# A unified adaptive event-triggered output feedback consensus for multi-agent systems with or without output constraints

Zixuan Huang[1], Hamid Reza Karimi[2], Ben Niu[3], Lun Li[4], and Xudong Zhao[5]

[1]Bohai University
[2]Politecnico di Milano
[3]Shandong Normal University
[4]Weifang University
[5]Dalian University of Technology

August 30, 2024

**Abstract**

This paper focuses on the issue of adaptive event-triggered leader-follower consensus for multi-agents systems with output constraints and dead-zone inputs. By introducing an advanced nonlinear mapping technique to obtain the unconstrained auxiliary variables of constrained system states, a new systems model without output constraints is constructed. Unlike existing schemes, the proposed strategy can be used in both constrained and unconstrained situations without requiring changes to the control structure. Moreover, a state estimator is constructed to observe the unavailable states. To conserve communication resources, an event-triggered rule with a dynamic threshold is designed to decrease superfluous information transmissions from the controller to the actuator. It is proven that all signals in closed-loop systems are ultimately bounded and the system output does not violate the given constraint range. At last, a numerical simulation example is provided to confirm the correctness and efficiency of the proposed method.

figures/FIGURE-1/FIGURE-1-eps-converted-to.pdf

figures/FIGURE-2/FIGURE-2-eps-converted-to.pdf

figures/FIGURE-3/FIGURE-3-eps-converted-to.pdf

figures/FIGURE-4/FIGURE-4-eps-converted-to.pdf

figures/FIGURE-5/FIGURE-5-eps-converted-to.pdf

figures/FIGURE-6/FIGURE-6-eps-converted-to.pdf

figures/FIGURE-7/FIGURE-7-eps-converted-to.pdf

figures/FIGURE-8/FIGURE-8-eps-converted-to.pdf

Received ; | Revised ; | Accepted
DOI: xxx/xxxx

ARTICLE TYPE

# A unified adaptive event-triggered output feedback consensus for multi-agent systems with or without output constraints

Zixuan Huang*[1] | Hamid Reza Karimi[2] | Ben Niu[3] | Lun Li[4] | Xudong Zhao[5]

[1]College of Control Science and Engineering, Bohai University, Jinzhou, 121013, China
[2]Department of Mechanical Engineering, Politecnico di Milano, Milan, 20156 , Italy
[3]School of Information Science and Engineering, Shandong Normal University, Jinan 250014, China
[4]College of Machinery and Automation, Weifang University, Weifang 261061, China
[5]Faculty of Electronic Information and Electrical Engineering, Dalian University of Technology, Dalian, 116024, China

**Correspondence**
*Corresponding author name, Corresponding address. Email: huangzixuan0301@163.com

**Summary**

This paper focuses on the issue of adaptive event-triggered leader-follower consensus for multi-agents systems with output constraints and dead-zone inputs. By introducing an advanced nonlinear mapping technique to obtain the unconstrained auxiliary variables of constrained system states, a new systems model without output constraints is constructed. Unlike existing schemes, the proposed strategy can be used in both constrained and unconstrained situations without requiring changes to the control structure. Moreover, a state estimator is constructed to observe the unavailable states. To conserve communication resources, an event-triggered rule with a dynamic threshold is designed to decrease superfluous information transmissions from the controller to the actuator. It is proven that all signals in closed-loop systems are ultimately bounded and the system output does not violate the given constraint range. At last, a numerical simulation example is provided to confirm the correctness and efficiency of the proposed method.

**KEYWORDS:**
Output constraints, nonlinear mapping technique, state estimator, event-triggered control, consensus control

## 1 | INTRODUCTION

In recent years, multi-agent systems (MASs) have become a hot topic in the control field due to their wide applications in practical engineering, such as manufacturing [1], robotics [2], unmanned aerial vehicles [3,4,5], formation control of spacecraft [6], sensor networks [7,8], etc. Compared with single control systems, the multi-agent systems, which are composed of a finite number of agents with perception, reasoning and decision-making capabilities, can perform large-scale complex tasks by designing a suitable control protocol based on mutual communications between the agents, and it has outstanding characteristics, including high reliability, resource sharing, good real-time performance, strong flexibility, and so on. For the multi-agent systems, a fundamental problem is how to design an appropriate control strategy such that each agent continuously adjusts its own behaviours in light of information from a leader and its neighbour agents, and make the state of all agents eventually reach a identical value or agreement, which is called as the consensus problem. Moreover, the consensus issue commonly can be classified into leader-following consensus problem [9] and leaderless consensus problem [10]. In [11], a leader-following consensus tracking issue was addressed for linear multi-agent systems with external disturbances. Considering a fact that most real systems contain nonlinearities, a distributed consensus scheme for nonlinear multi-agent systems was reported in [12]. It should be noticed that control signals are transmitted by using a periodic sampling way in the above consensus algorithms, which probably results in the excessive consumption of both communication and computation resources of the network. In addition, some significant information may be missed when communication resources are limited.

To deal with the above-mentioned problems, event-triggered control strategies have been reported in [13,14,15], which is distinct from the time-triggered control strategy. Specifically, control signals under the latter one are updated at the end or beginning of each time interval (time-triggered control has the same trigger interval), whereas control signals of the former are updated only when the deviation between the desired states and the current states violates a predefined threshold. In [13], the superiority of event-triggered control strategy was verified, where the triggered condition was set as a fixed threshold value. Note that this event-triggered control strategy is not suitable for some practical cases especially when the present state deviates from the expected state in a short time interval. In [16], an improved event-triggered control strategy with dynamic triggered condition was reported, which can adjust dynamically the time to update controller and enhance system performances. Nevertheless, the above event-triggered control strategies are developed with the assumption that the closed-loop systems have input-to-state stability with respect to the measurement errors, and to ensure this assumption is difficult. By designing simultaneously triggering events and adaptive controllers, the authors in [17] proposed an event-triggered fuzzy adaptive control strategy for nonlinear systems, which successfully avoided such an assumption. Following the work [17], several fruitful and crucial results were developed for multi-agents systems [18,19,20,21,22,23], which can well balance communication resources and system performances. For example, in [21], an adaptive event-triggered dynamic surface consensus control strategy was reported for uncertain multi-agents systems with unknown external disturbances, and good tracking abilities was gained. In [22], the filtering errors in [21] were eliminated by constructing a compensation signal.

On the other hand, due to the requirements of safety regulations and physical restrictions, the output constraints problem is often encountered in real engineering, and poses a challenge to the design of controllers. A widely used approach to solving the output constraints problem is the barrier Lyapunov function (BLF) strategy [24,25,26,27,28,29], which creates a scalar function and transforms the output constraints into error constraints. In fact, there exist two restrictions in the above-mentioned BLF-based schemes: (I) These BLF-based schemes depend on the the feasibility conditions of virtual controllers, namely, such schemes need to find a set of optimal parameters such that virtual controllers are maintained within predefined constraints areas, which increases design difficulties. Besides, such optimal parameters may not exist when the predefined constraints areas are small. (II) These schemes can only deal with constrained systems and cannot be directly applied to unconstrained ones. A specific example is a robot sometimes working in a free (unconstrained) area and sometimes working in a narrow (constrained) realm, requiring that both constrained and unconstrained cases need to be handled under a unified framework. To remove the feasibility conditions, in [30,31], two different types of mapping techniques were developed. By applying such techniques, the unconstrained systems models of the original constrained systems are constructed, which reduced the design difficulties. However, the above methods are still subject the restriction (II). Recently, some improved mapping techniques were developed in [32,33], and the proposed strategy can be applied to both constrained and unconstrained control systems without changing the control structure, where the unconstrained case is considered as a constraint function tends to infinity. Although these methods are not limited by restrictions (I) and (II), they are not applicable to multi-agent systems, due to the existence of interactions between agents and between agents and leaders, which inspires this study.

Inspired by the above discussions, this paper investigates an adaptive event-triggered leader-following consensus control issue for nonlinear multi-agent systems with unmeasured states and output constraints under undirected communication networks. Firstly, a fuzzy state observer is constructed to estimate the unmeasurable state. Then, based on this observer, the nonlinear mapping technique, the command filtering technique and the event-triggered mechanism are combined into the backstepping design process such that the proposed control scheme is able to ensure that the system output remains within the given constraint interval. Compared with the existing methods, the main contributions of this paper are concluded as follows:

(1) An adaptive event-triggered output-feedback consensus control scheme for nonlinear multi-agent systems is proposed, which can handle both cases with and without output constraints under a unified framework and guarantee the system to be eventually stable. Besides, an event-triggered mechanism with time-varying thresholds is introduced to balance communication resources and system performances, and the assumption that the closed-loop systems have input-to-state stability with respect to measurement errors is removed.

(2) Different from existing methods in literature [28,29,30,31] for dealing with state constraints, an improved nonlinear mapping method is used to solve the output constraint problem, which allows the designed controllers to be applied in both the cases without altering the control structures and the feasibility conditions of virtual controllers are eliminated in this work.

The remainder of this essay is organized as follows. In **Section** 2, the MASs system is described in detail, and the problem under study is formulated. The state estimator, the adaptive event-based triggered consensus controller and the stability analysis are given in **Section** 3. In **Section** 4 gives a Numerical simulation example. The important results are summarized in **Section** 5.

## 2 | PROBLEM STATEMENT AND PRELIMINARIES

### 2.1 | Undirected Graph Theory and Notations

To address the distributed consensus problems and model the interaction networks between agents and between agents and leaders, a graph theory is used in this article similar to the work[?] . To facilitate, some symbols and basic graph theorems are repeated here.

The undirected graph that contains one leader and N followers is set as $G = (V, E, \Delta)$, where $V = \{V_1, V_2, ..., V_n\}$ expresses the nonempty set of nodes (agents), $E \subseteq \mathrm{V} \times \mathrm{V}$ stands for an edge set of $G$, $\Delta = [a_{i,j}] \in R^{n\times n}$ represents a weighted adjacency matrix. Because $G$ is an undirected graph, the edge $(V_i, V_j) \in E$ is equivalent to $(V_j, V_i) \in E$. The aggregation of neighbors of nodes $i$ can be denoted as $N_i = \{V_j|(V_j, V_i) \in E, i \neq j\}$. Each element of the matrix $\Delta$ can be specified as $a_{i,j} = 0$ or $a_{i,j} = 1$, if $a_{i,j} = 1$, which mean that the information can be transferred from node $i$ to node $j$, otherwise $a_{i,j} = 0$. Obviously, it easily can be get that $a_{i,j} = a_{j,i} = 1$ or $a_{i,j} = a_{j,i} = 0$ for undirected graph. By setting the degree matrix as $D = diag\{d_1, d_2, ...d_n\} \in R^{n\times n}$ and letting its each elements as $d_i = \sum_{j=1}^{n} a_{i,j}$, then the graph Laplacian matrix can be defined as $L = D - A$. Moreover, the pinning matrix is configured as $B = diag\{\mu_1, \mu_2, ..., \mu_n\}$. The node $i$ (follower) can communicate with the leader when $\mu_i = 1$, if can not, then $\mu_i = 0$.

