Prescribed-Time Fault-Tolerant Control of Uncertain Linear Time-Varying Systems by Smooth Linear Time-Varying Feedback

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Keywords: Global prescribed-time control, fault-tolerant, linear time-varying system, non-lexicographically fixed, uncertainties, elliptical orbital rendezvous system

1 Introduction

Linear time-varying (LTV) systems have always received widespread attention in the past few decades [1, 2, 3, 4]. It has many practical engineering applications, such as the nonholonomic car [5] and the magnetic torque spacecraft [6]. Compared to the control of time-invariant systems, the control of time-varying systems is a rather arduous and challenging task. We lack effective mathematical tools to handle LTV systems, and even so, there is still a significant amount of literature on the control of LTV systems [2, 3, 7, 8]. In the previous studies, researchers have primarily concentrated on the asymptotic stability of LTV systems, wherein the system state converges to zero after a substantial duration. However, practical applications often require faster convergence speeds.

Prescribed-time control has garnered considerable attention in recent years for its capability to set the convergence time arbitrarily and its robustness against unknown disturbances [9, 10, 11, 12, 13]. Firstly, concepts including finite-time, fixed-time, and prescribed-time convergence have been proposed. In the finite-time scheme, the state converges to zero within a defined time interval, rather than indefinitely. The convergence time, however, is related to the initial value [14, 15]. In cases where the upper bound of convergence time remains constant regardless of the initial value, the finite-time scheme transitions into the fixed-time scheme [16, 17]. In the prescribed-time scheme, the upper bound of the system's settling time is presumed, meaning that the convergence time is not only known but can also be arbitrarily specified as a parameter. In recent years, criteria and stabilization methods for prescribed-time control have been developed for a large number of systems, such as nonlinear systems [11, 18, 19] , linear time-invariant systems [20, 21], stochastic systems [10, 22], time-delay systems [9, 23] and multi-agent systems [24].

On the one hand, controlling LTV systems is quite difficult, and on the other hand, prescribed-time control can significantly improve system performance. Accordingly, achieving prescribed-time control for LTV sys-

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tems represents a significant challenge. However, to the author's knowledge, there is currently no literature to address this problem.

In this paper, the problem of prescribed-time control for LTV systems has been investigated. Firstly, uniformly controllable LTV systems are transformed into their controllable canonical forms. When the uniformly controllable system is lexicographically fixed, we will directly convert it. However, when the uniformly controllable system is non-lexicographically fixed, it should first be expanded into an augmented lexicographically fixed LTV system with the assistance of an auxiliary LTV. Subsequently, the augmented lexicographically fixed LTV system will be converted into the controllable canonical form. Secondly, on the basis of the obtained controllable canonical form, by employing the inherent properties of a certain type of parametric Lyapunov equations (PLEs) [25, 26, 27], a smooth linear time-varying high-gain state feedback controller is designed. Then, by choosing a suitable Lyapunov-like function, it will be demonstrated that the state approaches zero within a prescribed time. Additionally, the uncertainties and the faults in the actuator [28, 29] are also taken into account. Ultimately, our contribution to the design of prescribed-time fault-tolerant controller for uncertain LTV systems is manifold.

1. There is no need to impose slow changing constraints on the coefficient matrix or adopt approximate methods for LTV systems. In addition, the system's state transition matrix is not needed to be known. This paper only requires that the LTV system is uniformly controllable.
2. The global prescribed-time control for LTV systems is achieved. The uniformly prescribed-time input-to-state stability in sense of Definition 2 and the globally uniformly prescribed-time stability in sense of Definition 3 are considered. Moreover, the convergence time is independent of the initial condition. In addition, our controller maintains a linear, concise, smooth form, and it is bounded.
3. The designed controller is capable of fault tolerance and robustness. The prescribed-time stability of the LTV system can be maintained when the upper bound of the model uncertainties satisfies a specific condition.

Notation: Consider $\mathbf{R}^{k\times l}$ as sets of $k\times l$ matrices. $0_{p\times q}$ denotes the all-zero $p\times q$ matrices. $\mathbf{R}_{\geq t_a}$ refers to the interval $[t_a,\infty)$ and $\|\cdot\|$ denotes the Euclidean norm. $M\oplus N$ represents the matrix with diagonal elements being matrix M and matrix N . Let A^{T}, $\mathrm{rank}(A)$, $\sigma_{\max}(A)$, $\sigma_{\min}(A)$ denote, respectively, the transpose, rank, maximal and minimal eigenvalue of matrix A. $\mathcal{J}^k(\mathbf{Z},\mathbf{O})$, $\mathcal{J}^\infty(\mathbf{Z},\mathbf{O})$ and $\mathcal{S}^\infty(\mathbf{Z},\mathbf{O})$ refer to the set of k-times differentiable functions, smooth functions and bounded functions $f:\mathbf{Z}\to\mathbf{O}$, respectively. $\mathbf{N}^+$ represents the set of positive integers. $\rho_i^{[n]}$ represents a row vector with dimensions $1\times n$, where only the i-th element is 1, and the rest are 0. The phrase "unique positive definite solution" is abbreviated as "UPDS".

2 Preliminaries

Consider the following MIMO LTV system with actuator faults and uncertainties

$$\begin{cases} \dot{x}=(1+\varpi(t))A(t)x+B(t)((1-\eta(t))u+\delta(t)), \\ y=C(t)x, \quad t,t_0\in\mathbf{R}_{\geq 0}, \quad t\geq t_0, \end{cases} \tag{1}$$

where $x\in\mathbf{R}^n$ represents the state, $u\in\mathbf{R}^m$ denotes the input, $y\in\mathbf{R}^p$ signifies the output, $A(t)\in(\mathcal{J}^\infty,\mathcal{S}^\infty)(\mathbf{R}_{\geq 0},\mathbf{R}^{n\times n})$ and $B(t)\in\mathcal{J}^\infty(\mathbf{R}_{\geq 0},\mathbf{R}^{n\times m})$. $\varpi(t)$, $\eta(t)$ and $\delta(t)\in\mathbf{R}^m$ denote the model uncertainty, the fault gain and float fault, respectively.

In this paper, we make following assumptions on the LTV systems (1).

Assumption 1 *The model uncertainty $\varpi(t)$ is unknown but bounded, satisfying, $\|\varpi(t)\|\leq w_{\max}<\infty,\forall t\in\mathbf{R}_{\geq 0}$, where $w_{\max}$ is a known finite positive constant.*

Assumption 2 *The fault gain $\eta(t)\in[0,1)$ is unknown but bounded, satisfying $\|\eta(t)\|\leq\eta_{\max}<\infty,\forall t\in\mathbf{R}_{\geq 0}$, where $\eta_{\max}$ is a known finite positive constant. The float fault $\delta(t)$ is unknown and its norm has an unknown finite upper bound on the interval $\mathbf{R}_{\geq 0}$.*

Remark 1 *In Assumption 2, $\eta(t) \in [0,1)$ guarantees that the input of the LTV system (1) is not infinitely close to zero. Otherwise, if $\eta(t) = 1$, the control function could be completely lost, causing the system to become unforced. Similar constraints on fault are also found in [28, 29].*

For simplicity, we define the following two operations. One operator $\mathscr{J}(\cdot)$ is defined as

$$\left\{ \begin{array}{l} \mathscr{J}^0(M(t)) \triangleq M(t), \\ \mathscr{J}(M(t)) \triangleq M(t)A(t) + \frac{\mathrm{d}}{\mathrm{d}t}M(t), \\ \mathscr{J}^i(M(t)) \triangleq \mathscr{J}(\mathscr{J}^{i-1}(M(t))), \quad i = 1, 2, \ldots, \end{array} \right. \tag{2}$$

and other operator $\mathscr{R}(\cdot)$ is defined as

$$\left\{ \begin{array}{l} \mathscr{R}^0(M(t)) \triangleq M(t), \\ \mathscr{R}(M(t)) \triangleq A(t)M(t) - \frac{\mathrm{d}}{\mathrm{d}t}M(t), \\ \mathscr{R}^i(M(t)) \triangleq \mathscr{R}(\mathscr{R}^{i-1}(M(t))), \quad i = 1, 2, \ldots, \end{array} \right. \tag{3}$$

where $A(t)$ is defined in LTV system (1). Specifically, $\mathscr{J}_{A_1}(\cdot)$ and $\mathscr{R}_{A_1}(\cdot)$ denote the similar operations in (2) and (3) with the only difference being that $A(t)$ is replaced by $A_1(t)$.

Definition 1 *[30] The LTV system (1) is said to be uniformly controllable on the interval $\mathbf{R}_{\geq 0}$, if its controllability matrix*

$$Q(t) = \begin{bmatrix} B^{\mathrm{T}}(t) \\ (\mathscr{R}(B(t)))^{\mathrm{T}} \\ \vdots \\ (\mathscr{R}^{n-1}(B(t)))^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \in \mathbf{R}^{n \times (nm)}, \tag{4}$$

satisfies $\operatorname{rank}(Q_i(t)) = n, \forall t \in \mathbf{R}_{\geq 0}$.

Consider the following partitions

$$B(t) = \begin{bmatrix} b_1(t) & b_2(t) & \cdots & b_m(t) \end{bmatrix}.$$

The controllability matrix $Q(t)$ is screened for independent columns in the order from left to right. First, at step 1, the columns of the matrix $B(t)$, $b_i(t), i = 1, 2, \ldots, m$, are investigated. At step $j, j \in \{1, 2, ..., n\}$, $\mathscr{R}^{j-1}(b_i(t)), i = 1, 2, \ldots, m$ are investigated for their dependence on all previous ones. If $\mathscr{R}^j(b_i(t)), i = 1, 2, ..., m, j = 1, 2, ..., n-1$ can be represented by its previous vectors in a linear combination, we will discard this vector. Otherwise, we will keep this vector. Finally, a new nonsingular matrix $Q_{\mathrm{c}}(t)$ is constructed as follows

$$\begin{array}{l} Q_{\mathrm{c}}(t) = \begin{bmatrix} Q_{\mathrm{c}}^1(t) & Q_{\mathrm{c}}^2(t) & \cdots & Q_{\mathrm{c}}^{n-1}(t) \end{bmatrix} \in \mathbf{R}^{n \times n}, \\ Q_{\mathrm{c}}^i(t) = \begin{bmatrix} b_i(t) & \mathscr{R}(b_i(t)) & \cdots & \mathscr{R}^{\mu_i - 1}(b_i(t)) \end{bmatrix} \in \mathbf{R}^{n \times \mu_i}, \end{array} \tag{5}$$

where $i = 1, 2, \ldots, m$. Then, the ordered set $\{\mu_1, \mu_2, \ldots, \mu_m\}$ is called the controllability indices. The controllability indices may change over time. Then, we give following two assumptions.

Assumption 3 *There exists controllability indices $\{\mu_1, \mu_2, \ldots, \mu_m\}$ that does not change over time such that the LTV system (1) is uniformly controllable. In this case, the LTV system (1) is said to be lexicographically fixed.*

Assumption 4 *There exists controllability indices $\{\mu_1(t), \mu_2(t), \ldots, \mu_m(t)\}$ that change over time such that the LTV system (1) is uniformly controllable. In this case, the LTV system (1) is said to be non-lexicographically fixed.*

Remark 2 *Assumption 3 was widely presented, such as [31, 32, 33]. However, Assumption 3 is restrictive, and Assumption 4 is more weak but has received less attention. (See [34] and the references therein.) If the LTV system (1) is uniformly controllable, it must be non-lexicographically fixed with controllability indices being $\{\mu_1(t), \mu_2(t), \ldots, \mu_m(t)\}$. (Here we consider the lexicographically fixed LTV system as a special type of non-lexicographically fixed LTV system.)*

If the LTV system is lexicographically fixed, the controllability indices $\{\mu_1, \mu_2, \ldots, \mu_m\}$ are some constants. Denote $\sum_{i=1}^{m} \mu_i = n$.

If the LTV system is non-lexicographically fixed, the controllability indices $\{\mu_1(t), \mu_2(t), \ldots, \mu_m(t)\}$ are time-varying numbers. For arbitrary given $t \in \mathbf{R}_{\geq 0}$, we have $\sum_{i=1}^{m} \mu_i(t) = n$. Define

$$v_i = \max_{t \in \mathbf{R}_{\geq 0}} \{\mu_i(t)\}, \quad i = 1, 2, \ldots, m. \tag{6}$$

Set

$$n_{\mathrm{g}} \triangleq \sum_{i=1}^{m} v_i = \sum_{i=1}^{m} \max_{t \in \mathbf{R}_{\geq 0}} \{\mu_i(t)\} > \sum_{i=1}^{m} \mu_i(t) = n. \tag{7}$$

Finally, we will introduce the relevant knowledge on prescribed-time stability.

