effects of noise; nevertheless, the Fuzzy-PID technique shows a better regulation but is insufficient in noisy environments. Additionally, the Ly-PID controller demonstrates good robustness in handling this situation, reiterating its suitability as an option for industrial applications.

5 Experimental Results

The PWM approach has been used for the converter's topology to drive the switches. The control and power components of the Inverter, as well as the real-time hardware for the device, are shown in Fig. 11.

The Ly-PID controller's efficacy was tested using the prototype machine in Fig. 12. For this module, several parts and gadgets have been utilised. The primary component is the Arduino processor, which is in charge of converting and creating the control signal and incorporating sensors to create control loops. The Arduino board uses a DUE-based CPU to transform the observed voltage utilising
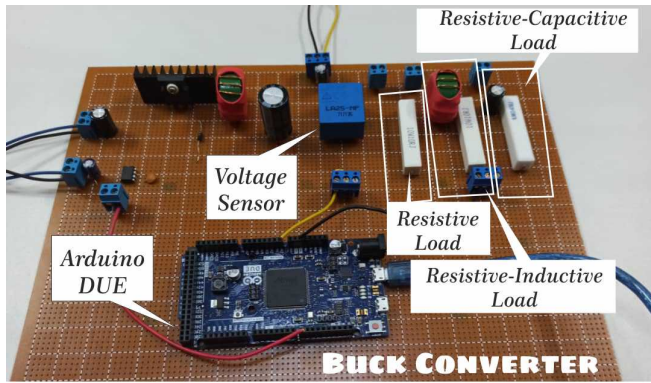
Fig. 11: Topology of the Buck converter.

sensors to digital data using analog-to-digital conversion. The generated average values are then inputs to the controllers. Additionally, the supply voltage value is chosen to be between 12V and 15V with the intention of controlling the output voltage between 5V and 11V. To assess the effectiveness of this structure in real-time scenarios, the Ly-PID controller continuously tracks the 5V and 8V references (Fig. 12).

Table 2 Real-time Components.

Components	Type
Voltage Sensor	LV25-P
Processor	Arduino (DUE)
Diodes	UF4007
Switches	IRF9630
MOSFET	TC4427
Switching Frequency	10kHz

There is no perfect working environment in the industrial settings where converters are utilised; yet, output load fluctuations or parametric variations may occur. To examine this condition, the load-type will be altered and replaced with other amounts with the controller connected to RC and RL loads. Fig.13 provides evidence that the proposed management technique can effectively regulate these fluctuations.

The system will next be put to the test by taking noise with 0.1 and 0.01 variances into account. If you look at Fig. 14, you can see that the detrimental effect of noise has been offset by a decent response. One can infer from Fig. 14 that a Ly-PID controller with a resilient structure can operate effectively when noise is applied, as shown in the simulation result of Fig. 10.

6 Conclusion

This study used a Buck converter to construct the Lyapunov-based PID controller using adaptive mechnism. Two additional contemporary controllers—fuzzy-based and PSO-based PID controllers—have also been developed for comparison with the suggested approach. Both adaptive approaches have been demonstrated to offer higher performance in terms of their adaptive dynamics, but the Lyapunov-based PID strategy is more resilient under a range of load situations. More importantly, though, the proposed strategy is a beneficial alternative against various disturbances, particularly in noisy cases and load uncertainties for real-time applications. It is suggested that thee conventional schemes are appropriate solutions for reference tracking, but to present a proper operation for power Converters against disturbances, improved control methods are needed reaching higher robustness. Because of its straight-forward structure and quick dynamics, the Lyapunov-based PID controller may satisfy this need in both simulation and experiment.