### 2.2 | System Descriptions

Consider the MASs including a leader and $M$ agents (followers), and each follower is described as

$$\begin{cases} \dot{x}_{i,p} = x_{i,p+1} + \phi_{i,p}(\underline{x}_{i,p}) + d_{i,p} \\ \dot{x}_{i,n} = m_i(u_i) + \phi_{i,n}(\underline{x}_{i,n}) + d_{i,n} \\ y_i = x_{i,1}, \qquad p = 1, 2, \dots, n-1, \end{cases} \tag{1}$$

where $i = \{1, 2, ..., M\}$ denotes the i-th agent and $M$ is the sum of the used agents, $\underline{x}_{i,n} = [x_{i,1}, x_{i,2}, ..., x_{i,n}]^T \in R^n$ and system states $x_{i,p}$ is unavailable, and $y_i \in R$ stands for the system output of the i-th agent. $\phi_{i,p}(\bullet) : R^p \to R$ represents the unknown and smooth function, and $d_{i,p} \in R$ expresses the unknown exogenous disturbance. The control input $u_i$ is subjected to the nonlinear asymmetric dead-zone $m_i(u_i)$. Following work[34], the relation between $m_i(u_i)$ and $u_i$ is described as

$$m_i(u_i) = \begin{cases} \rho_{i,r}(u_i - h_{i,r}) & \text{if} \quad u_i \geq h_{i,r} \\ 0 & \text{if} \quad -h_{i,l} < u_i < h_{i,r} \\ \rho_{i,l}(u_i + h_{i,l}) & \text{if} \quad u_i \leq -h_{i,l} \end{cases} \tag{2}$$

where $h_{i,l} > 0$ and $h_{,ir} > 0$ express the breakpoints, and $\rho_{i,r} > 0$ and $\rho_{i,l} > 0$ are respectively left and right slope characteristics meeting $\rho_{i,r} \neq \rho_{i,l}$. To facilitate analysis, the dead-zone model can be rewritten as

$$m_i(u_i) = \rho_i(t)u_i(t) + H_i(t) \tag{3}$$

where

$$\rho_i(t) = \begin{cases} \rho_{i,l} & u_i \leq 0 \\ \rho_{i,r} & u_i > 0 \end{cases} \tag{4}$$

and

$$H_i(t) = \begin{cases} -\rho_{i,r}h_{i,r} & u_i \geq h_{i,r} \\ -\rho_i(t)u_i(t) & -h_{i,l} < u_i < h_{i,r} \\ \rho_{i,l}h_{i,l} & u_i \leq -h_{i,l} \end{cases} \tag{5}$$

Note that, there exits a constant $\overline{H}_i$ satisfying $H_i(t) \leq \overline{H}_i$, and $\overline{H}_i = max\{\rho_{i,l}h_{i,l}, \rho_{i,r}h_{i,r}\}$. Define $\overline{\rho}_i = max\{\rho_{i,r}, \rho_{i,l}\}$ and $\underline{\rho}_i = min\{\rho_{i,r}, \rho_{i,l}\}$. Also, from work[34], we can find that

$$\frac{\rho_i(t)}{\underline{\rho}_i} = 1 + r_i(t) \tag{6}$$

where $r_i(t)$ is a positive piecewise function satisfying $r_i(t) \leq \frac{\overline{\rho}_i}{\underline{\rho}_i} - 1$. Combining (3) and (6), the dead-zone model can be reformulated as

$$m_i(u_i) = \underline{\rho}_i(1 + r_i(t))u_i(t) + H_i(t). \tag{7}$$

In addition, the system state $x_{i,1}$ is limited to a certain zone, which is depicted as follows:

$$\Omega_{x_{i,1}} = \{x_{i,1}(t) \in R : -L_{i,1}(t) < x_{i,1}(t) < L_{i,2}(t)\}, \quad \forall t > 0; \tag{8}$$

where $L_{i,1}(t)$ and $L_{i,2}(t)$ are known functions, and represent the upper and lower boundaries of the system state $x_{i,1}$, respectively. The initial condition of the system state $x_{i,1}$ meets the condition $-L_{i,1}(0) < x_{i,1}(0) < L_{i,2}(0)$.

The main objective of this paper is to design an adaptive consensus control strategy that can uniformly handle both the situations with and without output constraints, which not only ensures that the system outputs are maintained within the limiting intervals during the operation of system, but also balances control performances and communication resources of system. For this, the following theorems and assumptions are essential.

**Assumption 1.** $M$ agents are connected and their communications protocol is fixed. Besides, in whole controlled systems, at least one follower is able to communicate with the reference signal $y_d(t)$, i.e, $\sum_{i=1}^{M} \mu_i > 0$

**Assumption 2.** The expected trajectory function $y_d$ and its derivative functions $\dot{y}_d$, $\ddot{y}_d$ are bounded and available. Besides, $y_d$ meets $|y_d| < \min\{|L_{i,1}|, |L_{i,2}|\}$ . Without loss of generality, there is a constant meeting $\Omega_d = \{(y_d, \dot{y}_d, \ddot{y}_d) : y_d + \dot{y}_d + \ddot{y}_d \leq Y_c\}$.

**Assumption 3.** For the restriction functions for each agent, there exist constants $\overline{L}_{i,1} > 0$, $\overline{L}_{i,2} > 0$ to meet $|L_{i,1}(t)| \leq \overline{L}_{i,1}$ and $|L_{i,2}(t)| \leq \overline{L}_{i,2}$, respectively.

**Assumption 4.** If the unknown function $\phi_{i,p}(\bullet)$ meets the global Lipschitz condition, then, for $\forall \underline{x}_{i,p} \in R^p$ and $\forall \hat{\underline{x}}_{i,p} \in R^p$, one can find a constant $H_{i,p}$ satisfying the following inequality

$$\left|\phi_{i,p}(\underline{x}_{i,p}) - \hat{\phi}_{i,p}(\hat{\underline{x}}_{i,p})\right| \leq H_{i,p} \left\|\underline{x}_{i,p} - \hat{\underline{x}}_{i,p}\right\|, \quad p = 1, 2, ..., n \tag{9}$$

where $\|\bullet\|$ stands for the two-norm of a vector, and $\hat{\underline{x}}_{i,p} = [\hat{x}_{i,1}, \hat{x}_{i,2}, ..., \hat{x}_{i,p}]^T \in R^p$ denotes the estimation of vector $\underline{x}_{i,p} = [x_{i,1}, x_{i,2}, ..., x_{i,p}]^T \in R^p$.

**Lemma 1** ([35]). Suppose that $g(\chi)$ denotes a smooth function defined on a compact set $\Omega$. Then it can be approached by a fuzzy logic system, which is expressed as

$$\sup_{\chi \in \Omega} \left|g(\chi) - \theta^T \varphi(\chi)\right| \leq \beta \tag{10}$$

where $\beta$ denotes the approximation errors and it is a constant. Meanwhile, $\theta = [\theta_1, \theta_2, ..., \theta_l]^T$ is the weighted vector and $\varphi(\chi) = [s_1(\chi), s_2(\chi), ...., s_l(\chi)]^T / \sum_{i=1}^{l} s_i(\chi)$ is the basis function vector, in which $l$ represents the amount of fuzzy rules and $s_i(\chi)$ expresses the Gaussian function. In general, the Gaussian function can be selected as $s_i(\chi) = \exp(\frac{-(\chi-\omega_i)^T(\chi-\omega_i)}{\ell_i^2})$, where $\ell_i$ is the breadth of the Gaussian function and $\omega_i = [\omega_{i,1}, \omega_{i,2}, ..., \omega_{i,l}]^T$ indicates the center vector.

**Lemma 2** ([33]). If $b > 0$ and $\ell \in R$, then ones have

$$\begin{cases} 0 \leq |\ell| - \ell \tanh(\frac{\ell}{b}) \leq 0.2785b \\ 0 \leq \ell \tanh(\frac{\ell}{b}) \end{cases} \tag{11}$$

## 2.3 | Nonlinear Mapping Technique

To handle both the situations with and without output constraints uniformly in a comprehensive control framework, a new nonlinear mapping approach provided in work[33] is introduced as follows:

$$\xi_{i,1} = \frac{L_{i,1}x_{i,1}}{2(L_{i,1}+x_{i,1})} + \frac{L_{i,2}x_{i,1}}{2(L_{i,2}-x_{i,1})}. \tag{12}$$

The discussions on function $\xi_{i,1}$ can be divided into the following two situations:

**Situation 1 :** Based on **Assumption** 3, if the system state $x_{i,1}$ meet $x_{i,1}(0) \in \Omega_{x_{i,1}}$, then we can deduce that $\xi_{i,1} \to \pm\infty$ when $x_{i,1} \to -L_{i,1}(t)$ or $x_{i,1} \to L_{i,2}(t)$. In such case, the system output has not overstep its restriction intervals for all time.

**Situation 2 :** According to L'Hospital's rule, if $L_{i,1}(t) = L_{i,2}(t) \to \infty$, there existing

$$\lim_{L_{i,1}=L_{i,2}\to\infty} \xi_{i,1} = \frac{L_{i,1}x_{i,1}}{2(L_{i,1}+x_{i,1})} + \frac{L_{i,2}x_{i,1}}{2(L_{i,2}-x_{i,1})} = x_{i,1}, \tag{13}$$

which mean that the auxiliary variable $\xi_{i,1}$ become the system state $x_{i,1}$ and the state $x_{i,1}$ is unconstrained.

*Remark 1.* The current control schemes dealing with output constraints in literatures[31] relies on the boundedness of $L_{i,1}(t)$ and $L_{i,2}(t)$. If there existing $t_c$ such that $L_{i,1}(t_c) = L_{i,2}(t_c) \to \infty$, then the above control schemes would fail, namely, these schemes are only suitable for the constrained case. Nevertheless, the mapping approach provided in work[33] allows $L_{i,1}(t)$ and $L_{i,2}(t)$ to be unbounded, which can meet both the **Situation 1** and **2** and thus the reported scheme in work[33] can be applied to the systems with and without output constraints while the control frame is not needed to be modified.