Definition 2 *[19, 35] (UPT-ISS+C) The time-varying system $\dot{x} = f(x, \delta, t, t_0), t, t_0 \in \mathbf{R}_{\geq 0}, t \geq t_0$, is referred to as uniformly prescribed-time input-to-state stable and converges to zero at the prescribed time T (represented by UPT-ISS+C), if there exists two $\mathcal{K}$ functions τ and ϱ and a $\mathcal{KL}$ function β_{c} such that for any initial moment t_0, all $t \in [t_0, t_0 + T]$ and the bounded function $\delta(t)$,*

$$\|x(t)\| \leq \beta_{\mathrm{c}} \left(\tau(\|x(t_0)\|) + \varrho\Big(\sup_{s \in [t_0, t_0+T)} \|\delta(s)\| \Big), \gamma_1(t) \right), \tag{8}$$

where $\gamma_1(t)$ is a T-finite-time escaping function ($\lim_{t \to t_0+T} \gamma_1(t) = \infty$).

Definition 3 *[36] The time-varying system $\dot{x} = f(x, t, t_0)$, $t \geq t_0, t, t_0 \in \mathbf{R}_{\geq 0}$ is referred to as globally uniformly prescribed-time stable (GUPTS) with the predetermined time T, if it is uniformly stable and, for any given initial state value and initial moment, $\lim_{t \to t_0+T} \|x(t)\| = 0$.*

3 Controllable Canonical Forms for LTV Systems

In this section, employing a nonsingular time-varying transformation, the uniformly controllable LTV system is converted into its controllable canonical form, regardless of whether it is lexicographically fixed or non-lexicographically fixed. For lexicographically fixed LTV systems, we directly transform them into their controllable canonical forms. Regarding non-lexicographically fixed LTV systems, we first introduce an auxiliary system based on the original system to ensure that the extended LTV system becomes lexicographically fixed. Subsequently, the augmented lexicographically fixed LTV system can be transformed into its controllable canonical form.

3.1 Controllable Canonical Forms of Lexicographically Fixed LTV Systems

Theorem 1 *Suppose that the LTV system (1) satisfies Assumption 3, then there exists a nonsingular transformation*

$$\bar{x} = T_{\mathrm{c}}(t)x, \tag{9}$$

such that the LTV system (1) is converted into the following controllable canonical form

$$\begin{cases} \dot{\bar{x}} = (1 + \varpi(t))\bar{A}(t)\bar{x} + \bar{B}(t)((1 - \eta(t))u + \delta(t)), \\ y = \bar{C}(t)x, \quad t, t_0 \in \mathbf{R}_{\geq 0}, \quad t \geq t_0, \end{cases} \tag{10}$$

where $(\bar{A}(t), \bar{B}(t)) = ((T_{\mathrm{c}}(t)A(t) + \dot{T}_{\mathrm{c}}(t))T_{\mathrm{c}}^{-1}(t), T_{\mathrm{c}}(t)B(t))$. Moreover, $\bar{A}(t)$ and $\bar{B}(t)$ take the following form:

$$\bar{A}(t) = \begin{bmatrix} A_{11}(t) & A_{12}(t) & \cdots & A_{1m}(t) \\ A_{21}(t) & A_{22}(t) & \cdots & A_{2m}(t) \\ \vdots & \vdots & & \vdots \\ A_{m1}(t) & A_{m2}(t) & \cdots & A_{mm}(t) \end{bmatrix} \in \mathbf{R}^{n \times n}, \tag{11}$$

$$\bar{B}(t)=\begin{bmatrix} b_1 & & \\ & \ddots & \\ & & b_m \end{bmatrix}\Gamma(t)\in\mathbf{R}^{n\times m}, \tag{12}$$

where $\Gamma(t)\in\mathbf{R}^{m\times m}$ is an upper triangular matrix with diagonal elements all being ones and, for $i,j=1,2,\ldots,m$,

$$A_{ii}(t)=\left[\begin{array}{c|ccc} 0 & 1 & & \\ \vdots & & \ddots & \\ 0 & & & 1 \\ \hline \alpha_{ii,1} & \alpha_{ii,2} & \cdots & \alpha_{ii,\mu_i} \end{array}\right]\in\mathbf{R}^{\mu_i\times\mu_i},$$

$$A_{ij}(t)=\left[\begin{array}{cccc} 0 & \cdots & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & \cdots & \cdots & 0 \\ \hline \alpha_{ij,1} & \alpha_{ij,2} & \cdots & \alpha_{ij,\mu_j} \end{array}\right]\in\mathbf{R}^{\mu_i\times\mu_j},\quad i\neq j, \tag{13}$$

$$b_i=\begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix}^{\mathrm{T}}\in\mathbf{R}^{\mu_i\times 1}, \tag{14}$$

in which $\alpha_{ij,k}$, $k=1,2,\ldots,\mu_j$ are time-varying known parameters.

Proof. The conversion process to controllable canonical form has been given in many previous literatures [37]. We next mainly provide the construction method of the transformation matrix $T_{\mathrm{c}}(t)$. First, the inverse of matrix $Q_c(t)$ in (5) is divided into

$$\hat{Q}(t)=Q_c^{-1}(t)=\begin{bmatrix} \hat{Q}_1^{\mathrm{T}}(t) & \hat{Q}_2^{\mathrm{T}}(t) & \cdots & \hat{Q}_m^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}},$$

where, for $i=1,2,\ldots,m$,

$$\hat{Q}_i(t)=\begin{bmatrix} \varkappa_{i,0}^{\mathrm{T}}(t) & \varkappa_{i,1}^{\mathrm{T}}(t) & \cdots & \varkappa_{i,\mu_i-1}^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}.$$

Denote

$$\varkappa_k(t)\triangleq\varkappa_{k,\mu_k-1}(t),\qquad k=1,2,\ldots,m. \tag{15}$$

Next, the transformation matrix $T_{\mathrm{c}}(t)$ is constructed as follows

$$T_{\mathrm{c}}(t)\triangleq\begin{bmatrix} T_{\mathrm{c}1}(t) \\ T_{\mathrm{c}2}(t) \\ \vdots \\ T_{\mathrm{c}m}(t) \end{bmatrix}\in\mathbf{R}^{n\times n}, \tag{16}$$

$$T_{\mathrm{c}k}(t)\triangleq\begin{bmatrix} \varkappa_k(t) \\ \mathscr{J}(\varkappa_k(t)) \\ \vdots \\ \mathscr{J}^{(\mu_k-1)}(\varkappa_k(t)) \end{bmatrix}\in\mathbf{R}^{\mu_k\times n},\quad k=1,2,\ldots,m. \tag{17}$$

Then, by using the transformation $\bar{x}=T_{\mathrm{c}}(t)x$,, $\bar{A}(t)$ and $\bar{B}(t)$ yield the expressions as shown in (11) and (12). Moreover, $\Gamma(t)$ in (12) takes the following form

$$\Gamma(t)=\begin{bmatrix} \mathscr{J}^{(\mu_1-1)}(\varkappa_1(t)) \\ \mathscr{J}^{(\mu_2-1)}(\varkappa_2(t)) \\ \vdots \\ \mathscr{J}^{(\mu_m-1)}(\varkappa_m(t)) \end{bmatrix}B(t)=\begin{bmatrix} 1 & * & \cdots & * \\ & 1 & \ddots & \vdots \\ & & \ddots & * \\ & & & 1 \end{bmatrix}, \tag{18}$$

where $\Gamma(t)\in\mathbf{R}^{m\times m}$ and $*$ represents known time-varying elements. ■

Remark 3 *The theorem mentioned above shares similarities with the result in [33]. However, in Theorem 1, The constraint*

$$|\mu_i-\mu_j|\leq 1,\quad i,j\in\{1,2,\ldots,m\}, \tag{19}$$

imposed on the controllability indices $\{\mu_1, \mu_2, \ldots, \mu_m\}$ has been relaxed during the conversion to the controllable canonical form. Traditionally, many works including [31, 32, 33, 38] consider the condition (19) as a requirement for the conversion to controllable canonical form. However, it has been noted by [39, 40] that this condition is not essential. They highlighted that even without such condition, the LTV system can still be converted to its controllable canonical form in form of (11) and (12), albeit without providing a detailed proof. In our forthcoming research, we will provide a detailed proof that the condition (19) is redundant for the conversion to controllable canonical form.

3.2 Controllable Canonical Forms of Non-lexicographically Fixed LTV Systems

In the previous subsection, employing the transformation characterized by the system data, the lexicographically fixed LTV system with actuator faults and uncertainties is transformed into its controllable canonical form. Next we will introduce the method of transforming the non-lexicographically fixed LTV system with precisely known model into its controllable canonical form. The non-lexicographically fixed LTV system can be expanded into an augmented lexicographically fixed LTV system with the assistance of an auxiliary LTV system. This idea was initially proposed in [34]. We will provide a direct method for obtaining the augmented lexicographically fixed LTV system in this subsection and apply it into the analysis of global prescribed-time fault-tolerant control in the next section. Initially, we introduce the time-varying base extension theorem, inspired by the Doležar's theorem [41], with its proof relocated to Appendix A1 for clarity.

Lemma 1 *Let $l, n \in \mathbf{N}^+$, $r \in \{1, 2, \ldots, n\}$ and $M(t) \in \mathcal{J}^k(\mathbf{R}_{\geq 0}, \mathbf{R}^{r\times n})$. Suppose that* $\operatorname{rank}(M(t)) = r$, *for all $t \in \mathbf{R}_{\geq 0}$, then there exists $S(t) \in (\mathcal{J}^k, \mathcal{S}^\infty)(\mathbf{R}_{\geq 0}, \mathbf{R}^{(n-r)\times n})$ such that*

$$\operatorname{rank}\begin{bmatrix} M(t) \\ S(t) \end{bmatrix} = n, \quad \forall t \in \mathbf{R}_{\geq 0}.$$

Then, assume that the LTV system model depicted in (1) is precisely known, that is $\varpi(t) = 0, \forall t \in \mathbf{R}_{\geq 0}$. Denote $n_{\mathrm{e}} \triangleq n_{\mathrm{g}} - n$, where n_{g} is defined in (7). The following result regarding the augmented lexicography fixed LTV system is given, whose proof has been moved to Appendix A2 for clarity.

Lemma 2 *Let Assumption 4 be fulfilled. Then, there exists suitable matrices $A_1(t) \in \mathbf{R}^{n_{\mathrm{e}}\times n}$, $A_2(t) \in \mathbf{R}^{n_{\mathrm{e}}\times(n_{\mathrm{g}}-n)}$ and $B_{\mathrm{e}}(t) \in \mathbf{R}^{n_{\mathrm{e}}\times m}$ such that the LTV system (1) with $\varpi(t) = 0, \forall t \in \mathbf{R}_{\geq 0}$ can be expanded into the following augmented lexicographically fixed LTV system*

$$\begin{cases} \dot{x}_{\mathrm{g}} = A_{\mathrm{g}}(t)x_{\mathrm{g}} + B_{\mathrm{g}}(t)((1-\eta(t))u + \delta(t)), \\ y_{\mathrm{g}} = C_{\mathrm{g}}(t)x_{\mathrm{g}}, \quad t, t_0 \in \mathbf{R}_{\geq 0}, \quad t \geq t_0, \end{cases} \tag{20}$$

where

$$x_{\mathrm{g}} = \begin{bmatrix} x \\ x_{\mathrm{e}} \end{bmatrix} \in \mathbf{R}^{n_{\mathrm{g}}},$$
$$A_{\mathrm{g}}(t) = \begin{bmatrix} A(t) & 0 \\ A_2(t) & A_1(t) \end{bmatrix} \in \mathbf{R}^{n_{\mathrm{g}}\times n_{\mathrm{g}}},$$
$$B_{\mathrm{g}}(t) = \begin{bmatrix} B(t) \\ B_{\mathrm{e}}(t) \end{bmatrix} \in \mathbf{R}^{n_{\mathrm{g}}\times m}, C_{\mathrm{g}}(t) = \begin{bmatrix} C(t) & 0 \end{bmatrix} \in \mathbf{R}^{p\times n_{\mathrm{g}}}.$$

Additionally, the augmented lexicographically fixed LTV system (20) has controllability indices $\{v_1, v_2, \ldots, v_m\}$.

Such widespread non-lexicographically fixed LTV systems are only considered in the problem of pole placement [34, 42, 43, 44, 45] and lack attention on other interesting issues. Utilizing the augmented system, the conversion to the controllable canonical form can be generalized to non-lexicographically fixed LTV systems.