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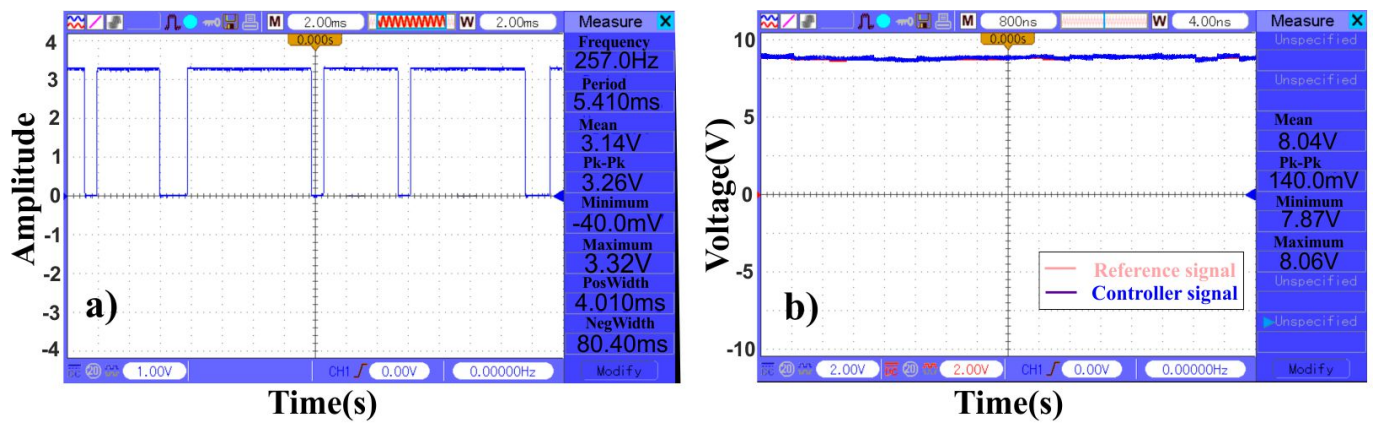
Fig. 12: The Ly-PID controller with reference of 8V .a) resulted PWM signal, b) tracking signal of the controller.
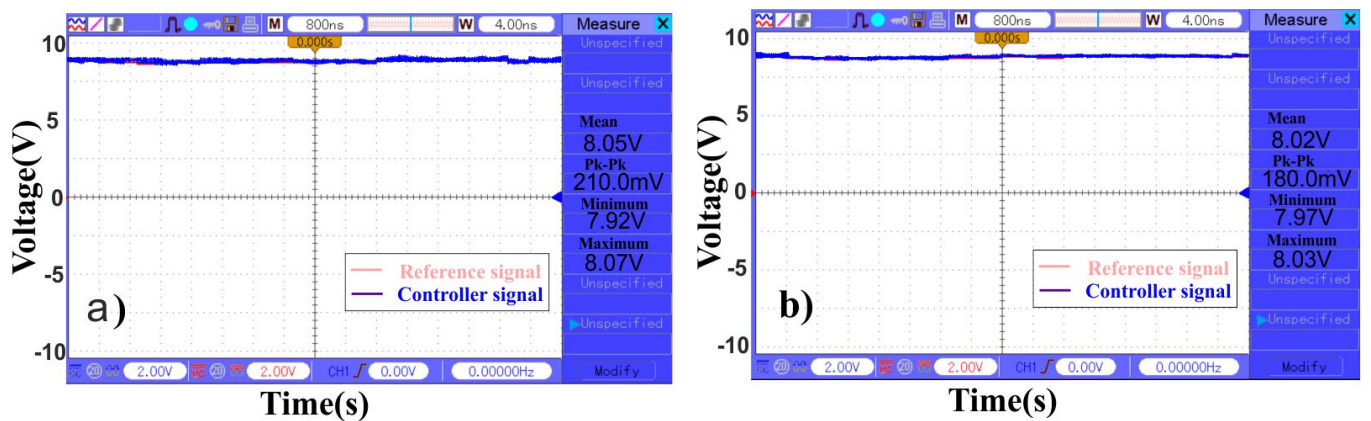

Fig. 13: The performance of the Ly-PID controller on Buck converter in the connection with different loads. a)tracking reference signal in resistive-inductive load ($R = 30\Omega, L = 10\mu H$, b) tracking reference signal by controller in resistive-capacitive load ($R = 50\Omega, C = 100\mu F$
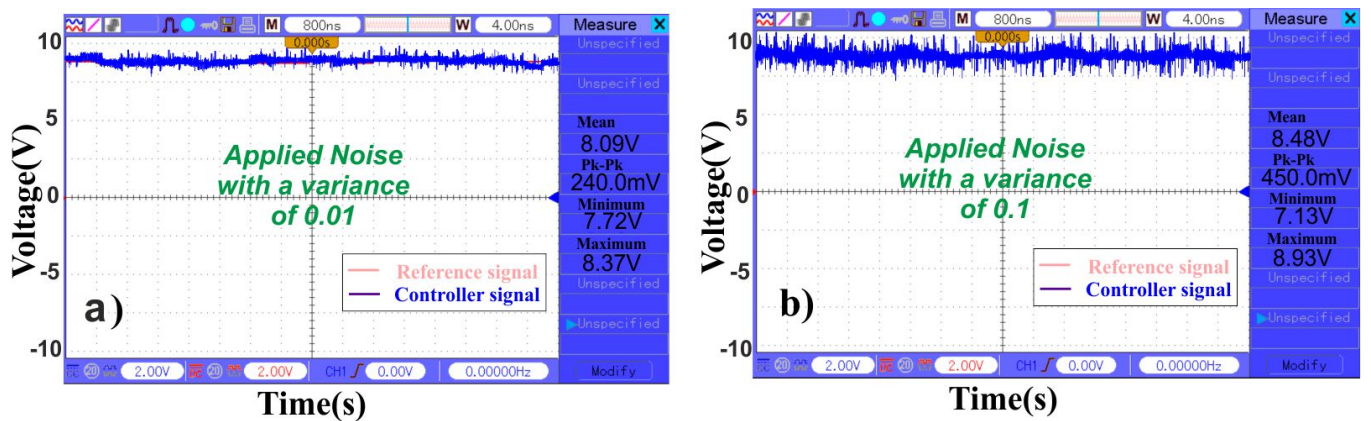

Fig. 14: the Lyapunov-based PID controller on Buck converter with applied noise ; a) applied noise with a variance of 0.01, b) applied noise with a variance of 0.1.