# 3 | MAIN RESULTS

In this chapter, an adaptive consensus control strategy would be developed, which can be applied to the systems with and without output constraints under a unified control framework. Firstly, considering the system state to be unavailable, a state observer is designed. Then, by using a nonlinear mapping approach to construct auxiliary variables, the influence of output constraints on the controller design is completely eliminated. In order to conserve communication resources, an event-triggered technique with a relative threshold is designed to reduce the superfluous information transmissions. Finally, the related stability analysis is given.

## 3.1 | Design of State Estimator

In this subsection, a fuzzy state estimator (FSE) would be designed to observe the unmeasured system states by utilizing the system output and FLSs, which frees the developed control scheme from the restriction that the system states must be measurable. The details are as follows:

In the first place, following the works[36], rewrite each agent model (1) as

$$\begin{cases} \dot{x}_{i,p} = x_{i,p+1} + \Delta\phi_{i,p} + \phi_{i,p}(\hat{\underline{x}}_{i,p}) + \beta_{i,p} + d_{i,p} \\ \dot{x}_{i,n} = m(u_i) + \Delta\phi_{i,n} + \phi_{i,n}(\hat{\underline{x}}_{i,n}) + \beta_{i,n} + d_{i,n}, \\ y_i = x_{i,1} \end{cases} \tag{14}$$

where $\Delta\phi_{i,p} = \phi_{i,p}(\underline{x}_{i,p}) - \phi_{i,p}(\hat{\underline{x}}_{i,p})$, and $\hat{\underline{x}}_{i,p}$ is the estimation of $\underline{x}_{i,p}$.

According to **Lemma** 1, using the FLSs as an approximator to approach the smooth function $\phi_{i,p}(\hat{\underline{x}}_{i,p})$, one has

$$\hat{\phi}_{i,p}(\hat{\underline{x}}_{i,p}|\theta_{i,p}) = \theta_{i,p}^T \varphi_{i,p}(\hat{\underline{x}}_{i,p}). \tag{15}$$

Design the optimal parameter vectors $\theta_{i,p}^*$ as

$$\theta_{i,p}^* = \arg\min_{\theta_{i,p}\in\Omega_{i,p}}[\sup_{\hat{\underline{x}}_{i,p}\in\Xi_{i,p}} |\hat{\phi}_{i,p}(\hat{\underline{x}}_{i,p}|\theta_{i,p}) - \phi_{i,p}(\hat{\underline{x}}_{i,p})|]. \tag{16}$$

where $\Omega_{i,p}$ and $\Xi_{ip}$ are the bounded compact regions of $\theta_{i,p}$ and $\hat{\underline{x}}_{i,p}$, respectively. Define the approximation error of FLSs to approach the function $\phi_{i,p}(\hat{\underline{x}}_{i,p})$ as

$$\beta_{i,p} = \phi_{i,p}(\hat{\underline{x}}_{i,p}) - \hat{\phi}_{i,p}(\hat{\underline{x}}_{i,p}|\theta_{i,p}^*), |\beta_{i,p}| \le \beta_{i,p}^*, \tag{17}$$

where $\beta_{i,p}^{*} > 0$ is a constant.

Then, the state observer can be built as

$$\begin{cases} \dot{\hat{x}}_{i,1} = \hat{x}_{i,2} + \theta_{i,1}^{T}\varphi_{i,1}(\underline{\hat{x}}_{i,1}) + l_{i,1}(y_i - \hat{x}_{i,1}) \\ \dot{\hat{x}}_{i,p} = \hat{x}_{i,p+1} + \theta_{i,p}^{T}\varphi_{i,p}(\underline{\hat{x}}_{i,p}) + l_{i,p}(y_i - \hat{x}_{i,1}) \\ \dot{\hat{x}}_{i,n} = m(u_i) + \theta_{i,n}^{T}\varphi_{i,n}(\underline{\hat{x}}_{i,n}) + l_{i,n}(y_i - \hat{x}_{i,1}), \\ \hat{y}_i = \hat{x}_{i,1} \end{cases} \tag{18}$$

where $l_{i,p} > 0$ is a constant.

By constructing the matrix $A = \begin{bmatrix} -l_{i,1} & & \\ & I_{n-1} & \\ -l_{i,n} & & 0 \end{bmatrix}$ and the vectors $\hat{F}_i = \left[\theta_{i,1}^{T}\varphi_{i,1}(\underline{\hat{x}}_{i,1}), \theta_{i,2}^{T}\varphi_{i,p}(\underline{\hat{x}}_{i,2}), ..., \theta_{i,n}^{T}\varphi_{i,n}(\underline{\hat{x}}_{i,n})\right]^{T}$, $L_i = \left[l_{i,1}, l_{i,2}, ..., l_{i,n}\right]^{T}$, $B_i = \left[0, 0, ...., 1\right]^{T}$, $C_i = \left[1, 0, ..., 0\right]^{T}$ and $\underline{\hat{x}}_i = \left[\hat{x}_{i,1}, \hat{x}_{i,2}, ..., \hat{x}_{i,n}\right]^{T}$, one has

$$\begin{cases} \dot{\underline{\hat{x}}}_i = A_i\underline{\hat{x}}_i + K_i y_i + \hat{F}_i + b_i m(u_i) \\ \hat{y}_i = C_i^{T}\underline{\hat{x}}_i. \end{cases} \tag{19}$$

Note that, the selection about the parameter $l_{i,p}$ should make the matrix $A_i$ to be a Hurwitz. Consequently, for an arbitrarily given matrix $Q_i^{T} = Q_i > 0$, there exists a matrix $P_i^{T} = P_i > 0$ to satisfy the equation $A_i^{T}P_i + P_iA_i = -2Q_i$.

By defining the observation errors as $\underline{\tilde{x}}_i = \underline{x}_i - \underline{\hat{x}}_i = \left[\tilde{x}_{i,1}, \tilde{x}_{i,2}, ...., \tilde{x}_{i,n}\right]^{T}$, one has

$$\dot{\underline{\tilde{x}}}_i = A_i\underline{\tilde{x}}_i + \varepsilon_i + \Delta f_i + \tilde{F}_i + d_i \tag{20}$$

where the vectors $\beta_i = \left[\beta_{i,1}, \beta_{i,2}, ..., \beta_{i,n}\right]^{T}$, $\Delta\phi_i = \left[\Delta\phi_{i,1}, \Delta\phi_{i,2}, ...., \Delta\phi_{i,n}\right]^{T}$, $\tilde{F}_i = \left[\tilde{\theta}_{i,1}^{T}\varphi_{i,1}(x_{i,1}), \tilde{\theta}_{i,2}^{T}\varphi_{i,2}(\underline{\hat{x}}_{i,2}), ..., \tilde{\theta}_{i,n}^{T}\varphi_{i,n}(\underline{\hat{x}}_{i,n})\right]^{T}$ and $d_i = \left[d_{i,1}, d_{i,2}, ..., d_{i,n}\right]^{T}$, and the variable $\tilde{\theta}_{i,p} = \theta_{i,p}^{*} - \theta_{i,p}$.

Construct the Lyapunov function as

$$V_0 = \sum_{i=1}^{N} V_{i,0} = \sum_{i=1}^{N} \frac{1}{2}\underline{\tilde{x}}_i^{T}P\underline{\tilde{x}}_i \tag{21}$$

By differentiating $V_0$, we can gain

$$\begin{aligned} \dot{V}_{i,0} &= \tfrac{1}{2}\dot{\underline{\tilde{x}}}_i^{T}P\underline{\tilde{x}}_i + \tfrac{1}{2}\underline{\tilde{x}}^{T}P\dot{\underline{\tilde{x}}}_i \\ &= -\underline{\tilde{x}}_i^{T}Q_i\underline{\tilde{x}}_i + \underline{\tilde{x}}_i^{T}P_i(\varepsilon_i + \Delta\phi_i + \tilde{F}_i + d_i) \end{aligned} \tag{22}$$

By applying the Young's inequality and **Assumption** 4, we can achieve

$$\begin{aligned} \underline{\tilde{x}}_i^{T}P_i\Delta\phi_i &\leq \tfrac{1}{2}\left\|\underline{\tilde{x}}_i\right\|^2 + \tfrac{1}{2}\|P_i\|^2\|\Delta\phi_i\|^2 \\ &\leq \tfrac{1}{2}\|P_i\|^2\Big(\sum_{p=2}^{n} H_{i,p}^2\left\|\underline{\tilde{x}}_i\right\|^2\Big) + \tfrac{1}{2}\left\|\underline{\tilde{x}}_i\right\|^2 \end{aligned} \tag{23}$$

$$\underline{\tilde{x}}_i^{T}P_i\varepsilon_i + \underline{\tilde{x}}^{T}P_id_i \leq \left\|\underline{\tilde{x}}_i\right\| + \tfrac{1}{2}\|P_i\|^2\|\varepsilon_i^{*}\|^2 + \tfrac{1}{2}\|P_i\|^2\|d_i^{*}\|^2 \tag{24}$$

$$\underline{\tilde{x}}_i^{T}P_i\tilde{F}_i \leq \tfrac{1}{2}\left\|\underline{\tilde{x}}_i\right\|^2\|P_i\|^2 + \tfrac{1}{2}\sum_{p=1}^{n}\tilde{\theta}_{i,p}^{T}\tilde{\theta}_{i,p} \tag{25}$$

where $\beta_i^{*} = \left[\beta_{i,1}^{*}, \beta_{i,2}^{*}, ...., \beta_{i,n}^{*}\right]^{T}$ and $d_i^{*} = \left[d_{i,1}^{*}, d_{i,2}^{*}, ..., d_{i,n}^{*}\right]^{T}$. Meanwhile, According to definition of $\varphi_{i,p}$, it is true that $0 < \varphi_{i,p}^{T}\varphi_{i,p} \leq 1$.