Theorem 2 *Let Assumption 4 be fulfilled. Suppose that $\varpi(t) = 0, \forall t \geq t_0 \in \mathbf{R}_{\geq 0}$ in the LTV system (1), then there exists a nonsingular transformation $\bar{x}_{\mathrm{g}} = T_{\mathrm{g}}(t)x_{\mathrm{g}}$, such that the augmented system (20) is* *transformed into the following augmented controllable canonical form*

$$\begin{cases} \dot{x}_{\mathrm{g}}(t) = \bar{A}_{\mathrm{g}}(t)x_{\mathrm{g}} + \bar{B}_{\mathrm{g}}(t)((1-\eta(t))u + \delta(t)), \\ y_{\mathrm{g}} = \bar{C}_{\mathrm{g}}(t)x_{\mathrm{g}}, \quad t, t_0 \in \mathbf{R}_{\geq 0}, \quad t \geq t_0, \end{cases} \tag{21}$$

where $(\bar{A}_{\mathrm{g}}(t), \bar{B}_{\mathrm{g}}(t)) = ((T_{\mathrm{g}}(t)A_{\mathrm{g}}(t) + \dot{T}_{\mathrm{g}}(t))T_{\mathrm{g}}^{-1}(t), T_{\mathrm{g}}(t)B(t))$*. Moreover,* $\bar{A}_{\mathrm{g}}(t)$ *takes the similar form to (11) and*

$$\bar{B}_{\mathrm{g}}(t) = \begin{bmatrix} b_{\mathrm{g}1} & & \\ & \ddots & \\ & & b_{\mathrm{g}m} \end{bmatrix} \mathit{\Gamma}_{\mathrm{g}}(t) \in \mathbf{R}^{n_{\mathrm{g}} \times m}, \tag{22}$$

where, for each $i \in \{1, 2, \cdots, m\}$*,*

$$b_{\mathrm{g}i} = \begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix}^{\mathrm{T}} \in \mathbf{R}^{v_i \times 1}, \tag{23}$$

and $\Gamma_{\mathrm{g}}(t)$ *takes the similar form to (18).*

4 Global Prescribed-Time Fault-Tolerant Control by Linear Time-Varying Feedback

In this section, employing the clear structure of the controllable canonical form and the characteristic of the PLE, a linear time-varying feedback is designed to achieve global prescribed-time fault-tolerant control for uniformly controllable LTV systems. Both uncertain lexicographically fixed and non-lexicographically fixed LTV systems with precisely known models have been taken into account. We first consider the prescribed-time fault-tolerant control for the lexicographically fixed LTV system.

4.1 Lexicographically Fixed LTV Systems

In this subsection, the prescribed-time fault-tolerant controller is designed for uncertain lexicographically fixed LTV systems. We first design a preliminary controller. Then, based on the preliminary controller, we redesign the overall controller.

Assume that $T_{\mathrm{c}}(t)$ constructed in (16) and (17) is a Lyapunov transformation matrix. Therefore, the stability of the original system can be preserved after the transformation $\bar{x} = T_{\mathrm{c}}(t)x$. Accordingly, the controller design can be continued based on the controllable canonical form derived in the previous subsection.

For $i \in \{1, 2, \ldots, m\}$, denote

$$K_i(t) \triangleq \begin{bmatrix} K_{i,1}(t) & K_{i,2}(t) & \cdots & K_{i,m}(t) \end{bmatrix} \in \mathbf{R}^{1 \times n}, \tag{24}$$

where, for each $j \in \{1, 2, \ldots, m\}$,

$$K_{i,j}(t) \triangleq \begin{bmatrix} \alpha_{ij,1} & \alpha_{ij,2} & \cdots & \alpha_{ij,\mu_j} \end{bmatrix} \in \mathbf{R}^{1 \times \mu_j}, \tag{25}$$

in which $\alpha_{ij,k}$, $i, j = 1, 2, \ldots, m, k = 1, 2, \ldots, \mu_j$ are defined in (13). Set

$$\delta_{\mathit{\Gamma}}(t) \triangleq \begin{bmatrix} \delta_{\mathit{\Gamma}1}(t) \\ \delta_{\mathit{\Gamma}2}(t) \\ \vdots \\ \delta_{\mathit{\Gamma}m}(t) \end{bmatrix} = \mathit{\Gamma}(t)\delta(t) \in \mathbf{R}^m. \tag{26}$$

where $\mathit{\Gamma}(t)$ is defined in (12). Since $T_{\mathrm{c}}(t)$ is a Lyapunov transformation matrix and $A(t)$ as well as $B(t)$ are bounded for all $t \in \mathbf{R}_{\geq 0}$, then both $\bar{A}(t)$ and $\bar{B}(t)$, respectively, depicted in (11) and (12) are bounded for all $t \in \mathbf{R}_{\geq 0}$. Therefore, based on the special structure of matrices $\bar{A}(t)$ and $\bar{B}(t)$ outlined in (11), (12) and the definition of $K_i(t), i = 1, 2, \ldots, m$ depicted in (24) and (25), along with the depiction of $\delta_{\mathit{\Gamma}}(t)$ in (26), we can conclude that, for every $i, j \in \{1, 2, \ldots, m\}$, $\varGamma(t)$, $\delta_{\varGamma i}(t)$ and $K_{i,j}(t)$ are all bounded for all $t \in \mathbf{R}_{\geq 0}$. Set

$$\delta_{\max} \triangleq \sup_{t \geq t_0} \left\{ \sum_{i=1}^{m} \delta_{\varGamma_i}^2(t) \right\}. \tag{27}$$

In addition, since $\varGamma(t)$ depicted in (18) is bounded and $\det(\varGamma(t)) = 1, \forall t \in \mathbf{R}_{\geq 0}$, according to Lemma 5, we can infer that $\varGamma^{-1}(t)$ is also bounded for all $t \in \mathbf{R}_{\geq 0}$.

Partition the state of the controllable canonical form (10) into

$$\bar{x} = \begin{bmatrix} \bar{x}_1^{\mathrm{T}} & \bar{x}_2^{\mathrm{T}} & \cdots & \bar{x}_m^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, \tag{28}$$

where $\bar{x}_i \in \mathbf{R}^{\mu_i}, i = 1, 2, \ldots, m$. Define the following preliminary controller

$$u = \varGamma^{-1}(t) v, \quad v = \begin{bmatrix} v_1^{\mathrm{T}} & v_2^{\mathrm{T}} & \cdots & v_m^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, \tag{29}$$

where $\varGamma(t)$ is defined in (18) and $v_i, i = 1, 2, \ldots, m$ are the remaining part of the controller to be designed subsequently. Inserting the preliminary controller (29) into the controllable canonical form (10) yields

$$\begin{aligned} \dot{\bar{x}}_i =& (1 + \varpi(t)) \bar{A}_i \bar{x}_i + (1 + \varpi(t)) b_i \sum_{j=1}^{m} K_{i,j}(t) \bar{x}_j + b_i \delta_{\varGamma i}(t) \\ &+ (1 - \eta(t)) b_i v_i, \quad i = 1, 2, \ldots, m, \end{aligned} \tag{30}$$

where, for each $i \in \{1, 2, \ldots, m\}$,

$$\bar{A}_i = \left[\begin{array}{c|ccc} 0 & 1 & & \\ \vdots & & \ddots & \\ 0 & & & 1 \\ \hline 0 & 0 & \cdots & 0 \end{array}\right] \in \mathbf{R}^{\mu_i \times \mu_i},$$

and b_i, $K_{i,j}(t), j = 1, 2, \ldots, m$, and $\delta_{\varGamma i}(t)$ are defined in (14), (25) and (26), respectively. Next, we give the following properties on the parametric Lyapunov equation (PLE).

Lemma 3 *[25, 26] Let*

$$A = \left[\begin{array}{c|ccc} 0 & 1 & & \\ \vdots & & \ddots & \\ 0 & & & 1 \\ \hline 0 & 0 & \cdots & 0 \end{array}\right], \quad b = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \tag{31}$$

where $A \in \mathbf{R}^{n \times n}$ and $b \in \mathbf{R}^{n \times 1}$. The PLE

$$A^{\mathrm{T}} P + PA - Pbb^{\mathrm{T}} P = -\gamma P, \quad \gamma > 0, \tag{32}$$

possesses a UPDS $P(\gamma)$ to (31), exhibiting the following properties:

1. *The UPDS $P(\gamma)$ is provided by*
$$P(\gamma) = \gamma L_n P_n L_n, \tag{33}$$
where $P_n = P(1)$ and $L_n = L_n(\gamma) = \gamma^{n-1} \oplus \gamma^{n-2} \oplus \cdots \oplus 1$.

2. *There exists a constant $\theta(n) \geq 1$ independent of γ such that*
$$\frac{P(\gamma)}{n\gamma} \leq \frac{\mathrm{d}P(\gamma)}{\mathrm{d}\gamma} \leq \frac{\theta(n) P(\gamma)}{n\gamma}, \tag{34}$$
where $\theta(n) = n(1 + \sigma_{\max}(M_n + P_n M_n P_n^{-1}))$, with $M_n = n - 1 \oplus n - 2 \oplus \cdots \oplus 1 \oplus 0$.

3. There holds

$$A^{\mathrm{T}}P(\gamma)A \leq 3n^2\gamma^2 P(\gamma). \tag{35}$$

$$b^{\mathrm{T}}P(\gamma)b = n\gamma. \tag{36}$$

$$\sigma_{\max}(P_n) = \sigma_{\min}^{-1}(P_n) \triangleq \Lambda_n. \tag{37}$$

We then give the assumption on the upper bound of the model uncertainty $\varpi(t)$.

Assumption 5 *For every $i \in \{1, 2, \ldots, m\}$, the known finite upper bound of the model uncertainty $w_{\max}$ satisfies*

$$2\sqrt{3}\mu_i w_{\max} < 1.$$

By utilizing the properties of the PLE, we redesign the preliminary controller. Then, we present the following main result.

Theorem 3 *Let Assumption 3 and Assumption 5 be fulfilled. Assume that $T_{\mathrm{c}}(t)$ constructed in (16) and (17) is a Lyapunov transformation. Let $T > 0$ be a prescribed time. Then, there exist positive constants α and β, such that the closed-loop system composed of the LTV system (1) and the linear time-varying state feedback (29) with*

$$v_i = -fb_i^{\mathrm{T}}P_i(\gamma)\begin{bmatrix} \rho_{\bar{\mu}_{i,1}}^{[n]} \\ \rho_{\bar{\mu}_{i,2}}^{[n]} \\ \vdots \\ \rho_{\bar{\mu}_{i,\mu_i}}^{[n]} \end{bmatrix} T_{\mathrm{c}}(t)x, \quad i = 1, 2, \ldots, m, \tag{38}$$

where f is a constant satisfying $f > \frac{1}{2(1-\eta_{\max})}$, b_i is defined in (14), $\bar{\mu}_{i,j} = \sum_{k=1}^{i-1}\mu_k + j, j = 1, 2, \ldots, \mu_i$, and

$$\gamma = \frac{\mathrm{e}^{\alpha\beta(t_0+T)} - \mathrm{e}^{\alpha\beta t_0}}{\mathrm{e}^{\alpha\beta(t_0+T)} - \mathrm{e}^{\alpha\beta t}}\gamma_0, \quad \gamma_0 = \max\left\{\frac{k\beta \mathrm{e}^{\alpha\beta(t_0+T)}}{\mathrm{e}^{\alpha\beta(t_0+T)} - \mathrm{e}^{\alpha\beta t_0}}, 1\right\}, \tag{39}$$

in which $k \geq 1$ is a constant, and $P_i(\gamma)$ is the UPDS to the PLE (32) with dimensions μ_i, satisfies

$$\|x(t)\|^2 \leq \chi(1 - \mathrm{e}^{-\alpha\beta(t_0+T-t)})(\|x(t_0)\|^2 + \delta_{\max}),$$

for all $t \in [t_0, t_0 + T)$ and any $x(t_0) \in \mathbf{R}^n$, in which χ is a finite positive constant. Moreover, the overall controller $u(t)$ composed of (29) and (38) satisfies, for all $t \in [t_0, t_0 + T)$,

$$\|u(t)\|^2 \leq \vartheta_{\mathrm{u}}((\mathrm{e}^{\alpha\beta(t_0+T-t)} - 1)^{\varsigma}\|x(t_0)\|^2 + \delta_{\max}),$$

where ϑ_{u} and ς are finite positive constants. In addition, when the float fault $\delta(t)$ exists in the LTV system (1), namely, $\delta(t) \neq 0$, the closed-loop system is UPT-ISS+C in sense of Definition 2 and the overall controller $u(t)$ is bounded. When the float fault $\delta(t)$ does not exist in the LTV system (1), namely, $\delta(t) = 0$, the closed-loop system is GUPTS in sense of Definition 3 and the overall controller $u(t)$ satisfies $\lim_{t \to t_0+T} u(t) = 0$.

Proof. Let k_1 and $\epsilon_i, i = 1, 2, \ldots, m$ represent positive constants such that, for $i = 1, 2, \ldots, m$,

$$\alpha_i \triangleq \frac{\mu_i}{\mu_i + \theta(\mu_i)}(1 - k_1\mu_i - 2\sqrt{3}\mu_i w_{\max} - \epsilon_i) > 0. \tag{40}$$

Choose

$$0 < \alpha \leq \min_{i=1,2,\ldots,m}\{\alpha_i\}. \tag{41}$$

For $i = 1, 2, \ldots, m$, define

$$\beta_i \triangleq \frac{\mu_i}{\mu_i + \theta(\mu_i)}\left(m(1 + w_{\max})\mu_i + (1 + w_{\max})\sum_{j=1}^{m}\frac{k_{ji}^2}{p_i}\right), \tag{42}$$

where

$$k_{ji} = \sup_{t\geq 0}\{\|K_{j,i}(t)\|\}, \quad j = 1, 2, \ldots, m, \tag{43}$$

and $p_i \triangleq \sigma_{\min}(P_{\mu_i})$. Choose

$$\beta \geq \frac{\max\limits_{i=1,2,\ldots,m}\{\beta_i\}}{\alpha} \tag{44}$$

According to (9) and (28), (38) can be rewritten as

$$v_i = -f b_i^{\mathrm{T}} P_i(\gamma)\bar{x}_i, \quad i = 1, 2, \ldots, m. \tag{45}$$

Consider the Lyapunov-like function

$$V_i(t, \bar{x}_i) = \gamma\bar{x}_i^{\mathrm{T}} P_i(\gamma)\bar{x}_i, \quad i = 1, 2, \ldots, m.$$

For arbitrary given $i \in \{1, 2, \ldots, m\}$, taking the time-derivative of $V_i(t, \bar{x}_i)$ along system (30) with controller (45) yields

$$\begin{aligned}
\dot{V}_i(t, \bar{x}_i) =& \dot{\gamma}\bar{x}_i^{\mathrm{T}} P_i(\gamma)\bar{x}_i + \dot{\gamma}\gamma\bar{x}_i^{\mathrm{T}}\frac{\mathrm{d}P_i(\gamma)}{\mathrm{d}\gamma}\bar{x}_i + \gamma\Bigg((1+\varpi(t))\bar{x}_i^{\mathrm{T}}\bar{A}_i^{\mathrm{T}} + (1+\varpi(t))\sum_{j=1}^{m}\bar{x}_j^{\mathrm{T}} K_{i,j}^{\mathrm{T}}(t) b_i^{\mathrm{T}} + \delta_{\Gamma i}(t) b_i^{\mathrm{T}} \\
& - (1-\eta(t)) f\bar{x}_i^{\mathrm{T}} P_i(\gamma) b_i b_i^{\mathrm{T}}\Bigg) P_i(\gamma)\bar{x}_i + \gamma\bar{x}_i^{\mathrm{T}} P_i(\gamma)\Bigg((1+\varpi(t))\bar{A}_i\bar{x}_i + (1+\varpi(t)) b_i\sum_{j=1}^{m} K_{i,j}(t)\bar{x}_j \\
& + b_i\delta_{\Gamma_i}(t) - (1-\eta(t)) f b_i b_i^{\mathrm{T}} P_i(\gamma)\bar{x}_i\Bigg) \\
=& \dot{\gamma}\bar{x}_i^{\mathrm{T}} P_i(\gamma)\bar{x}_i + \dot{\gamma}\gamma\bar{x}_i^{\mathrm{T}}\frac{\mathrm{d}P_i(\gamma)}{\mathrm{d}\gamma}\bar{x}_i + \gamma\bar{x}_i^{\mathrm{T}}(\bar{A}_i^{\mathrm{T}} P_i(\gamma) + P_i(\gamma)\bar{A}_i - 2(1-\eta(t)) f P_i(\gamma) b_i b_i^{\mathrm{T}} P_i(\gamma))\bar{x}_i \\
& + 2(1+\varpi(t))\gamma\sum_{j=1}^{m}\bar{x}_j^{\mathrm{T}} K_{i,j}^{\mathrm{T}}(t) b_i^{\mathrm{T}} P_i(\gamma)\bar{x}_i + 2\delta_{\Gamma i}(t)\gamma b_i^{\mathrm{T}} P_i(\gamma)\bar{x}_i + 2\varpi(t)\gamma\bar{x}_i^{\mathrm{T}} P_i(\gamma)\bar{A}_i\bar{x}_i.
\end{aligned} \tag{46}$$

According to (34), we have

$$\dot{\gamma}\gamma\bar{x}_i^{\mathrm{T}}\frac{\mathrm{d}P_i(\gamma)}{\mathrm{d}\gamma}\bar{x}_i \leq \dot{\gamma}\frac{\theta(\mu_i)}{\mu_i}\bar{x}_i^{\mathrm{T}} P_i(\gamma)\bar{x}_i. \tag{47}$$

Referring to (32) and $f > \frac{1}{2(1-\eta_{\max})}$, we can get

$$\begin{aligned}
&\gamma\bar{x}_i^{\mathrm{T}}(\bar{A}_i^{\mathrm{T}} P_i(\gamma) + P_i(\gamma)\bar{A}_i - 2(1-\eta(t)) f P_i(\gamma) b_i b_i^{\mathrm{T}} P_i(\gamma))\bar{x}_i \\
=& -\gamma^2\bar{x}_i^{\mathrm{T}} P_i(\gamma)\bar{x}_i + (1-2(1-\eta(t)) f)\gamma\bar{x}_i^{\mathrm{T}} P_i(\gamma) b_i b_i^{\mathrm{T}} P_i(\gamma)\bar{x}_i \\
\leq& -\gamma^2\bar{x}_i^{\mathrm{T}} P_i(\gamma)\bar{x}_i + (1-2(1-\eta_{max}) f)\gamma\bar{x}_i^{\mathrm{T}} P_i(\gamma) b_i b_i^{\mathrm{T}} P_i(\gamma)\bar{x}_i \\
\leq& -\gamma^2\bar{x}_i^{\mathrm{T}} P_i(\gamma)\bar{x}_i.
\end{aligned} \tag{48}$$

It follows from (36) that

$$\gamma\bar{x}_i^{\mathrm{T}} P_i(\gamma) b_i b_i^{\mathrm{T}} P_i(\gamma)\bar{x}_i \leq \gamma b_i^{\mathrm{T}} P_i(\gamma) b_i\bar{x}_i^{\mathrm{T}} P_i(\gamma)\bar{x}_i = \mu_i\gamma^2\bar{x}_i^{\mathrm{T}} P_i(\gamma)\bar{x}_i. \tag{49}$$

By using Young's inequality and (49), we can derive

$$2\delta_{\Gamma_i}(t)\gamma b_i^{\mathrm{T}} P_i(\gamma)\bar{x}_i \leq k_1\gamma\bar{x}_i^{\mathrm{T}} P_i(\gamma) b_i b_i^{\mathrm{T}} P_i(\gamma)\bar{x}_i + \frac{1}{k_1}\gamma\delta_{\Gamma_i}^2(t) \leq k_1\mu_i\gamma^2\bar{x}_i^{\mathrm{T}} P_i(\gamma)\bar{x}_i + \frac{1}{k_1}\gamma\delta_{\Gamma_i}^2(t), \tag{50}$$

where $k_1 > 0$ is a constant. Using Young's inequality and (35) yields

$$\begin{aligned}
2\varpi(t)\gamma\bar{x}_i^{\mathrm{T}} P_i(\gamma)\bar{A}_i\bar{x}_i \leq& k_2 w_{\max}\gamma^2\bar{x}_i^{\mathrm{T}} P_i(\gamma)\bar{x}_i + \frac{1}{k_2} w_{\max}\bar{x}_i^{\mathrm{T}}\bar{A}_i^{\mathrm{T}} P_i(\gamma)\bar{A}_i\bar{x}_i \\
\leq& k_2 w_{\max}\gamma^2\bar{x}_i^{\mathrm{T}} P_i(\gamma)\bar{x}_i + \frac{3\mu_i^2 w_{\max}}{k_2}\gamma^2\bar{x}_i^{\mathrm{T}} P(\gamma)\bar{x}_i
\end{aligned}$$

$$=\left(k_2+\frac{3\mu_i^2}{k_2}\right)w_{\max}\gamma^2\bar{x}_i^{\mathrm{T}}P_i(\gamma)\bar{x}_i. \tag{51}$$

Notice that, for $i\in\{1,2,\ldots,m\}$, there exists positive scalar p_i such that

$$p_iI_{\mu_i}\triangleq\sigma_{\min}(P_{\mu_i})I_{\mu_i}=\sigma_{\min}(P_{\mu_i})L_{\mu_i}^2(1)\le\sigma_{\min}(P_{\mu_i})L_{\mu_i}^2(\gamma_0)\le\sigma_{\min}(P_{\mu_i})L_{\mu_i}^2\le L_{\mu_i}P_{\mu_i}L_{\mu_i}, \tag{52}$$

where (33) and (39) are used. Using Young's inequality, (33), (49) and (52) gives

$$\begin{aligned}
&2(1+\varpi(t))\gamma\bar{x}_i^{\mathrm{T}}P_i(\gamma)b_i\sum_{j=1}^{m}K_{i,j}(t)\bar{x}_j\\
=&\sum_{j=1}^{m}2(1+\varpi(t))\gamma\bar{x}_i^{\mathrm{T}}P_i(\gamma)b_iK_{i,j}(t)\bar{x}_j\\
\le&\sum_{j=1}^{m}k_3(1+w_{\max})\bar{x}_i^{\mathrm{T}}P_i(\gamma)b_ib_i^{\mathrm{T}}P_i(\gamma)\bar{x}_i+\sum_{j=1}^{m}\frac{1}{k_3}(1+w_{\max})\gamma^2\bar{x}_j^{\mathrm{T}}K_{i,j}^{\mathrm{T}}(t)K_{i,j}(t)\bar{x}_j\\
\le&mk_3g(\varpi)\mu_i\gamma\bar{x}_i^{\mathrm{T}}P_i(\gamma)\bar{x}_i+\frac{g(\varpi)\gamma^2}{k_3}\sum_{j=1}^{m}k_{ij}^2\bar{x}_j^{\mathrm{T}}\bar{x}_j\\
\le&mk_3g(\varpi)\mu_i\gamma\bar{x}_i^{\mathrm{T}}P_i(\gamma)\bar{x}_i+\frac{g(\varpi)\gamma}{k_3}\sum_{j=1}^{m}k_{ij}^2\bar{x}_j^{\mathrm{T}}\frac{\gamma L_{\mu_j}P_{\mu_j}L_{\mu_j}}{p_j}\bar{x}_j\\
\le&mk_3g(\varpi)\mu_i\gamma\bar{x}_i^{\mathrm{T}}P_i(\gamma)\bar{x}_i+\frac{g(\varpi)\gamma}{k_3}\sum_{j=1}^{m}\frac{k_{ij}^2}{p_j}\bar{x}_j^{\mathrm{T}}P_j(\gamma)\bar{x}_j,
\end{aligned} \tag{53}$$

where k_3 is a positive constant, $g(\varpi)=1+w_{\max}$ and $k_{ij},j=1,2,\ldots,m$ are defined in (43). Combining (47), (48), (50), (51) and (53), (46) can be further written as

$$\begin{aligned}
\dot{V}_i(t,\bar{x}_i)\le&\dot{\gamma}\bar{x}_i^{\mathrm{T}}P_i(\gamma)\bar{x}_i+\dot{\gamma}\frac{\theta(\mu_i)}{\mu_i}\bar{x}_i^{\mathrm{T}}P_i(\gamma)\bar{x}_i-\gamma^2\bar{x}_i^{\mathrm{T}}P_i(\gamma)\bar{x}_i+mk_3g(\varpi)\mu_i\gamma\bar{x}_i^{\mathrm{T}}P_i(\gamma)\bar{x}_i+\frac{g(\varpi)\gamma}{k_3}\sum_{j=1}^{m}\frac{k_{ij}^2}{p_j}\bar{x}_j^{\mathrm{T}}P_j(\gamma)\bar{x}_j\\
&+k_1\mu_i\gamma^2\bar{x}_i^{\mathrm{T}}P_i(\gamma)\bar{x}_i+\frac{1}{k_1}\gamma\delta_{\Gamma_i}^2(t)+\left(k_2+\frac{3\mu_i^2}{k_2}\right)w_{\max}\gamma^2\bar{x}_i^{\mathrm{T}}P_i(\gamma)\bar{x}_i\\
=&\left(\left(1+\frac{\theta(\mu_i)}{\mu_i}\right)\dot{\gamma}+\left(-1+k_1\mu_i+\left(k_2+\frac{3\mu_i^2}{k_2}\right)w_{\max}+\epsilon_i\right)\gamma^2\right)\bar{x}_i^{\mathrm{T}}P_i(\gamma)\bar{x}_i-\epsilon_i\gamma^2\bar{x}_i^{\mathrm{T}}P_i(\gamma)\bar{x}_i\\
&+mk_3g(\varpi)\mu_i\gamma\bar{x}_i^{\mathrm{T}}P_i(\gamma)\bar{x}_i+\frac{g(\varpi)\gamma}{k_3}\sum_{j=1}^{m}\frac{k_{ij}^2}{p_j}\bar{x}_j^{\mathrm{T}}P_j(\gamma)\bar{x}_j+\frac{1}{k_1}\gamma\delta_{\Gamma_i}^2(t)\\
=&\varphi_i(\gamma)\bar{x}_i^{\mathrm{T}}P_i(\gamma)\bar{x}_i-\epsilon_i\gamma^2\bar{x}_i^{\mathrm{T}}P_i(\gamma)\bar{x}_i+mk_3g(\varpi)\mu_i\gamma\bar{x}_i^{\mathrm{T}}P_i(\gamma)\bar{x}_i+\frac{g(\varpi)\gamma}{k_3}\sum_{j=1}^{m}\frac{k_{ij}^2}{p_j}\bar{x}_j^{\mathrm{T}}P_j(\gamma)\bar{x}_j+\frac{1}{k_1}\gamma\delta_{\Gamma_i}^2(t),
\end{aligned} \tag{54}$$

where

$$\varphi_i(\gamma)\triangleq\left(1+\frac{\theta(\mu_i)}{\mu_i}\right)\dot{\gamma}+\left(-1+k_1\mu_i+\left(k_2+\frac{3\mu_i^2}{k_2}\right)w_{\max}+\epsilon_i\right)\gamma^2 \tag{55}$$

and ϵ_i is a positive constant to be designed.