Substituting (23)-(25) into (22), we have

$$\begin{aligned} \dot{V}_{i,0} \leq& -\underline{\tilde{x}}_i^{T}Q_i\underline{\tilde{x}}_i + \Big(\tfrac{3}{2} + \tfrac{1}{2}\|P_i\|^2 + \tfrac{1}{2}\|P_i\|^2\sum_{p=2}^{n}H_{i,p}^2\Big)\left\|\underline{\tilde{x}}_i\right\|^2 \\ &+ \tfrac{1}{2}\|P_i\|^2\big(\|\varepsilon_i^{*}\|^2 + \|d_i^{*}\|^2\big) + \tfrac{1}{2}\sum_{p=1}^{n}\tilde{\theta}_{i,p}^{T}\tilde{\theta}_{i,p} \end{aligned} \tag{26}$$

In view of (21) and (26), we can obtain

$$\begin{aligned}\dot{V}_0 \leq & \sum_{i=1}^{N}\{-\underline{\tilde{x}}_i^T Q_i \underline{\tilde{x}}_i + (\tfrac{3}{2} + \tfrac{1}{2}\|P_i\|^2 + \tfrac{1}{2}\|P_i\|^2 \sum_{p=2}^{n} H_{i,p}^2)\left\|\underline{\tilde{x}}_i\right\|^2 \\ & + \tfrac{1}{2}\|P_i\|^2(\|\varepsilon_i^*\|^2 + \|d_i^*\|^2) + \tfrac{1}{2}\sum_{p=1}^{n}\tilde{\theta}_{i,p}^T\tilde{\theta}_{i,p}\} \\ \leq & \sum_{i=1}^{N}\{-[\lambda_{\min}(Q_i) - \tfrac{3}{2} - \tfrac{1}{2}\|P_i\|^2 - \tfrac{1}{2}\|P_i\|^2 \sum_{p=2}^{n} H_{i,p}^2]\left\|\underline{\tilde{x}}_i\right\|^2 \\ & + \tfrac{1}{2}\|P_i\|^2(\|\varepsilon_i^*\|^2 + \|d_i^*\|^2) + \tfrac{1}{2}\sum_{p=1}^{n}\tilde{\theta}_{i,p}^T\tilde{\theta}_{i,p}\}\end{aligned} \tag{27}$$

where $-\underline{\tilde{x}}_i^T Q_i \underline{\tilde{x}}_i \leq -\lambda_{\min}(Q_i)\left\|\underline{\tilde{x}}_i\right\|^2$.

## 3.2 | Controller Design

In this part, an adaptive controller would be constructed by using the backstepping techniques and the fuzzy state estimator. To handle the output constraints, a nonlinear mapping approach is introduced. Meanwhile, an event-triggered mechanism with dynamic threshold is utilized to conserve communication resources.

Before designing the controller, an auxiliary variable $\xi_{i,1}$ described in (11) would be employed. By differentiating $\xi_{i,1}$, we have

$$\dot{\xi}_{i,1} = \eta_{i,1}\dot{x}_{i,1} + \eta_{i,2} \tag{28}$$

where $\eta_{i,1} = \frac{L^2{}_{i,1}}{2(L_{i,1}+x_{i,1})^2} + \frac{L^2{}_{i,2}}{2(L_{i,2}-x_{i,1})^2}$ and $\eta_{i,2} = \frac{\dot{L}_{i,1}x^2{}_{i,1}}{2(L_{i,1}+x_{i,1})^2} + \frac{-\dot{L}_{i,2}x^2{}_{i,1}}{2(L_{i,2}-x_{i,1})^2}$.

In addition, the reference signal is required to perform same nonlinear mapping, which is shown as follows

$$y_r = \frac{L_{i,1}y_d}{2(L_{i,1}+y_d)} + \frac{L_{i,2}y_d}{2(L_{i,2}-y_d)} \tag{29}$$

Differentiate $y_r$, one has

$$\dot{y}_r = \eta_{i,d1}\dot{y}_d + \eta_{i,d2} \tag{30}$$

where $\eta_{i,d1} = \frac{L^2{}_{i,1}}{2(L_{i,1}+y_d)^2} + \frac{L^2{}_{i,2}}{2(L_{i,2}-y_d)^2}$ and $\eta_{i,d2} = \frac{\dot{L}_{i,1}y^2{}_d}{2(L_{i,1}+y_d)^2} + \frac{-\dot{L}_{i,2}y^2{}_d}{2(L_{i,2}-y_d)^2}$.

Based on $\xi_{i,1}$ and $y_r$, the detailed process of design is given here.

**Step 1**: Firstly, the following coordinate transformations are introduced

$$s_{i,1} = \sum_{j\in N_i} a_{i,j}(\xi_{i,1} - \xi_{j,1}) + \mu_i(\xi_{i,1} - y_r), \tag{31}$$

$$s_{i,p} = \hat{x}_{i,p} - \hbar_{i,p}, \tag{32}$$

where $\hbar_{i,p}$ will be explained later.

Calculating the derivative of $s_{i,1}$ gives

$$\begin{aligned}\dot{s}_{i,1} &= \Big(\sum_{j\in N_i} a_{i,j} + \mu_i\Big)\dot{\xi}_{i,1} - \sum_{j\in N_i} a_{i,j}\dot{\xi}_{j,1} - \mu_i\dot{y}_r \\ &= (b_i + \mu_i)\big[\tilde{x}_{i,2} + \eta_{i,1}\big(\hbar_{i,2} + \alpha_{i,2} - \alpha_{i,2} + s_{i,2} \\ &\quad + \tilde{\theta}_{i,1}^T\varphi_{i,1}\big(\underline{\hat{x}}_{i,1}\big) + \theta_{i,1}^T\varphi_{i,1}(\underline{\hat{x}}_{i,1}) + \varepsilon_{i,1} + d_{i,1}\big) + \eta_{i,2}\big] \\ &\quad - \sum_{j\in N_i} a_{i,j}\big[\eta_{j,1}\big(\hat{x}_{j,2} + \tilde{x}_{j,2} + \varepsilon_{j,1} + d_{j,1} \\ &\quad + \tilde{\theta}_{j,1}^T\varphi_{j,1}(\underline{\hat{x}}_{j,1}) + \theta_{j,1}^T\varphi_{j,1}(\underline{\hat{x}}_{j,1})\big) + \eta_{j,2}\big] - \mu_i\dot{y}_r\end{aligned} \tag{33}$$

where $b_i = \sum_{j\in N_i} a_{i,j}$, $\theta_{i,1}^* = \theta_{i,1} - \tilde{\theta}_{i,1}$ and $\theta_{j,1}^* = \theta_{j,1} - \tilde{\theta}_{j,1}$. In addition, $\alpha_{i,2}$ is virtual controller, and $\hbar_{i,2}$ is an auxiliary variable of first-order filter as follows:

$$\begin{aligned}&\partial_{i,2}\dot{\hbar}_{i,2} + \hbar_{i,2} = \alpha_{i,2}, \\ &\hbar_{i,2}(0) = \alpha_{i,2}(0)\end{aligned} \tag{34}$$

where $\partial_{i,2}$ is a constant, and the output and input signals of the filter are $\hbar_{i,2}$ and $\alpha_{i,2}$, respectively. To eliminate the filtering errors generated by filter, a compensation signal is constructed as follows:

$$\dot{q}_{i,1} = -g_{i,1}q_{i,1} + \eta_{i,1}(b_i + \mu_i)q_{i,2} + \eta_{i,1}(b_i + \mu_i)(\hbar_{i,2} - \alpha_{i,2}) - r_{i,1}\tanh(q_{i,1}) \tag{35}$$

where $r_{i,1}$ and $g_{i,1}$ are positive constants.

Constructing the compensation tracking errors as $\upsilon_{i,1} = s_{i,1} - q_{i,1}$ and differentiating it, we have

$$\begin{aligned}\dot{\upsilon}_{i,1} &= (b_i + \mu_i)\big[\eta_{i,1}\big(s_{i,2} + \tilde{x}_{i,2} + \hbar_{i,2} + \alpha_{i,2} - \alpha_{i,2} + \varepsilon_{i,1} + d_{i,1} \\ &\quad + \theta_{i,1}^T\varphi_{i,1}(\underline{\hat{x}}_{i,1})\big) + \tilde{\theta}_{i,1}^T\varphi_{i,1}(\underline{\hat{x}}_{i,1}) + \eta_{i,2}\big] - \sum_{j\in N_i} a_{i,j}\big[\eta_{j,1}\big(\hat{x}_{j,2} + \tilde{x}_{j,2} \\ &\quad + \varepsilon_{j,1} + d_{j,1} + \tilde{\theta}_{j,1}^T\varphi_{j,1}(\underline{\hat{x}}_{j,1}) + \theta_{j,1}^T\varphi_{j,1}(\underline{\hat{x}}_{j,1})\big) + \eta_{j,2}\big] - \mu_i\dot{y}_r - \big[-g_{i,1}q_{i,1} \\ &\quad + \eta_{i,1}(b_i + \mu_i)q_{i,2} + \eta_{i,1}(b_i + \mu_i)(\hbar_{i,2} - \alpha_{i,2}) - r_{i,1}\tanh(q_{i,1})\big] \\ &= (b_i + \mu_i)\eta_{i,1}\big[\tilde{x}_{i,2} + \alpha_{i,2} + d_{i,1} + \varepsilon_{i,1} + \tfrac{\eta_{i,2}}{\eta_{i,1}} + \tilde{\theta}_{i,1}^T\varphi_{i,1}\big(\underline{\hat{x}}_{i,1}\big) + \theta_{i,1}^T\varphi_{i,1}\big(\underline{\hat{x}}_{i,1}\big)\big] \\ &\quad - \sum_{j\in N_i} a_{i,j}\eta_{j,1}\big[\hat{x}_{j,2} + \tilde{x}_{j,2} + d_{j,1} + \varepsilon_{j,1} + \tfrac{\eta_{j,2}}{\eta_{j,1}} + \tilde{\theta}_{j,1}^T\varphi_{j,1}(\underline{\hat{x}}_{j,1}) \\ &\quad + \theta_{j,1}^T\varphi_{j,1}(\underline{\hat{x}}_{j,1})\big] - \mu_i\dot{y}_r + g_{i,1}q_{i,1} + r_{i,1}\tanh(q_{i,1}) + \eta_{i,1}(b_i + \mu_i)\upsilon_{i,2}\end{aligned} \tag{36}$$

Determine the Lyapunov-function as

$$V_1 = V_0 + \frac{1}{2}[\sum_{i=1}^{N}(\upsilon_{i,1}^2 + \frac{1}{\lambda_{i,1}}\tilde{\theta}_{i,1}^T\tilde{\theta}_{i,1} + \sum_{j\in N_i}\frac{a_{i,j}}{\beta_{j,1}}\tilde{\theta}_{j,1}^T\tilde{\theta}_{j,1})] \tag{37}$$