Notice that

$$\sum_{i=1}^{m}\sum_{j=1}^{m}\frac{k_{ij}^2}{p_j}\bar{x}_j^{\mathrm{T}}P_j(\gamma)\bar{x}_j=\sum_{i=1}^{m}\sum_{j=1}^{m}\frac{k_{ji}^2}{p_i}\bar{x}_i^{\mathrm{T}}P_i(\gamma)\bar{x}_i. \tag{56}$$

Let

$$V(t,\bar{x})=\sum_{i=1}^{m}V_i(t,\bar{x}_i)=\sum_{i=1}^{m}\gamma\bar{x}_i^{\mathrm{T}}P_i(\gamma)\bar{x}_i. \tag{57}$$

By using (27), (54) and (56), $V(t, \bar{x})$ can be further written as

$$\begin{aligned}
\dot{V}(t,\bar{x}) \leq & \sum_{i=1}^{m}\Bigg(\varphi_i(\gamma)\bar{x}_i^{\mathrm{T}}P_i(\gamma)\bar{x}_i - \epsilon_i\gamma^2\bar{x}_i^{\mathrm{T}}P_i(\gamma)\bar{x}_i + mk_3 g(\varpi)\mu_i\gamma\bar{x}_i^{\mathrm{T}}P_i(\gamma)\bar{x}_i \\
& + \frac{g(\varpi)\gamma}{k_3}\sum_{j=1}^{m}\frac{k_{ij}^2}{p_j}\bar{x}_j^{\mathrm{T}}P_j(\gamma)\bar{x}_j + \frac{1}{k_1}\gamma\delta_{\Gamma_i}^2(t)\Bigg) \\
= & \sum_{i=1}^{m}(\varphi_i(\gamma)\bar{x}_i^{\mathrm{T}}P_i(\gamma)\bar{x}_i - \epsilon_i\gamma^2\bar{x}_i^{\mathrm{T}}P_i(\gamma)\bar{x}_i + mk_3 g(\varpi)\mu_i\gamma\bar{x}_i^{\mathrm{T}}P_i(\gamma)\bar{x}_i) \\
& + \frac{g(\varpi)\gamma}{k_3}\sum_{i=1}^{m}\sum_{j=1}^{m}\frac{k_{ij}^2}{p_j}\bar{x}_j^{\mathrm{T}}P_j(\gamma)\bar{x}_j + \frac{1}{k_1}\gamma\sum_{i=1}^{m}\delta_{\Gamma_i}^2(t) \\
= & \sum_{i=1}^{m}(\varphi_i(\gamma)\bar{x}_i^{\mathrm{T}}P_i(\gamma)\bar{x}_i - \epsilon_i\gamma^2\bar{x}_i^{\mathrm{T}}P_i(\gamma)\bar{x}_i + mk_3 g(\varpi)\mu_i\gamma\bar{x}_i^{\mathrm{T}}P_i(\gamma)\bar{x}_i) \\
& + \frac{g(\varpi)\gamma}{k_3}\sum_{i=1}^{m}\sum_{j=1}^{m}\frac{k_{ji}^2}{p_i}\bar{x}_i^{\mathrm{T}}P_i(\gamma)\bar{x}_i + \frac{1}{k_1}\gamma\delta_{\max} \\
= & \sum_{i=1}^{m}\left(\varphi_i(\gamma) - \epsilon_i\gamma^2 + mk_3 g(\varpi)\mu_i\gamma + \frac{g(\varpi)\gamma}{k_3}\sum_{j=1}^{m}\frac{k_{ji}^2}{p_i}\right)\bar{x}_i^{\mathrm{T}}P_i(\gamma)\bar{x}_i + \frac{1}{k_1}\gamma\delta_{\max} \\
= & \sum_{i=1}^{m}(\phi_i(\gamma) - \epsilon_i\gamma^2)\bar{x}_i^{\mathrm{T}}P_i(\gamma)\bar{x}_i + \frac{1}{k_1}\gamma\delta_{\max},
\end{aligned}$$

where

$$\phi_i(\gamma) \triangleq \varphi_i(\gamma) + \left(mk_3 g(\varpi)\mu_i + \frac{g(\varpi)}{k_3}\sum_{j=1}^{m}\frac{k_{ji}^2}{p_i}\right)\gamma,$$

and $\delta_{\max}$ is defined in (27). By choosing

$$k_2 = \sqrt{3}\mu_i, \quad k_3 = 1,$$

$\phi_i(\gamma)$ can be written as

$$\begin{aligned}
\phi_i(\gamma) = & \left(1 + \frac{\theta(\mu_i)}{\mu_i}\right)\dot{\gamma} + \left(-1 + k_1\mu_i + \left(k_2 + \frac{3\mu_i^2}{k_2}\right)w_{\max} + \epsilon_i\right)\gamma^2 + \left(mk_3 g(\varpi)\mu_i + \frac{g(\varpi)}{k_3}\sum_{j=1}^{m}\frac{k_{ji}^2}{p_i}\right)\gamma \\
= & \left(1 + \frac{\theta(\mu_i)}{\mu_i}\right)\left(\dot{\gamma} - \frac{\mu_i}{\mu_i + \theta(\mu_i)}(1 - k_1\mu_i - 2\sqrt{3}\mu_i w_{\max} - \epsilon_i)\gamma^2 + \frac{\mu_i}{\mu_i + \theta(\mu_i)}\left(mg(\varpi)\mu_i + g(\varpi)\sum_{j=1}^{m}\frac{k_{ji}^2}{p_i}\right)\gamma\right) \\
= & \left(1 + \frac{\theta(\mu_i)}{\mu_i}\right)(\dot{\gamma} - \alpha_i\gamma^2 + \beta_i\gamma),
\end{aligned}$$

where $\alpha_i, \beta_i, i = 1, 2, \ldots, m$ are defined in (40) and (42).

According to (39), (41) and (44), for every $i \in \{1, 2, \ldots, m\}$ and all $t \in [t_0, t_0 + T)$, we have

$$\begin{aligned}
\dot{\gamma} - \alpha_i\gamma^2 + \beta_i\gamma \leq & \dot{\gamma} - \alpha\gamma^2 + \alpha\beta\gamma \\
= & \frac{(\mathrm{e}^{\alpha\beta(t_0+T)} - \mathrm{e}^{\alpha\beta t_0})\gamma_0}{(\mathrm{e}^{\alpha\beta(t_0+T)} - \mathrm{e}^{\alpha\beta t})^2}(-\alpha(\mathrm{e}^{\alpha\beta(t_0+T)} - \mathrm{e}^{\alpha\beta t_0})\gamma_0 + \alpha\beta\mathrm{e}^{\alpha\beta(t_0+T)}) \\
= & \frac{\alpha(\mathrm{e}^{\alpha\beta(t_0+T)} - \mathrm{e}^{\alpha\beta t_0})^2\gamma_0}{(\mathrm{e}^{\alpha\beta(t_0+T)} - \mathrm{e}^{\alpha\beta t})^2}\left(-\gamma_0 + \frac{\beta\mathrm{e}^{\alpha\beta(t_0+T)}}{\mathrm{e}^{\alpha\beta(t_0+T)} - \mathrm{e}^{\alpha\beta t_0}}\right) \\
\leq & 0.
\end{aligned}$$

Therefore, $\dot{V}(t,\bar{x})$ can be further written as

$$\dot{V}(t,\bar{x}) \leq -\sum_{i=1}^{m} \epsilon_i \gamma^2 \bar{x}_i{}^{\mathrm{T}} P_i(\gamma)\bar{x}_i + \frac{1}{k_1}\gamma\delta_{\max} \leq -\epsilon\gamma\left(V(t,\bar{x}) - \frac{1}{\epsilon k_1}\delta_{\max}\right), \tag{58}$$

where $\epsilon = \min_{i=1,2,\ldots,m}\{\epsilon_i\}$. By using the comparison lemma [46], $V(t,\bar{x})$, for all $t \in [t_0, t_0+T)$, satisfies

$$V(t,\bar{x}) \leq \lambda^{\varsigma}(t,t_0)V(t_0,\bar{x}(t_0)) + \frac{1}{\epsilon k_1}\delta_{\max}(1-\lambda^{\varsigma}(t,t_0)), \tag{59}$$

where $\varsigma = \epsilon(\mathrm{e}^{\alpha\beta T}-1)\gamma_0/\alpha\beta\mathrm{e}^{\alpha\beta T}$ is a positive constant and the scalar function $\lambda(t,t_0) = (\mathrm{e}^{\alpha\beta(t_0+T-t)} - 1)/(\mathrm{e}^{\alpha\beta T}-1) \in (0,1)$ is bounded. Using (33) and (37), we have

$$\begin{aligned} V(t,\bar{x}) &= \sum_{i=1}^{m}\gamma\bar{x}_i{}^{\mathrm{T}}P_i(\gamma)\bar{x}_i = \sum_{i=1}^{m}\gamma^2\bar{x}_i^{\mathrm{T}}L_{\mu_i}P_{\mu_i}L_{\mu_i}\bar{x}_i \\ &\geq \sum_{i=1}^{m}\gamma^2\Lambda_{\mu_i}^{-1}\bar{x}_i^{\mathrm{T}}L_{\mu_i}L_{\mu_i}\bar{x}_i \\ &\geq \chi_1(\gamma_0)\gamma\sum_{i=1}^{m}\bar{x}_i^{\mathrm{T}}\bar{x}_i \\ &= \chi_1(\gamma_0)\gamma\left\|\bar{x}(t)\right\|^2, \end{aligned} \tag{60}$$

where $\chi_1(\gamma_0) \triangleq \min_{i=1,2,\ldots,m}\{\gamma_0\Lambda_{\mu_i}^{-1}\}$. By employing (33), we have

$$\begin{aligned} V(t_0,\bar{x}(t_0)) &= \sum_{i=1}^{m}\gamma_0\bar{x}_i^{\mathrm{T}}(t_0)P_i(\gamma_0)\bar{x}_i(t_0) \\ &= \sum_{i=1}^{m}\gamma_0^2\bar{x}_i^{\mathrm{T}}(t_0)L_{\mu_i}(\gamma_0)P_{\mu_i}L_{\mu_i}(\gamma_0)\bar{x}_i(t_0) \\ &\leq \sigma\gamma_0^2\sum_{i=1}^{m}\bar{x}_i^{\mathrm{T}}(t_0)\bar{x}_i(t_0) \\ &= \chi_2(\gamma_0)\left\|\bar{x}(t_0)\right\|^2, \end{aligned} \tag{61}$$

where $\sigma = \sigma_{\max}(L_{\mu_i}(\gamma_0)P_{\mu_i}L_{\mu_i}(\gamma_0)), i = 1,2,\ldots,m$, and $\chi_2(\gamma_0) \triangleq \sigma\gamma_0^2$. Hence, it follows from (59), (60) and (61) that

$$\begin{aligned} \left\|\bar{x}(t)\right\|^2 \leq& \frac{1}{\chi_1(\gamma_0)\gamma}\left(\lambda^{\varsigma}(t,t_0)V(t_0,\bar{x}(t_0)) + \frac{1}{\epsilon k_1}\delta_{\max}(1-\lambda^{\varsigma}(t,t_0))\right) \\ \leq& \frac{\mathrm{e}^{\alpha\beta(t_0+T)}-\mathrm{e}^{\alpha\beta t}}{(\mathrm{e}^{\alpha\beta(t_0+T)}-\mathrm{e}^{\alpha\beta t_0})\gamma_0}\left(\frac{\chi_2(\gamma_0)}{\chi_1(\gamma_0)}\lambda^{\varsigma}(t,t_0)\left\|\bar{x}(t_0)\right\|^2 + \frac{1-\lambda^{\varsigma}(t,t_0)}{\epsilon k_1\chi_1(\gamma_0)}\delta_{\max}\right) \\ \leq& \chi_3(1-\mathrm{e}^{-\alpha\beta(t_0+T-t)})(\left\|\bar{x}(t_0)\right\|^2 + \delta_{\max}), \end{aligned} \tag{62}$$

where $t \in [t_0, t_0+T)$, χ_3 represents a finite positive constant. By using (62) and the assumption that $\bar{x} = T_{\mathrm{c}}(t)x$ is a Lyapunov transformation, we can get

$$\left\|x(t)\right\|^2 \leq \chi(1-\mathrm{e}^{-\alpha\beta(t_0+T-t)})(\left\|x(t_0)\right\|^2 + \delta_{\max}), \tag{63}$$

where $t \in [t_0, t_0+T)$, χ represents a finite positive constant. Next, we will prove the boundedness of $v(t), \forall t \in [t_0, t_0+T)$. According to (36), (45), (57), (59) and (61), for all $t \in [t_0, t_0+T)$, we have

$$\begin{aligned} \left\|v(t)\right\|^2 &= \sum_{i=1}^{m}\left\|v_i(t)\right\|^2 = \sum_{i=1}^{m} f^2\bar{x}_i^{\mathrm{T}}P_i(\gamma)b_ib_i^{\mathrm{T}}P_i(\gamma)\bar{x}_i \\ &\leq \sum_{i=1}^{m} f^2 b_i^{\mathrm{T}}P_i(\gamma)b_i\bar{x}_i^{\mathrm{T}}P_i(\gamma)\bar{x}_i \end{aligned}$$

$$
\begin{aligned}
&= \sum_{i=1}^{m} \mu_i f^2 \gamma \bar{x}_i^{\mathrm{T}} P_i(\gamma) \bar{x}_i \\
&\le \bar{\mu} V(t, \bar{x}) \\
&\le \bar{\mu} \lambda^{\varsigma}(t, t_0) V(t_0, \bar{x}(t_0)) + \frac{\bar{\mu}}{\epsilon k_1} \delta_{\max}(1 - \lambda^{\varsigma}(t, t_0)) \\
&\le \bar{\mu} \lambda^{\varsigma}(t, t_0) \chi_2(\gamma_0) \left\| \bar{x}(t_0) \right\|^2 + \frac{\bar{\mu}}{\epsilon k_1} \delta_{\max}(1 - \lambda^{\varsigma}(t, t_0)) \\
&\le \vartheta((\mathrm{e}^{\alpha\beta(t_0+T-t)} - 1)^{\varsigma} \left\| x(t_0) \right\|^2 + \delta_{\max}),
\end{aligned} \tag{64}
$$

where $\bar{\mu} = \max_{i=1,2,\ldots,m} \{\mu_i f^2\}$ and ϑ is a finite positive constant. Finally, we will analyze the boundedness of the overall controller composed of (29) and (38). Since in (29), $\Gamma^{-1}(t)$ is bounded for all $t \in \mathbf{R}_{\ge 0}$, according to (64), for all $t \in [t_0, t_0 + T)$, we have

$$
\left\| u(t) \right\|^2 \le \vartheta_{\mathrm{u}}((\mathrm{e}^{\alpha\beta(t_0+T-t)} - 1)^{\varsigma} \left\| x(t_0) \right\|^2 + \delta_{\max}),
$$

where ϑ_{u} is a finite positive constant. As of this point, the proof is finished. ■

Remark 4 *To prevent singularity issues that when t approaches $t_0 + T$, (39) can be updated to*

$$
\gamma = \begin{cases} \frac{\mathrm{e}^{\alpha\beta(t_0+T)} - \mathrm{e}^{\alpha\beta t_0}}{\mathrm{e}^{\alpha\beta(t_0+T)} - \mathrm{e}^{\alpha\beta t}} \gamma_0, & t \in [t_0, t_0 + T_{\#}], \\ \frac{\mathrm{e}^{\alpha\beta(t_0+T)} - \mathrm{e}^{\alpha\beta t_0}}{\mathrm{e}^{\alpha\beta(t_0+T)} - \mathrm{e}^{\alpha\beta(t_0+T_{\#})}} \gamma_0, & t \ge t_0 + T_{\#}, \end{cases}
$$

in which $T_{\#} = T - \delta_T$ with δ_T being a small enough value. [26].

Remark 5 *In Theorem 3, we only utilize the special form of the controllable canonical form of LTV systems during the controller design. The feedback linearization technique is not employed, as it would result in increased energy consumption for the controller.*

4.2 Non-lexicographically Fixed LTV Systems

Assume that the LTV system model depicted in (1) is precisely known, that is $\varpi(t) = 0, \forall t \in \mathbf{R}_{\ge 0}$. Using the controllable canonical form of non-lexicographically fixed LTV systems obtained from Theorem 2 and following the steps of designing the global prescribed-time fault-tolerant controller for lexicographically fixed LTV systems depicted in Theorem 3, the following result can be directly obtained.

Theorem 4 *Let Assumption 4 be fulfilled. Assume that $\bar{x}_{\mathrm{g}} = T_{\mathrm{g}}(t) x_{\mathrm{g}}$ such that the augmented system (20) is transformed into the augmented controllable canonical form (21) is a Lyapunov transformation. Let $T > 0$ be a prescribed time. Then, there exist positive constants α and β, such that the closed-loop system composed of the augmented system (20) and the linear time-varying state feedback*

$$
u = \Gamma_{\mathrm{g}}(t)^{-1} v_{\mathrm{g}}, \quad v_{\mathrm{g}} = \begin{bmatrix} v_{\mathrm{g}1}^{\mathrm{T}} & v_{\mathrm{g}2}^{\mathrm{T}} & \cdots & v_{\mathrm{g}m}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, \tag{65}
$$

$$
v_{\mathrm{g}i} = -f_{\mathrm{g}} b_{\mathrm{g}i}^{\mathrm{T}} P_i(\gamma) \begin{bmatrix} \rho_{\bar{v}_{i,1}}^{[n]} \\ \rho_{\bar{v}_{i,2}}^{[n]} \\ \vdots \\ \rho_{\bar{v}_{i,v_i}}^{[n]} \end{bmatrix} T_{\mathrm{g}}(t) x_{\mathrm{g}}, \quad i = 1, 2, \ldots, m, \tag{66}
$$

where f_{g} is a constant satisfying $f_{\mathrm{g}} > \frac{1}{2(1-\eta_{\max})}$, $\Gamma_{\mathrm{g}}(t)$ and $b_{\mathrm{g}i}$ are defined in (22) and (23), $\bar{v}_{i,j} = \sum_{k=1}^{i-1} v_k + j, j = 1, 2, \ldots, v_i$, and

$$
\gamma = \frac{\mathrm{e}^{\alpha\beta(t_0+T)} - \mathrm{e}^{\alpha\beta t_0}}{\mathrm{e}^{\alpha\beta(t_0+T)} - \mathrm{e}^{\alpha\beta t}} \gamma_0, \quad \gamma_0 = \max \left\{ \frac{k\beta \mathrm{e}^{\alpha\beta(t_0+T)}}{\mathrm{e}^{\alpha\beta(t_0+T)} - \mathrm{e}^{\alpha\beta t_0}}, 1 \right\},
$$

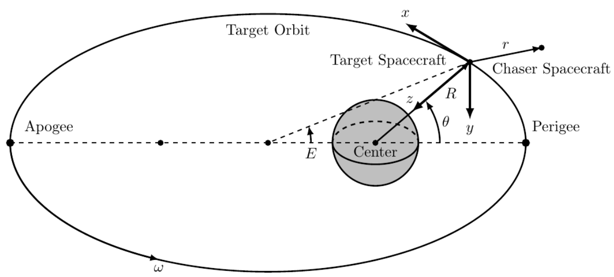

Figure 1: Spacecraft rendezvous system and coordinates

Parameters	Symbol	Values
True anomaly	θ	
The θ-derivative of variable α	$\mathring{\alpha}$	$\mathrm{d}\alpha\ /\mathrm{d}\theta$
Semimajor axis	a	2.4616×10^7 m
Eccentricity	e	0.73074
Geocentric gravitational constant	μ	$3.986 \times 10^{14}\ \mathrm{m}^3/\mathrm{s}^2$
Constant k	k	$2.267 \times 10^{-2}/\mathrm{s}^{1/2}$
Specific angular momentum	h	$6.762 \times 10^{10}\ \mathrm{m}^2/\mathrm{s}$

Table 1: The parameters of the elliptical orbital rendezvous system

in which $k \geq 1$ is a constant, and $P_i(\gamma)$ is the UPDS to the PLE (32) with dimensions v_i, satisfies

$$\|x_{\mathrm{g}}(t)\|^2 \leq \chi_{\mathrm{g}}(1 - \mathrm{e}^{-\alpha\beta(t_0+T-t)})(\|x_{\mathrm{g}}(t_0)\|^2 + \delta_{\mathrm{g}\max}),$$

for all $t \in [t_0, t_0 + T)$ and any $x_{\mathrm{g}}(t_0) \in \mathbf{R}^n$, in which χ_{g} is a finite positive constant and $\delta_{\mathrm{g}\max}$ takes the similar form to (26) and (27) with the only difference being that $\Gamma(t)$ is replaced by $\Gamma_{\mathrm{g}}(t)$. Moreover, the overall controller $u(t)$ composed of (65) and (66) satisfies, for all $t \in [t_0, t_0 + T)$,

$$\|u(t)\|^2 \leq \vartheta_{\mathrm{ug}}((\mathrm{e}^{\alpha\beta(t_0+T-t)} - 1)^{\varsigma_{\mathrm{g}}} \|x_{\mathrm{g}}(t_0)\|^2 + \delta_{\mathrm{g}\max}),$$

where ϑ_{ug} and ς_{g} are finite positive constants. In addition, when the float fault $\delta(t)$ exists in the LTV system (1), namely, $\delta(t) \neq 0$, the closed-loop system is UPT-ISS+C in sense of Definition 2 and the controller $u(t)$ is bounded. When the float fault $\delta(t)$ does not exist in the LTV system (1), namely, $\delta(t) = 0$, the closed-loop system is GUPTS in sense of Definition 3 and the controller $u(t)$ satisfies $\lim_{t \to t_0+T} u(t) = 0$.

5 Applications to the Elliptical Orbital Rendezvous System

The spacecraft rendezvous system in elliptical orbit is shown in Figure 1. The parameters related to the target spacecraft is represented in Table 1. Denote

$$\begin{aligned} \xi &= \left[\begin{array}{cccccc} x & y & z & \dot{x} & \dot{y} & \dot{z} \end{array}\right]^{\mathrm{T}}, \\ u = a_{\mathrm{f}} &= \left[\begin{array}{ccc} a_{\mathrm{f}}^x(t) & a_{\mathrm{f}}^y(t) & a_{\mathrm{f}}^z(t) \end{array}\right]^{\mathrm{T}}, \end{aligned}$$

where a_{f} signifies the acceleration vector generated by the thrust of the pursuer spacecraft.