With the help of (36), the differential function of $V_1$ can be expressed as

$$\begin{aligned}\dot{V}_1 &= \dot{V}_0 + \sum_{i=1}^{N}\upsilon_{i,1}\{(b_i + \mu_i)\eta_{i,1}\big[\alpha_{i,2} + \tilde{x}_{i,2} + \tilde{\theta}_{i,1}^T\varphi_{i,1}(\underline{\hat{x}}_{i,1}) \\ &\quad + \theta_{i,1}^T\varphi_{i,1}(\underline{\hat{x}}_{i,1}) + \varepsilon_{i,1} + d_{i,1} + \tfrac{\eta_{i,2}}{\eta_{i,1}}\big] - \sum_{j\in N_i} a_{i,j}\eta_{j,1}\big[\hat{x}_{j,2} \\ &\quad + \tilde{x}_{j,2} + \varepsilon_{j,1} + d_{j,1} + \tfrac{\eta_{j,2}}{\eta_{j,1}} + \tilde{\theta}_{j,1}^T\varphi_{j,1}(\underline{\hat{x}}_{j,1}) + \theta_{j,1}^T\varphi_{j,1}(\underline{\hat{x}}_{j,1})\big] \\ &\quad - \mu_i\dot{y}_r + g_{i,1}q_{i,1} + r_{i,1}\tanh(q_{i,1}) + \eta_{i,1}(b_i + \mu_i)\upsilon_{i,2}\} \\ &\quad - \sum_{i=1}^{N}\tfrac{1}{\lambda_{i,1}}\tilde{\theta}_{i,1}^T\dot{\theta}_{i,1} - \sum_{i=1}^{N}\sum_{j\in N_i}\tfrac{a_{i,j}}{\beta_{j,1}}\tilde{\theta}_{j,1}^T\dot{\theta}_{j,1}\end{aligned} \tag{38}$$

By applying the Young's inequality, the following inequalities are gained

$$(b_i + \mu_i)\eta_{i,1}\upsilon_{i,1}(\tilde{x}_{i,2} + \tilde{x}_{j,2}) \leq [(b_i + \mu_i)\eta_{i,1}\upsilon_{i,1}]^2 + \frac{1}{2}(\tilde{x}_{i,2}^2 + \tilde{x}_{j,2}^2) \tag{39}$$

$$(b_i + \mu_i)\eta_{i,1}\upsilon_{i,1}(\varepsilon_{i,1} + \varepsilon_{j,1}) \leq [(b_i + \mu_i)\eta_{i,1}\upsilon_{i,1}]^2 + \frac{1}{2}(\|\varepsilon_{i,1}\|^2 + \|\varepsilon_{j,1}\|^2) \tag{40}$$

$$(b_i + \mu_i)\eta_{i,1}\upsilon_{i,1}(d_{i,1} + d_{j,1}) \leq \frac{1}{2}(\|d_{i,1}\|^2 + \|d_{j,1}\|^2) + [(b_i + \mu_i)\eta_{i,1}\upsilon_{i,1}]^2 \tag{41}$$

$$(b_i + \mu_i)\eta_{i,1}\upsilon_{i,1}r_{i,1}\tanh(q_{i,1}) \leq \frac{1}{2}[(b_i + \mu_i)\eta_{i,1}\upsilon_{i,1}]^2 + \frac{1}{2}r_{i,1}^2 \tag{42}$$

Substituting (39)-(42) into (38), we can get

$$
\begin{aligned}
\dot{V}_1 \leq & \dot{V}_0 + \sum_{i=1}^{N} v_{i,1}\big\{(b_i+\mu_i)\eta_{i,1}\big[\alpha_{i,2} + \tilde{\theta}_{i,1}^T\varphi_{i,1}(\underline{\hat{x}}_{i,1}) + \theta_{i,1}^T\varphi_{i,1}(\underline{\hat{x}}_{i,1}) + \tfrac{\eta_{i,2}}{\eta_{i,1}}\big] \\
& - \sum_{j\in N_i} a_{i,j}\eta_{j,1}\big[\hat{x}_{j,2} + \theta_{j,1}^{*T}\varphi_{j,1}(\underline{\hat{x}}_{j,1}) + \tfrac{\eta_{j,2}}{\eta_{j,1}}\big] - \mu_i\dot{y}_r + g_{i,1}q_{i,1} + \eta_{i,1}(b_i+\mu_i)v_{i,2}\big\} \\
& - \sum_{i=1}^{N}\tfrac{1}{\lambda_{i,1}}\tilde{\theta}_{i,1}^T\dot{\theta}_{i,1} - \sum_{i=1}^{N}\sum_{j\in N_i}\tfrac{a_{i,j}}{\beta_{j,1}}\tilde{\theta}_{j,1}^T\dot{\theta}_{j,1} + \sum_{i=1}^{N}\tfrac{7}{2}(b_i+\mu_i)^2\eta^2{}_{i,1}v^2{}_{i,1} \\
& + \sum_{i=1}^{N}\Big\{\big\|\underline{\tilde{x}}_i\big\|^2 + \big\|d_{*,1}^*\big\|^2 + \big\|\varepsilon_{*,1}^*\big\|^2\Big\} + \sum_{i=1}^{N}\tfrac{1}{2}r_{i,1}^2 \\
\leq & \dot{V}_0 + \sum_{i=1}^{N} v_{i,1}\big\{(b_i+\mu_i)\eta_{i,1}\big[\alpha_{i,2} + \theta_{i,1}^T\varphi_{i,1}(\underline{\hat{x}}_{i,1}) + \tfrac{\eta_{i,2}}{\eta_{i,1}}\big] \\
& - \sum_{j\in N_i} a_{i,j}\eta_{j,1}[\hat{x}_{j,2} + \theta_{j,1}^{T}\varphi_{j,1}(\underline{\hat{x}}_{j,1}) + \tfrac{\eta_{j,2}}{\eta_{j,1}}] - \mu_i\dot{y}_r + g_{i,1}q_{i,1} + \eta_{i,1}(b_i+\mu_i)v_{i,2}\big\} \\
& - \sum_{i=1}^{N}\tfrac{1}{\lambda_{i,1}}\tilde{\theta}_{i,1}^T\big(\dot{\theta}_{i,1} - \lambda_{i,1}(b_i+\mu_i)\eta_{i,1}v_{i,1}\varphi_{i,1}(\underline{\hat{x}}_{i,1})\big) \\
& - \sum_{i=1}^{N}\sum_{j\in N_i}\tfrac{a_{i,j}}{\beta_{j,1}}\tilde{\theta}_{j,1}^T\big(\dot{\theta}_{j,1} - \beta_{j,1}\eta_{j,1}v_{i,1}\varphi_{j,1}(\underline{\hat{x}}_{j,1})\big) \\
& + \sum_{i=1}^{N}\tfrac{7}{2}(b_i+\mu_i)^2\eta^2{}_{i,1}v^2{}_{i,1} + \sum_{i=1}^{N}\Big\{\big\|\underline{\tilde{x}}_i\big\|^2 + \big\|d_{*,1}^*\big\|^2 + \big\|\varepsilon_{*,1}^*\big\|^2\Big\} + \sum_{i=1}^{N}\tfrac{1}{2}r_{i,1}^2
\end{aligned}
\tag{43}
$$

where $d_{*,1}^* = \left[d_{1,1}^*, d_{2,1}^*, ...., d_{N,1}^*\right]^T$ and $\varepsilon_{*,1}^* = \left[\varepsilon_{1,1}^*, \varepsilon_{2,1}^*, ...., \varepsilon_{N,1}^*\right]^T$.

If the virtual controller is built as

$$
\begin{aligned}
\alpha_{i,2} = & -\tfrac{1}{(b_i+\mu_i)\eta_{i,1}}g_{i,1}s_{i,1} - \theta_{i,1}^T\varphi_{i,1}(\underline{\hat{x}}_{i,1}) - \tfrac{\eta_{i,2}}{\eta_{i,1}} + \tfrac{1}{(b_i+\mu_i)\eta_{i,1}}\mu_i\dot{y}_r \\
& + \tfrac{1}{(b_i+\mu_i)\eta_{i,1}}\Big(\sum_{j\in N_i} a_{i,j}\eta_{j,1}\big[\theta_{j,1}^T\varphi_{j,1}(\underline{\hat{x}}_{j,1}) + \tfrac{\eta_{j,2}}{\eta_{j,1}} + \hat{x}_{j,2}\big]\Big)
\end{aligned}
\tag{44}
$$

then, $\dot{V}_1$ can become as

$$
\begin{aligned}
\dot{V}_1 \leq & \dot{V}_0 + \sum_{i=1}^{N} v_{i,1}\big\{-g_{i,1}v_{i,1} + \tfrac{7}{2}(b_i+\mu_i)^2\eta^2{}_{i,1}v_{i,1} + \eta_{i,1}(b_i+\mu_i)v_{i,2}\big\} \\
& + \sum_{i=1}^{N}\Big\{\big\|\underline{\tilde{x}}_i\big\|^2 + \big\|d_{*,1}^*\big\|^2 + \big\|\varepsilon_{*,1}^*\big\|^2\Big\} + \sum_{i=1}^{N}\tfrac{1}{2}r_{i,1}^2 \\
& - \sum_{i=1}^{N}\tfrac{1}{\lambda_{i,1}}\tilde{\theta}_{i,1}^T(\dot{\theta}_{i,1} - \lambda_{i,1}(b_i+\mu_i)\eta_{i,1}v_{i,1}\varphi_{i,1}(\underline{\hat{x}}_{i,1})) \\
& - \sum_{i=1}^{N}\sum_{j\in N_i}\tfrac{a_{i,j}}{\beta_{j,1}}\tilde{\theta}_{j,1}^T(\dot{\theta}_{j,1} - \beta_{j,1}\eta_{j,1}v_{i,1}\varphi_{j,1}(\underline{\hat{x}}_{j,1}))
\end{aligned}
\tag{45}
$$