When the target spacecraft reaches perigee at time $t = 0$, the true anomaly is also equal to zero at the initial moment. In this case, there exists a one-to-one correspondence between t and the true anomaly θ. When $R \gg r$, the elliptical orbital rendezvous system is represented as the following LTV form with model uncertainties and actuator faults

$$\left\{ \begin{array}{l} \dot{\xi} = (1 + \varpi(t))\mathcal{A}(t)\xi + \mathcal{B}(t)((1 - \eta(t))u + \delta(t)) \\ y = \mathcal{C}(t)\xi, \end{array} \right. \tag{67}$$

where

$$\mathcal{A}(t)=\begin{bmatrix}0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\w^2-kw^{3/2}&0&\dot{w}&0&0&2w\\0&-kw^{3/2}&0&0&0&0\\-\dot{w}&0&w^2+2kw^{3/2}&-2w&0&0\end{bmatrix},$$

$$\mathcal{B}(t)=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\\1&0&0\\0&1&0\\0&0&1\end{bmatrix},\ \mathcal{C}(t)=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\\0&0&0\\0&0&0\\0&0&0\end{bmatrix}^{\mathrm{T}}$$

and $k=\mu/h^{\frac{3}{2}}=$ constant, w signifies the orbital angular velocity. Denote $\rho=1+e\cos(\theta)$. Applying the state transformation $[x(\theta),y(\theta),z(\theta)]^{\mathrm{T}}=\rho\,[x,y,z]^{\mathrm{T}}$ into the system (67) yields

$$\begin{aligned}\mathring{X}(\theta)=&(1+\varpi(\theta))A(\theta)X(\theta)+B(\theta)((1-\eta(\theta))U(\theta)+\delta(\theta))\\=&(1+\varpi(\theta))\begin{bmatrix}0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\\frac{3}{\rho}&0&0&0&2&0\\0&0&0&-2&0&0\\0&0&-1&0&0&0\end{bmatrix}X(\theta)+\frac{1}{k^4\rho^3}\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\\1&0&0\\0&1&0\\0&0&1\end{bmatrix}((1-\eta(\theta))U(\theta)+\delta(\theta)),\end{aligned}\tag{68}$$

$$\begin{aligned}Y(\theta)=&C(\theta)X(\theta)\\=&\begin{bmatrix}\frac{1}{\rho}&0&0&0&0&0\\0&\frac{1}{\rho}&0&0&0&0\\0&0&\frac{1}{\rho}&0&0&0\end{bmatrix}X(\theta),\end{aligned}\tag{69}$$

where $X(\theta)=[x(\theta),y(\theta),z(\theta),\mathring{x}(\theta),\mathring{y}(\theta),\mathring{z}(\theta)]^{\mathrm{T}}$, $U(\theta)=u(t)=[a_{\mathrm{f}}^{x}(\theta),a_{\mathrm{f}}^{y}(\theta),a_{\mathrm{f}}^{z}(\theta)]^{\mathrm{T}}$ and $Y(\theta)=y(t)$. It follows from (68) and (69) that the elliptical orbital rendezvous system depicted in the θ-domain is lexicographically fixed with controllability indices $\{2,2,2\}$. Then, according to (16) and (17), computing the transformation matrix $T_{\mathrm{c}}(\theta)$ yields

$$T_{\mathrm{c}}(\theta)=\begin{bmatrix}t_{\mathrm{c}1}(\theta)&0&0&0&0&0\\t_{\mathrm{c}2}(\theta)&0&0&t_{\mathrm{c}1}(\theta)&0&0\\0&t_{\mathrm{c}1}(\theta)&0&0&0&0\\0&t_{\mathrm{c}2}(\theta)&0&0&t_{\mathrm{c}1}(\theta)&0\\0&0&t_{\mathrm{c}1}(\theta)&0&0&0\\0&0&t_{\mathrm{c}2}(\theta)&0&0&t_{\mathrm{c}1}(\theta)\end{bmatrix},$$

where $t_{\mathrm{c}1}(\theta)=k_{\mathrm{c}1}\,\rho^3, t_{\mathrm{c}2}(\theta)=k_{\mathrm{c}2}\,e\,\sin(\theta)\,\rho^2, k_{\mathrm{c}1}=2.6412\times10^{-7}, k_{\mathrm{c}2}=-7.9237\times10^{-7}$. It is easy to verify that $T_{\mathrm{c}}(\theta)$ is a Lyapunov transformation matrix. Applying the transformation matrix $T_{\mathrm{c}}(\theta)$ to (68) and (69) gives the following controllable canonical form associated with θ

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}\theta}\bar{X}(\theta)&=(1+\varpi(\theta))\bar{A}(\theta)\bar{X}(\theta)+\bar{B}(\theta)((1-\eta(\theta))U(\theta)+\delta(\theta)),\\\bar{Y}(\theta)&=\bar{C}(\theta)\bar{X}(\theta),\end{aligned}$$

where

$$\bar{B}(\theta)=\begin{bmatrix}0&0&0\\1&0&0\\0&0&0\\0&1&0\\0&0&0\\0&0&1\end{bmatrix},$$

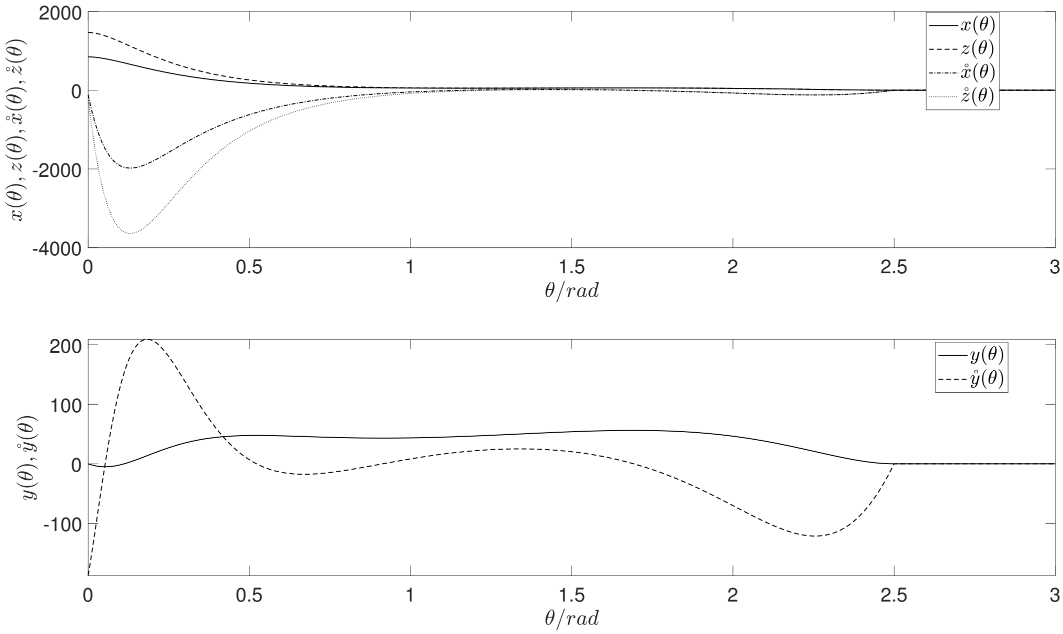

Figure 2: The state responses for elliptical orbital rendezvous system subject to float fault $\delta(\theta)$

in which $\Gamma(\theta) = I_3$. Following the steps in (29) and (38), the controller can be determined as

$$\begin{aligned} U(\theta) &= \left[\begin{array}{ccc} v_1^{\mathrm{T}} & v_2^{\mathrm{T}} & v_3^{\mathrm{T}} \end{array}\right]^{\mathrm{T}} x, \\ v_1 &= -f \left[\begin{array}{cc} \gamma^2 & 2\gamma \end{array}\right] \left[\begin{array}{c} \rho_1^{[6]} \\ \rho_2^{[6]} \end{array}\right] T_{\mathrm{c}}(\theta), \\ v_2 &= -f \left[\begin{array}{cc} \gamma^2 & 2\gamma \end{array}\right] \left[\begin{array}{c} \rho_3^{[6]} \\ \rho_4^{[6]} \end{array}\right] T_{\mathrm{c}}(\theta), \\ v_3 &= -f \left[\begin{array}{cc} \gamma^2 & 2\gamma \end{array}\right] \left[\begin{array}{c} \rho_5^{[6]} \\ \rho_6^{[6]} \end{array}\right] T_{\mathrm{c}}(\theta), \end{aligned}$$

Then, a numerical simulation will be employed to demonstrate the effectiveness of the proposed prescribed-time fault-tolerant controller for elliptical orbital rendezvous system. Let $\varpi(\theta) = 0.01\cos(\theta), \eta(\theta) = 0.01\sin(\theta) + 0.01, \delta(\theta) = 0.004\sin(\theta)$. Choose the initial state as $[847.5, 0, 1467.9, -112.9, -187.7, -195.6]^{\mathrm{T}}$. Set $T = 2.5$. The parameters of the proposed prescribed-time fault-tolerant controller are given in Table 2. Our simulation will be implemented on the nonlinear model of spacecraft rendezvous system [47]. The state responses and controller are plotted in Figure 2 and Figure 3, respectively. It is evident that the state responses converge to zero within the specified time and the controller is bounded. In addition, when the float fault $\delta(\theta)$ does not exist, namely, $\delta(\theta) = 0$, employing the same controller with parameters described in Table 2, the state responses and controller are plotted in Figure 4 and Figure 5, respectively. It can be observed that the state responses converge to zero within the specified time and the controller is bounded and approaches zero after the prescribed time.

6 Conclusion

This paper investigated the prescribed-time fault-tolerant control problem for uniformly controllable linear time-varying (LTV) systems, whether lexicographically fixed or non-lexicographically fixed. For lexicographically fixed LTV systems, we allow for uncertainties in the system, while for non-lexicographically fixed LTV systems, the system model is needed to be accurately known. By utilizing the clear structure of controllable

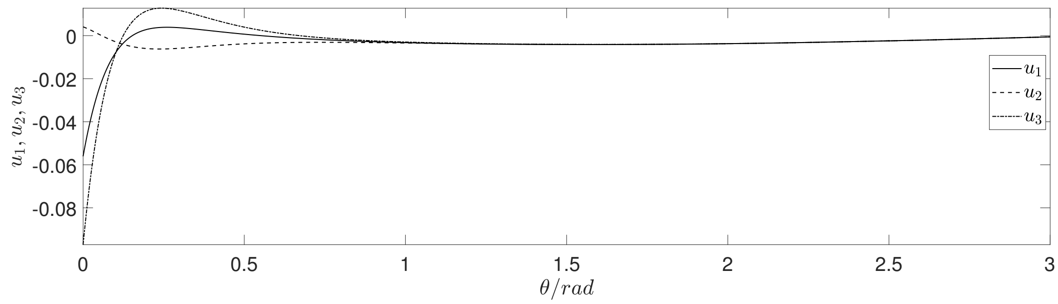

Figure 3: The controller for elliptical orbital rendezvous system subject to float fault $\delta(\theta)$

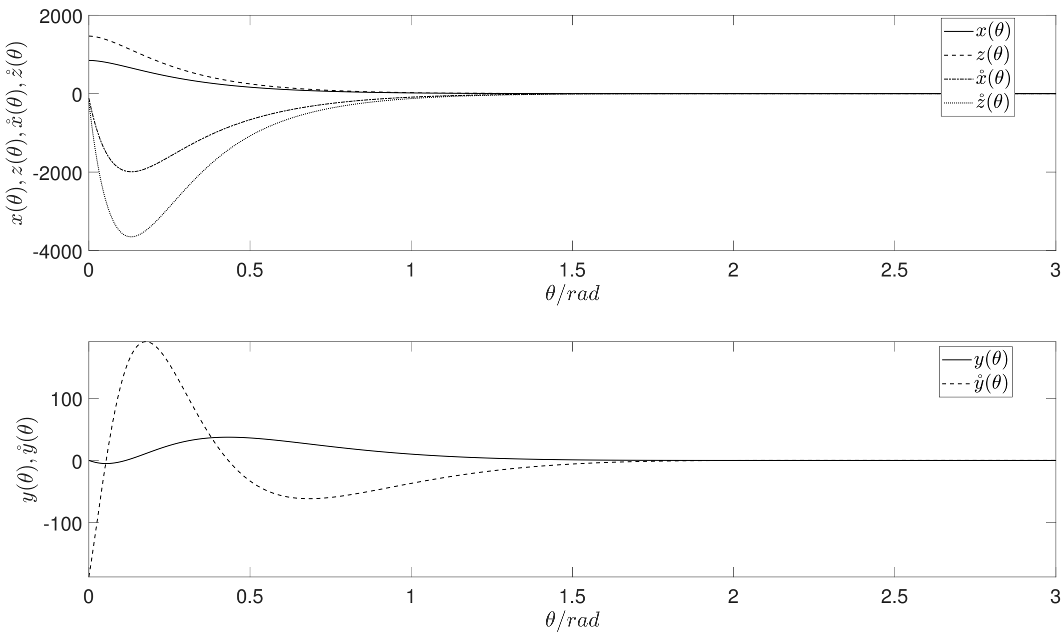

Figure 4: The state responses for elliptical orbital rendezvous system without float fault $\delta(\theta)$

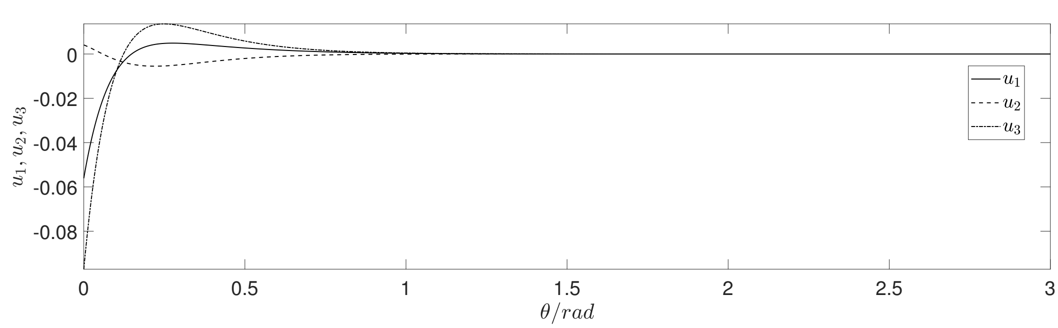

Figure 5: The controller for elliptical orbital rendezvous system without float fault $\delta(\theta)$

Parameters	Value
Initial moment	$\theta_0 = 0 \quad (t_0 = 0)$
α	0.0638
f	1.25
β	0.04
γ_0	$\frac{1.01\beta \mathrm{e}^{\alpha\beta T}}{\mathrm{e}^{\alpha\beta T}-1}$

Table 2: The parameters of the prescribed-time fault-tolerant controller designed for elliptical orbital rendezvous system

canonical form and employing the properties of the parametric Lyapunov equation, a global prescribed-time fault-tolerant controller is developed. The uncertainties present in LTV systems are reflected in the selection of controller parameters. By choosing a suitable Lyapunov-like function, the system state will be proven to converge to zero within the prescribed time. In addition, the designed controller is bounded and maintains a linear, concise and smooth form. Finally, the simulation of the elliptical orbital rendezvous system verified the effectiveness of the proposed controller.

Appendix

A1: The proof of Lemma 1

First, we introduce the Dolezar's theorem.

Lemma 4 *[41] Let $n, k \in \mathbf{N}^+$, $r \in \{1, 2, \dots, n\}$, and $M(t) \in \mathcal{J}^k(\mathbf{R}_{\geq 0}, \mathbf{R}^{n\times n})$. If $\operatorname{rank}(M(t)) = r$ for all $t \in \mathbf{R}_{\geq 0}$, then there exists a nonsingular matrix $T(t) \in \mathcal{J}^k(\mathbf{R}_{\geq 0}, \mathbf{R}^{n\times n})$ satisfying*

$$M(t)T(t) = \begin{bmatrix} V(t) & 0_{n\times(n-r)} \end{bmatrix},$$

where $\operatorname{rank}(V(t)) = r, \forall t \in \mathbf{R}_{\geq 0}$. In addition, for each $m \in \{1, 2, \dots, n\}$, if $T(t)$ is partitioned as

$$T(t) = \begin{bmatrix} \mathcal{E}(t) & \mathcal{D}(t) \end{bmatrix},$$

where $\mathcal{D}(t) \in \mathbf{R}^{n\times m}$, then $\mathcal{D}(t) \in \mathcal{S}^\infty(\mathbf{R}_{\geq 0}, \mathbf{R}^{n\times m})$ and $(\mathcal{D}^{\mathrm{T}}(t)\mathcal{D}(t))^{-1}\mathcal{D}^{\mathrm{T}}(t) \in \mathcal{S}^\infty(\mathbf{R}_{\geq 0}, \mathbf{R}^{m\times n})$.

Proof. According to Lemma 4, there exists a nonsingular matrix $T(t) = [\tau_1, \tau_2, \dots, \tau_n] \in \mathcal{J}^k(\mathbf{R}_{\geq 0}, \mathbf{R}^{n\times n})$ such that

$$\begin{bmatrix} M(t) \\ 0 \end{bmatrix} T(t) = \begin{bmatrix} F(t) & 0 \\ 0 & 0 \end{bmatrix} \in \mathcal{J}^k(\mathbf{R}_{\geq 0}, \mathbf{R}^{n\times n}), \tag{70}$$

where $\operatorname{rank}(F(t)) = r, \forall t \in \mathbf{R}_{\geq 0}$. Set

$$\mathcal{V}(t) = \begin{bmatrix} \tau_{r+1} & \tau_{r+2} & \cdots & \tau_n \end{bmatrix} \in \mathcal{J}^k(\mathbf{R}_{\geq 0}, \mathbf{R}^{n\times(n-r)}). \tag{71}$$

According to (70) and (71), we have

$$M(t)\mathcal{V}(t) = 0_{r\times(n-r)}. \tag{72}$$

Referring to Lemma 4, we have

$$\begin{aligned} \mathcal{V}(t) &\in (\mathcal{J}^k, \mathcal{S}^\infty)(\mathbf{R}_{\geq 0}, \mathbf{R}^{n\times(n-r)}), \\ (\mathcal{V}^{\mathrm{T}}(t)\mathcal{V}(t))^{-1}\mathcal{V}^{\mathrm{T}}(t) &\in (\mathcal{J}^k, \mathcal{S}^\infty)(\mathbf{R}_{\geq 0}, \mathbf{R}^{(n-r)\times n}). \end{aligned}$$

Set

$$S(t) \triangleq (\mathcal{V}(t)^{\mathrm{T}}\mathcal{V}(t))^{-1}\mathcal{V}^{\mathrm{T}}(t) \in (\mathcal{J}^k, \mathcal{S}^\infty)(\mathbf{R}_{\geq 0}, \mathbf{R}^{(n-r)\times n}). \tag{73}$$

Then, by using (72) and (73), we have

$$S(t)M^{\mathrm{T}}(t)(M(t)M^{\mathrm{T}}(t))^{-1} = 0_{(n-r)\times r},$$

$$S(t)\mathcal{V}(t) = I_{n-r}.$$

which imply

$$\begin{bmatrix} M(t) \\ S(t) \end{bmatrix} \begin{bmatrix} M^{\mathrm{T}}(t)(M(t)M^{\mathrm{T}}(t))^{-1} & \mathcal{V}(t) \end{bmatrix} = I_n. \tag{74}$$

■

A2: The proof of Lemma 2

Let $(P(t))_{\mathrm{e}}$ represent taking the last n_{e} rows of the matrix $P(t)$ to form a new matrix. Define the following generalized controllability matrix

$$Q_{\mathrm{g}}(t) \triangleq \begin{bmatrix} Q_{\mathrm{g}1}(t) & Q_{\mathrm{g}2}(t) & \cdots & Q_{\mathrm{g}m}(t) \end{bmatrix} \in \mathbf{R}^{n\times n_{\mathrm{g}}}, \tag{75}$$

$$Q_{\mathrm{g}i}(t) = \begin{bmatrix} b_i(t) & \mathscr{R}(b_i(t)) & \cdots & \mathscr{R}^{v_i-1}(b_i(t)) \end{bmatrix} \in \mathbf{R}^{n\times v_i},$$

where $i = 1, 2, \ldots, m$ and $\mathrm{rank}(Q_{\mathrm{g}}(t)) = n$, $\forall t \in \mathbf{R}_{\geq 0}$. According to Lemma 1, there exists $\tilde{Q}_{\mathrm{g}}(t) \in \mathbf{R}^{(n_{\mathrm{g}}-n)\times n_{\mathrm{g}}}$ such that

$$\mathrm{rank}\begin{bmatrix} Q_{\mathrm{g}}(t) \\ \tilde{Q}_{\mathrm{g}}(t) \end{bmatrix} = n_{\mathrm{g}}, \quad \forall t \in \mathbf{R}_{\geq 0}.$$

Define $\hat{Q}_{\mathrm{g}}(t) \triangleq \left[Q_{\mathrm{g}}^{\mathrm{T}}(t), \tilde{Q}_{\mathrm{g}}^{\mathrm{T}}(t)\right]^{\mathrm{T}}$. Consider the partition

$$B_{\mathrm{g}}(t) = \begin{bmatrix} b_{\mathrm{g}1}(t) & b_{\mathrm{g}2}(t) & \cdots & b_{\mathrm{g}m}(t) \end{bmatrix}.$$

It follows from the result in [34] that

$$\mathscr{R}^j_{A_{\mathrm{g}}}(b_{\mathrm{g}i}(t)) = \begin{bmatrix} \mathscr{R}^j(b_i(t)) \\ \left(\mathscr{R}^j_{A_{\mathrm{g}}}(b_{\mathrm{g}i}(t))\right)_{\mathrm{e}} \end{bmatrix} \tag{76}$$

$$\left(\mathscr{R}^j_{A_{\mathrm{g}}}(b_{\mathrm{g}i}(t))\right)_{\mathrm{e}} = \begin{bmatrix} A_2(t) & A_1(t) \end{bmatrix} \mathscr{R}^{j-1}_{A_{\mathrm{g}}}(b_{\mathrm{g}i}(t)) - \left(\frac{\mathrm{d}}{\mathrm{d}t}\mathscr{R}^{j-1}_{A_{\mathrm{g}}}(b_{\mathrm{g}i}(t))\right)_{\mathrm{e}}, \tag{77}$$

where $i = 1, 2, \ldots, m, j = 1, 2, \ldots$. Let $\hat{Q}_{\mathrm{g}}(t)$ be the controllability matrix of the augmented LTV system (20) with observability indices $\{v_1, v_2, \ldots, v_m\}$. Referring to (76) and (77), for $i = 1, 2, \ldots, m, j = 1, 2, \ldots, v_i$, we can get

$$\begin{bmatrix} A_2(t) & A_1(t) \end{bmatrix} \hat{Q}_{\mathrm{g}}(t) - \left(\frac{\mathrm{d}}{\mathrm{d}t}\hat{Q}_{\mathrm{g}}(t)\right)_{\mathrm{e}} = (\check{Q}_{\mathrm{g}}(t))_{\mathrm{e}}, \tag{78}$$

where

$$\check{Q}_{\mathrm{g}}(t) = \begin{bmatrix} \check{Q}_{\mathrm{g}1}(t) & \check{Q}_{\mathrm{g}2}(t) & \cdots & \check{Q}_{\mathrm{g}m}(t) \end{bmatrix} \in \mathbf{R}^{n_{\mathrm{g}}\times n_{\mathrm{g}}}, \tag{79}$$

$$\check{Q}_{\mathrm{g}i}(t) = \begin{bmatrix} \mathscr{R}^1_{A_{\mathrm{g}}}(b_{\mathrm{g}i}(t)) & \mathscr{R}^2_{A_{\mathrm{g}}}(b_{\mathrm{g}i}(t)) & \cdots & \mathscr{R}^{v_i}_{A_{\mathrm{g}}}(b_{\mathrm{g}i}(t)) \end{bmatrix}. \tag{80}$$

in which $\check{Q}_{\mathrm{g}i}(t) \in \mathbf{R}^{n_{\mathrm{g}}\times v_i}, i = 1, 2, \ldots, m$. It follows from (78), (79) and (80) that

$$\begin{bmatrix} A_2(t) & A_1(t) \end{bmatrix} = \left(\left(\frac{\mathrm{d}}{\mathrm{d}t}\hat{Q}_{\mathrm{g}}(t)\right)_{\mathrm{e}} + (\check{Q}_{\mathrm{g}}(t))_{\mathrm{e}}\right)\hat{Q}_{\mathrm{g}}^{-1}(t).$$

According to the fact that $\hat{Q}_{\mathrm{g}}(t)$ is the controllability matrix of the augmented LTV system (20), we can derive

$$B_{\mathrm{e}}(t) = \left(\hat{Q}_{\mathrm{g}}(t)\begin{bmatrix} \left(\rho^{[n]}_{\tilde{v}_1}\right)^{\mathrm{T}} & \left(\rho^{[n]}_{\tilde{v}_2}\right)^{\mathrm{T}} & \cdots & \left(\rho^{[n]}_{\tilde{v}_m}\right)^{\mathrm{T}} \end{bmatrix}\right)_{\mathrm{e}},$$

where $\tilde{v}_i = \sum\limits_{i=1}^{i-1} v_i + 1, i = 1, 2, \ldots, m$.

A3: Some Preliminaries

First, we recall the well-known Lyapunov transformation.

Definition 4 *The transformation $\bar{x} = L(t)x$ is referred to as a Lyapunov transformation if for all $t \in \mathbf{R}_{\geq 0}$*

1. *$L(t)$ is nonsingular and continuously differentiable.*
2. *$L^{-1}(t)$, $L(t)$ and $\dot{L}(t)$ are bounded.*

Then, we provide the following lemma to test the boundedness of inverse matrices.

Lemma 5 *[48] If the nonsingular matrix $L(t)$ is bounded and $|\det L(t)|$ has a lower bound on the interval $\mathbf{R}_{\geq 0}$, then, there exist a positive constant l such that*

$$\left\|L^{-1}(t)\right\| \leq l, \quad \forall t \in \mathbf{R}_{\geq 0}.$$

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