Constructing the following adaptive laws

$$
\dot{\theta}_{i,1} = \lambda_{i,1}\big((b_i+\mu_i)\eta_{i,1}v_{i,1}\varphi_{i,1}(\underline{\hat{x}}_{i,1}) - \gamma_{i,1}\theta_{i,1}\big) \tag{46}
$$

$$
\dot{\theta}_{j,1} = -\beta_{j,1}\big(\eta_{j,1}v_{j,1}\varphi_{j,1} - \sigma_{j,1}\theta_{j,1}\big) \tag{47}
$$

and substituting (46) and (47) into (45) , we can gain

$$\begin{aligned}\dot{V}_1 &\le \dot{V}_0 + \sum_{i=1}^{N} v_{i,1}\{-g_{i,1}v_{i,1} + \tfrac{7}{2}(b_i+\mu_i)^2\eta^2{}_{i,1}v_{i,1} + \eta_{i,1}(b_i+\mu_i)v_{i,2}\} \\ &\quad + \sum_{i=1}^{N}\{\left\|d^*_{*,1}\right\|^2 + \left\|\varepsilon^*_{*,1}\right\|^2 + \left\|\underline{\tilde{x}}_i\right\|^2\} + \sum_{i=1}^{N}\tfrac{1}{2}r^2_{i,1} \\ &\quad - \sum_{i=1}^{N}\tfrac{1}{\lambda_{i,1}}\tilde{\theta}^T_{i,1}\left(\dot{\theta}_{i,1} - \lambda_{i,1}(b_i+\mu_i)\eta_{i,1}v_{i,1}\varphi_{i,1}(\underline{\hat{x}}_{i,1})\right) \\ &\quad - \sum_{i=1}^{N}\sum_{j\in N_i}\tfrac{a_{i,j}}{\beta_{j,1}}\tilde{\theta}^T_{j,1}\left(\dot{\theta}_{j,1} - \beta_{j,1}\eta_{j,1}v_{i,1}\varphi_{j,1}(\underline{\hat{x}}_{j,1})\right) \\ &\le \sum_{i=1}^{N}\{-[\lambda_{\min}(Q_i) - \tfrac{5}{2} - \tfrac{1}{2}\|P_i\|^2 - \tfrac{1}{2}\|P_i\|^2\sum_{p=2}^{n}H^2_{i,p}]\left\|\underline{\tilde{x}}_i\right\|^2 \\ &\quad + \tfrac{1}{2}\|P_i\|^2(\|\varepsilon^*_i\|^2 + \|d^*_i\|^2) + \tfrac{1}{2}\sum_{p=1}^{n}\tilde{\theta}^T_{i,p}\tilde{\theta}_{i,p}\} \\ &\quad + \sum_{i=1}^{N} v_{i,1}\{-(g_{i,1} - \tfrac{7}{2}(b_i+\mu_i)^2\eta^2{}_{i,1})v_{i,1} + \eta_{i,1}(b_i+\mu_i)v_{i,2}\} \\ &\quad + \sum_{i=1}^{N}\{\left\|\underline{\tilde{x}}_i\right\|^2 + \left\|d^*_{*,1}\right\|^2 + \left\|\varepsilon^*_{*,1}\right\|^2\} + \sum_{i=1}^{N}\tfrac{1}{2}r^2_{i,1} \\ &\quad + \sum_{i=1}^{N}\gamma_{i,1}\tilde{\theta}^T_{i,1}\theta_{i,1} + \sum_{i=1}^{N}\sum_{j\in N_i}\sigma_{j,1}a_{i,j}\tilde{\theta}^T_{j,1}\theta_{j,1}\end{aligned} \tag{48}$$

**Step p ($2 \le p \le n-1$)**: From (17) and (31), we can acquire the derivatives of $s_{i,p}$

$$\begin{aligned}\dot{s}_{i,p} &= \dot{\hat{x}}_{i,p} - \dot{\hbar}_{i,p} \\ &= \hat{x}_{i,p+1} + l_{i,p}(y_i - \hat{x}_{i,1}) + \theta^T_{i,p}S_{i,p}(\underline{\hat{x}}_{i,p}) - \dot{\hbar}_{i,p} \\ &= \hbar_{i,p+1} + s_{i,p+1} + \alpha_{i,p+1} - \alpha_{i,p+1} + l_{i,p}(y_i - \hat{x}_{i,1}) + \theta^T_{i,p}S_{i,p}(\underline{\hat{x}}_{i,p}) - \dot{\hbar}_{i,p}\end{aligned} \tag{49}$$

By introducing the following filter similar to **Step 1**, the specific calculations about the derivative of $\alpha_{i,p}$ are avoided

$$\begin{aligned}&\partial_{i,p}\dot{\hbar}_{i,p} + \hbar_{i,p} = \alpha_{i,p}, \\ &\hbar_{i,p}(0) = \alpha_{i,p}(0),\end{aligned} \tag{50}$$

where $\partial_{i,p} > 0$ is a constant, and the output and input signals of the filter are $\hbar_{i,p}$ and $\alpha_{i,p}$, respectively. Besides, To remove the impact caused by the filter, the following compensation signal is designed

$$\dot{q}_{i,p} = \hbar_{i,p+1} - g_{i,p}q_{i,p} - q_{i,p-1} + q_{i,p+1} - \alpha_{i,p+1} - r_{i,p}\tanh(q_{i,p}) \tag{51}$$

By combining (49) and (51), the compensation tracking error can be formulated as

$$v_{i,p} = s_{i,p} - q_{i,p} \tag{52}$$

Differentiating $v_{i,p}$, results in

$$\begin{aligned}\dot{v}_{i,p} &= \dot{s}_{i,p} - \dot{q}_{i,p} \\ &= s_{i,p+1} + \hbar_{i,p+1} + \alpha_{i,p+1} - \alpha_{i,p+1} + \theta^T_{i,p}\varphi_{i,p}(\underline{\hat{x}}_{i,p}) \\ &\quad + l_{i,p}(y_i - \hat{x}_{i,1}) - \dot{\hbar}_{i,p} - [-g_{i,p}q_{i,p} + \hbar_{i,p+1} - \alpha_{i,p+1} \\ &\quad - q_{i,p-1} + q_{i,p+1} - r_{i,p}\tanh(q_{i,p})] \\ &= v_{i,p+1} + \alpha_{i,p+1} + l_{i,p}(y_i - \hat{x}_{i,1}) + \theta^T_{i,p}\varphi_{i,p}(\underline{\hat{x}}_{i,p}) \\ &\quad + q_{i,p-1} + g_{i,p}q_{i,p} + r_{i,p}\tanh(q_{i,p}) - \dot{\hbar}_{i,p}\end{aligned} \tag{53}$$

The Lyapunov function is selected as

$$V_p = V_{p-1} + \frac{1}{2}\Big[\sum_{i=1}^{N}(v^2_{i,p} + \frac{1}{\lambda_{i,p}}\tilde{\theta}^T_{i,p}\tilde{\theta}_{i,p})\Big] \tag{54}$$

Computing its derivative, we have

$$
\begin{aligned}
\dot{V}_p &= \dot{V}_{p-1} + \sum_{i=1}^{N} \upsilon_{i,p}\dot{\upsilon}_{i,p} + \sum_{i=1}^{N} \tfrac{1}{\lambda_{i,p}}\tilde{\theta}^T_{i,p}\dot{\theta}_{i,p} \\
&= \dot{V}_{p-1} + \sum_{i=1}^{N} \upsilon_{i,p}\big\{\upsilon_{i,p+1} + \alpha_{i,p+1} + \theta^T_{i,p}\varphi_{i,p}(\underline{\hat{x}}_{i,p}) + l_{i,p}(y_i - \hat{x}_{i,1}) \\
&\quad + g_{i,p}q_{i,p} + q_{i,p-1} + r_{i,p}\tanh(q_{i,p}) - \dot{\hbar}_{i,p} + \tilde{\theta}^T_{i,p}\varphi_{i,p}(\underline{\hat{x}}_{i,p}) \\
&\quad - \tilde{\theta}^T_{i,p}\varphi_{i,p}(\underline{\hat{x}}_{i,p})\big\} - \sum_{i=1}^{N} \tfrac{1}{\lambda_{i,p}}\tilde{\theta}^T_{i,p}\dot{\theta}_{i,p} \\
&= \dot{V}_{p-1} + \sum_{i=1}^{N} \upsilon_{i,p}\big\{\upsilon_{i,p+1} + \alpha_{i,p+1} + \theta^T_{i,p}\varphi_{i,p}(\underline{\hat{x}}_{i,p}) + l_{i,p}(y_i - \hat{x}_{i,1}) \\
&\quad + g_{i,p}q_{i,p} + q_{i,p-1} + r_{i,p}\tanh(q_{i,p}) - \dot{\hbar}_{i,p} - \tilde{\theta}^T_{i,p}\varphi_{i,p}(\underline{\hat{x}}_{i,p})\big\} \\
&\quad - \sum_{i=1}^{N} \tfrac{1}{\lambda_{i,p}}\tilde{\theta}^T_{i,p}(\dot{\theta}_{i,p} - \lambda_{i,p}\upsilon_{i,p}S_{i,p}(\underline{\hat{x}}_{i,p}))
\end{aligned} \tag{55}
$$

By utilizing Yong's inequality, we can find that

$$
\upsilon_{i,p}r_{i,p}\tanh(q_{i,p}) \le \frac{1}{2}\upsilon^2_{i,p} + \frac{1}{2}r^2_{i,p} \tag{56}
$$

$$
\upsilon_{i,p}(-\tilde{\theta}^T_{i,p}\varphi_{i,p}(\underline{\hat{x}}_{i,p})) \le \frac{1}{2}\upsilon^2_{i,p} + \frac{1}{2}\tilde{\theta}^2_{i,p} \tag{57}
$$

With the assist of (56) and (57), we can achieve

$$
\begin{aligned}
\dot{V}_p &\le \dot{V}_{p-1} + \sum_{i=1}^{N} \upsilon_{i,p}\big\{\upsilon_{i,p+1} + \alpha_{i,p+1} + i,p^T\varphi_{i,p}(\underline{\hat{x}}_{i,p}) + l_{i,p}(y_i - \hat{x}_{i,1}) \\
&\quad + g_{i,p}q_{i,p} + q_{i,p-1} - \dot{\hbar}_{i,p} + \tfrac{1}{2}\upsilon^2_{i,p} + \tfrac{1}{2}r^2_{i,p} + \tfrac{1}{2}\upsilon^2_{i,p} + \tfrac{1}{2}\tilde{\theta}^2_{i,p}\big\} \\
&\quad - \sum_{i=1}^{N} \tfrac{1}{\lambda_{i,p}}\tilde{\theta}^T_{i,p}(\dot{\theta}_{i,p} - \lambda_{i,p}\upsilon_{i,p}\varphi_{i,p}(\underline{\hat{x}}_{i,p}))
\end{aligned} \tag{58}
$$

By constructing the following virtual controller and adaptive law

$$
\alpha_{i,p+1} = -s_{i,p-1} - g_{i,p}s_{i,p} - l_{i,p}(y_i - \hat{x}_{i,1}) + \dot{\hbar}_{i,p} - \theta^T_{i,p}\varphi_{i,p}(\underline{\hat{x}}_{i,p}), \tag{59}
$$

$$
\dot{\theta}_{i,p} = \lambda_{i,p}\upsilon_{i,p}\varphi_{i,p}(\underline{\hat{x}}_{i,p}) - \lambda_{i,p}\gamma_{i,p}\theta_{i,p}. \tag{60}
$$

and substitute them into (58), we have

$$
\begin{aligned}
\dot{V}_p &\le \dot{V}_{p-1} + \sum_{i=1}^{N} \upsilon_{i,p}\big\{\upsilon_{i,p+1} - g_{i,p}\upsilon_{i,p} + \upsilon_{i,p-1} + \tfrac{1}{2}\upsilon^2_{i,p} \\
&\quad + \tfrac{1}{2}r^2_{i,p} + \tfrac{1}{2}\upsilon^2_{i,p} + \tfrac{1}{2}\tilde{\theta}^2_{i,p}\big\} - \sum_{i=1}^{N} \tilde{\theta}^T_{i,p}\gamma_{i,p}\theta_{i,p} \\
&\le \sum_{i=1}^{N}\big\{-\big[\lambda_{\min}(Q_i) - \tfrac{5}{2} - \tfrac{1}{2}\|P_i\|^2 - \tfrac{1}{2}\|P_i\|^2\sum_{p=2}^{n} H^2_{i,p}\big]\left\|\underline{\tilde{x}}_i\right\|^2 \\
&\quad + \tfrac{1}{2}\|P_i\|^2(\|\varepsilon^*_i\|^2 + \|d^*_i\|^2) + \tfrac{1}{2}\sum_{p=1}^{n} \tilde{\theta}^T_{i,p}\tilde{\theta}_{i,p}\big\} \\
&\quad - \sum_{i=1}^{N}(g_{i,1} - \tfrac{7}{2}(b_i + \mu_i)^2\eta^2{}_{i,1})\upsilon^2_{i,1} - \sum_{i=1}^{N}[\sum_{m=2}^{p}(g_{i,m} - 1)\upsilon^2_{i,m}] + \sum_{i=1}^{N}\upsilon_{i,p}\upsilon_{i,p+1}\} \\
&\quad + \sum_{i=1}^{N}\big\{\big[-\tfrac{\gamma_{i,1}}{2}\|\tilde{\theta}_{i,1}\|^2 - \tfrac{\gamma_{i,2}}{2}\|\tilde{\theta}_{i,2}\|^2 - \tfrac{1}{2}\sum_{j\in N_i}\sigma_{j,1}a_{i,j}\left\|\tilde{\theta}_{j,1}\right\|^2\big] \\
&\quad + \sum_{i=1}^{N}\big[\tfrac{\gamma_{i,1}}{2}\left\|\theta^*_{i,1}\right\|^2 + \tfrac{\gamma_{i,2}}{2}\left\|\theta^*_{i,2}\right\|^2 + \tfrac{1}{2}\sum_{j\in N_i}\sigma_{j,1}a_{i,j}\left\|\theta^*_{j,1}\right\|^2\big]\big\} \\
&\quad + \sum_{i=1}^{N}\{\left\|d^*_{*,1}\right\|^2 + \left\|\varepsilon^*_{*,1}\right\|^2\} + \sum_{i=1}^{N}\tfrac{1}{2}r^2_{i,1} + \sum_{i=1}^{N}\tilde{\theta}^T_{i,p}\gamma_{i,p}\theta_{i,p} + \sum_{i=1}^{N}(\sum_{m=2}^{p}\tfrac{1}{2}r^2_{i,m} + \tfrac{1}{2}\tilde{\theta}^2_{i,m})
\end{aligned} \tag{61}
$$

Using the Young's inequality again, there are

$$
\gamma_{i,p}\tilde{\theta}^T_{i,p}\theta_{i,p} \le \frac{1}{2}\gamma_{i,p}\left\|\theta^*_{i,p}\right\|^2 - \frac{1}{2}\gamma_{i,p}\left\|\tilde{\theta}_{i,p}\right\|^2. \tag{62}
$$

Simplify $\dot{V}_p$ yields

$$\begin{aligned}
\dot{V}_p \leq & \sum_{i=1}^{N}\{-[\lambda_{\min}(Q_i)-\tfrac{5}{2}-\tfrac{1}{2}\|P_i\|^2-\tfrac{1}{2}\|P_i\|^2\sum_{p=2}^{n}H_{i,p}^2]\left\|\underline{\tilde{x}}_i\right\|^2 \\
& +\tfrac{1}{2}\|P_i\|^2(\|\varepsilon_i^*\|^2+\|d_i^*\|^2)+\tfrac{1}{2}\sum_{p=1}^{n}\tilde{\theta}_{i,p}^T\tilde{\theta}_{i,p}\}-\sum_{i=1}^{N}\{(g_{i,1} \\
& -\tfrac{7}{2}(b_i+\mu_i)^2\eta^2{}_{i,1})v_{i,1}^2\}-\sum_{i=1}^{N}[\sum_{m=2}^{p}(g_{i,m}-1)v_{i,m}^2]+\sum_{i=1}^{N}v_{i,p}v_{i,p+1} \\
& +\sum_{i=1}^{N}[-\tfrac{\gamma_{i,1}}{2}\|\tilde{\theta}_{i,1}\|^2-\sum_{m=2}^{p-1}\tfrac{\gamma_{i,m}}{2}\|\tilde{\theta}_{i,m}\|^2-\tfrac{1}{2}\sum_{j\in N_i}\sigma_{j,1}a_{i,j}\left\|\tilde{\theta}_{j,1}\right\|^2] \\
& +\sum_{i=1}^{N}[\tfrac{\gamma_{i,1}}{2}\left\|\theta_{i,1}^*\right\|^2+\sum_{m=2}^{p-1}\tfrac{\gamma_{i,m}}{2}\left\|\theta_{i,m}^*\right\|^2+\tfrac{1}{2}\sum_{j\in N_i}\sigma_{j,1}a_{i,j}\left\|\theta_{j,1}^*\right\|^2] \\
& +\sum_{i=1}^{N}\{\left\|d_{*,1}^*\right\|^2+\left\|\varepsilon_{*,1}^*\right\|^2\}+\sum_{i=1}^{N}\tfrac{1}{2}r_{i,1}^2+\sum_{i=1}^{N}[\sum_{m=2}^{p}(\tfrac{1}{2}r_{i,m}^2+\tfrac{1}{2}\tilde{\theta}_{i,m}^2)] \\
& +\sum_{i=1}^{N}[-\tfrac{1}{2}\gamma_{i,p}\left\|\tilde{\theta}_{i,p}\right\|^2+\tfrac{1}{2}\gamma_{i,p}\left\|\theta_{i,p}^*\right\|^2] \\
\leq & \sum_{i=1}^{N}\{-[\lambda_{\min}(Q_i)-\tfrac{5}{2}-\tfrac{1}{2}\|P_i\|^2-\tfrac{1}{2}\|P_i\|^2\sum_{p=2}^{n}H_{i,p}^2]\left\|\underline{\tilde{x}}_i\right\|^2 \\
& +\tfrac{1}{2}\|P_i\|^2(\|\varepsilon_i^*\|^2+\|d_i^*\|^2)+\tfrac{1}{2}\sum_{p=1}^{n}\tilde{\theta}_{i,p}^T\tilde{\theta}_{i,p}\} \\
& -\sum_{i=1}^{N}(g_{i,1}-\tfrac{7}{2}(b_i+\mu_i)^2\eta^2{}_{i,1})v_{i,1}^2-\sum_{i=1}^{N}[\sum_{m=2}^{p}(g_{i,m}-1)v_{i,m}^2]+\sum_{i=1}^{N}v_{i,p}v_{i,p+1} \\
& +\sum_{i=1}^{N}[-\tfrac{\gamma_{i,1}}{2}\left\|\tilde{\theta}_{i,1}\right\|^2-\sum_{m=2}^{p}\tfrac{\gamma_{i,m}}{2}\left\|\tilde{\theta}_{i,m}\right\|^2-\tfrac{1}{2}\sum_{j\in N_i}\sigma_{j,1}a_{i,j}\left\|\tilde{\theta}_{j,1}\right\|^2] \\
& +\sum_{i=1}^{N}[\sum_{m=1}^{p}\tfrac{\gamma_{i,1}}{2}\left\|\theta_{i,1}^*\right\|^2+\tfrac{1}{2}\sum_{j\in N_i}\sigma_{j,1}a_{i,j}\left\|\theta_{j,1}^*\right\|^2] \\
& +\sum_{i=1}^{N}\{\left\|d_{*,1}^*\right\|^2+\left\|\varepsilon_{*,1}^*\right\|^2\}+\sum_{i=1}^{N}\sum_{m=1}^{p}\tfrac{1}{2}r_{i,m}^2+\sum_{i=1}^{N}\sum_{m=2}^{p}\tfrac{1}{2}\tilde{\theta}_{i,m}^2
\end{aligned} \tag{63}$$

**Step n**: In this final step, an adaptive controller suitable for both with and without output constraints would be established, in which an event-triggered control strategy with relative threshold is applied to decrease the communication from controller to actuators. Finding the derivative of $s_{i,n}=\hat{x}_{i,n}-\hbar_{i,n}$ yields

$$\dot{s}_{i,n}=\dot{\hat{x}}_{i,n}-\dot{\hbar}_{i,n}=m_i(u_i)+l_{i,n}(y_i-\hat{x}_{i,1})+\theta_{i,n}^T S_{i,n}(\underline{\hat{x}}_{i,n})-\dot{\hbar}_{i,n} \tag{64}$$

Construct the following compensation signal

$$\dot{q}_{i,n}=-g_{i,n}q_{i,n}-q_{i,n-1}-r_{i,n}\tanh(q_{i,n}) \tag{65}$$

Considering (64) and (65) results in the following compensation tracking signal

$$v_{i,n}=s_{i,n}-q_{i,n} \tag{66}$$

and its differentiation can be obtained

$$\begin{aligned}
\dot{v}_{i,n} &= \dot{s}_{i,n}-\dot{q}_{i,n} \\
&= m_i(u_i)+\theta_{i,n}^T\varphi_{i,n}(\underline{\hat{x}}_{i,n})+l_{i,n}(y_i-\hat{x}_{i,1})-\dot{\hbar}_{i,n} \\
&\quad -[-g_{i,n}q_{i,n}-q_{i,n-1}-r_{i,n}\tanh(q_{i,n})] \\
&= m_i(u_i)+\theta_{i,n}^T\varphi_{i,n}(\underline{\hat{x}}_{i,n})+l_{i,n}(y_i-\hat{x}_{i,1}) \\
&\quad +g_{i,n}q_{i,n}+q_{i,n-1}+r_{i,n}\tanh(q_{i,n})-\dot{\hbar}_{i,n}
\end{aligned} \tag{67}$$

The Lyapunov function is designed as

$$V_n=V_{n-1}+\frac{1}{2}[\sum_{i=1}^{N}(v_{i,n}^2+\frac{1}{\lambda_{i,n}}\tilde{\theta}_{i,n}^T\tilde{\theta}_{i,n}^)] \tag{68}$$

Calculating the derivative of $V_n$ leads to

$$\begin{aligned}\dot{V}_n &= \dot{V}_{n-1} + \sum_{i=1}^{N} v_{i,n}\dot{v}_{i,n} - \sum_{i=1}^{N} \tfrac{1}{\lambda_{i,n}}\tilde{\theta}_{i,n}^T\dot{\theta}_{i,n} \\ &= \dot{V}_{n-1} + \sum_{i=1}^{N} v_{i,n}\big\{m_i(u_i) + \theta_{i,n}^T\varphi_{i,n}(\underline{\hat{x}}_{i,n}) + l_{i,n}(y_i - \hat{x}_{i,1}) + g_{i,n}q_{i,n} \\ &\quad + q_{i,n-1} + r_{i,n}\tanh(q_{i,n}) - \dot{\hbar}_{i,n}\big\} - \sum_{i=1}^{N} \tfrac{1}{\lambda_{i,n}}\tilde{\theta}_{i,n}^T\dot{\theta}_{i,n} \\ &= \dot{V}_{n-1} + \sum_{i=1}^{N} v_{i,n}\big\{m_i(u_i) + \theta_{i,n}^T\varphi_{i,n}(\underline{\hat{x}}_{i,n}) + l_{i,n}(y_i - \hat{x}_{i,1}) \\ &\quad + g_{i,n}q_{i,n} + q_{i,n-1} + r_{i,n}\tanh(q_{i,n}) - \dot{\hbar}_{i,n} - \tilde{\theta}_{i,n}^T\varphi_{i,n}(\underline{\hat{x}}_{i,n})\big\} \\ &\quad - \sum_{i=1}^{N} \tfrac{1}{\lambda_{i,n}}\tilde{\theta}_{i,n}^T\big(\dot{\theta}_{i,n} - \lambda_{i,n}v_{i,n}\varphi_{i,n}(\underline{\hat{x}}_{i,n})\big)\end{aligned} \tag{69}$$

In aid of the Young's inequality, we can deduce that

$$v_{i,n}r_{i,n}\tanh(q_{i,n}) \le \frac{1}{2}v_{i,n}^2 + \frac{1}{2}r_{i,n}^2 \tag{70}$$

$$v_{i,n}(-\tilde{\theta}_{i,n}^T\varphi_{i,n}(\underline{\hat{x}}_{i,n})) \le \frac{1}{2}v_{i,n}^2 + \frac{1}{2}\tilde{\theta}_{i,n}^2 \tag{71}$$

Considering the existence of input dead-zone model, there exists

$$m_i(u_i) = \underline{\rho}_i(1 + r_i(t))u_i(t) + H_i(t) \le \underline{\rho}_i u_i + (\overline{\rho}_i - \underline{\rho}_i)\Upsilon_i + \overline{H}_i \tag{72}$$

By replacing (70)-(72) into (69), $\dot{V}_n$ can be simplified to

$$\begin{aligned}\dot{V}_n &\le \dot{V}_{n-1} + \sum_{i=1}^{N} v_{i,n}\big\{\underline{\rho}_i u_i + (\overline{\rho}_i - \underline{\rho}_i)\Upsilon_i + \overline{H}_i + \theta_{i,n}^T\varphi_{i,n}(\underline{\hat{x}}_{i,n}) \\ &\quad + l_{i,n}(y_i - \hat{x}_{i,1}) + g_{i,n}q_{i,n} + q_{i,n-1} - \dot{\hbar}_{i,n} + v_{i,n} \\ &\quad + \tfrac{1}{2}r_{i,n}^2 + \tfrac{1}{2}\tilde{\theta}_{i,n}^2\big\} - \sum_{i=1}^{N} \tfrac{1}{\lambda_{i,n}}\tilde{\theta}_{i,n}^T(\dot{\theta}_{i,n} - \lambda_{i,n}v_{i,n}\varphi_{i,n}(\underline{\hat{x}}_{i,n})\end{aligned} \tag{73}$$

The event-based triggered adaptive controller with time-varying threshold is designed as follows

$$\omega_i = -(1 + q_i)[\alpha_{i,n}\tanh(\frac{v_{i,n}\alpha_{i,n}}{\varsigma_i}) + \bar{z}_i\tanh(\frac{v_{i,n}\bar{m}}{\varsigma_i})] \tag{74}$$

$$u_i = \omega_i, \forall t \in \big[t_k, t_{k+1}\big) \tag{75}$$

$$t_{k+1} = \inf\{t \in R|\ |\omega_i - u_i| \ge q_i\,|u_i| + z_i\} \tag{76}$$

where $\alpha_{i,n}$ is actual controller and $\alpha_{i,n} = -\frac{1}{\underline{\rho}_i}\big(g_{i,n}s_{i,n} - s_{i,n-1} - l_{i,n}(y_i - \hat{x}_{i,1}) + \dot{\hbar}_{i,n} - \theta_{i,n}^T\varphi_{i,n}(\underline{\hat{x}}_{i,n})\big) - \frac{1}{\underline{\rho}_i}\big(\overline{\rho}_i - \underline{\rho}_i\big)\Upsilon_i - \frac{1}{\underline{\rho}_i}\overline{H}_i$, $\varsigma_i > 0$, $0 < q_i < 1$, $z_i > 0$ and $\bar{z}_i > \frac{z_i}{1-q_i}$ are constants. $t_{k+1}$ is the triggering time and the control input will be updated in this time.

Based on (76), we can get

$$\omega_i = (1 + c_{i,1}(t)q_i)u_i + c_{i,2}(t)z_i, \forall t \in \big[t_k, t_{k+1}\big] \tag{77}$$

where $c_{i,1}(t) \le 1$ and $c_{i,2}(t) \le 1$. Moreover, $u_i$ can be further obtained

$$u_i = \frac{\omega_i}{(1 + c_{i,1}(t)q_i)} - \frac{c_{i,2}(t)z_i}{(1 + c_{i,1}(t)q_i)} \tag{78}$$

According to **Lemma** 2, it can be gained that $v_{i,n}\omega_i \le 0$. Therefore, the following inequalities are true

$$\begin{aligned}\tfrac{v_{i,n}\omega_i\underline{\rho}_i}{(1+c_{i,1}(t)q_i)} &\le -v_{i,n}\underline{\rho}_i\tfrac{1+q_i}{(1+c_{i,1}(t)q_i)}(\alpha_{i,n}\tanh(\tfrac{v_{i,n}\alpha_{i,n}}{\varsigma_i}) + \bar{m}_i\tanh(\tfrac{v_{i,n}\bar{m}_i}{\varsigma_i})) \\ &\le v_{i,n}\underline{\rho}_i\alpha_{i,n} - \underline{\rho}_i\bar{m}_i\,|v_{i,n}| + 0.557\underline{\rho}_i\varsigma_i\end{aligned} \tag{79}$$

$$-v_{i,n}\underline{\rho}_i\frac{c_{i,2}(t)m_i}{(1 + c_{i,1}(t)q_i)} \le |v_{i,n}|\,\underline{\rho}_i\frac{m_i}{(1 - q_i)} \le |v_{i,n}|\,\underline{\rho}\bar{m}_i \tag{80}$$

$$v_{i,n}\underline{\rho}_i(\frac{w_i}{(1 + c_{i,1}(t)q_i)} - \frac{c_{i,2}(t)m_i}{(1 + c_{i,1}(t)q_i)}) \le v_{i,n}\underline{\rho}_i\alpha_{i,n} + 0.557\underline{\rho}_i\varsigma_i \tag{81}$$

Considering (74)-(81), $\dot{V}_n$ can become

$$\begin{aligned} \dot{V}_n \leq & \dot{V}_{n-1} + \sum_{i=1}^{N} \upsilon_{i,n} \big\{ \underline{\rho}_i \alpha_{i,n} + \theta_{i,n}^T \varphi_{i,n}(\underline{\hat{x}}_{i,n}) + l_{i,n}(y_i - \hat{x}_{i,1}) + \overline{H}_i \\ & + (\overline{\rho}_i - \underline{\rho}_i)\Upsilon_i + g_{i,n} q_{i,n} + q_{i,n-1} - \dot{\hbar}_{i,n} + \upsilon_{i,n} + \tfrac{1}{2} r_{i,n}^2 + \tfrac{1}{2}\tilde{\theta}_{i,n}^2 \big\} \\ & - \sum_{i=1}^{N} \tfrac{1}{\lambda_{i,n}} \tilde{\theta}_{i,n}^T (\dot{\theta}_{i,n} - \lambda_{i,n} \upsilon_{i,n} \varphi_{i,n}(\underline{\hat{x}}_{i,n}) + \sum_{i=1}^{N} 0.557 \underline{\rho}_i \varsigma_i \end{aligned} \tag{82}$$

Meanwhile, the adaptive laws are designed as

$$\dot{\theta}_{i,n} = \lambda_{i,n} \upsilon_{i,n} \varphi_{i,n}(\underline{\hat{x}}_{i,n}) - \lambda_{i,n} \gamma_{i,n} \theta_{i,n} \tag{83}$$

Substituting $\alpha_{i,n}$ and $\dot{\theta}_{i,n}$ into (82), we can achieve

$$\begin{aligned} \dot{V}_n \leq & \dot{V}_{n-1} + \sum_{i=1}^{N} \big\{ \upsilon_{i,n} \big[ -g_{i,n} \upsilon_{i,n} + \upsilon_{i,n-1} + \upsilon_{i,n} + \tfrac{1}{2} r_{i,n}^2 \\ & + \tfrac{1}{2} \tilde{\theta}_{i,n}^2 \big] \big\} - \sum_{i=1}^{N} \tfrac{1}{\lambda_{i,n}} \tilde{\theta}_{i,n}^T (\dot{\theta}_{i,n} - \lambda_{i,n} \upsilon_{i,n} \varphi_{i,n}(\underline{\hat{x}}_{i,n}) + \sum_{i=1}^{N} 0.557 \underline{\rho}_i \varsigma_i \\ \leq & \sum_{i=1}^{N} \{ -[\lambda_{\min}(Q_i) - \tfrac{5}{2} - \tfrac{1}{2} \|P_i\|^2 - \tfrac{1}{2} \|P_i\|^2 \sum_{p=2}^{n} H_{i,p}^2] \left\| \underline{\tilde{x}}_i \right\|^2 \\ & + \tfrac{1}{2} \|P_i\|^2 (\|\varepsilon_i^*\|^2 + \|d_i^*\|^2) + \tfrac{1}{2} \sum_{p=1}^{n} \tilde{\theta}_{i,p}^T \tilde{\theta}_{i,p} \} \\ & - \sum_{i=1}^{N} (g_{i,1} - \tfrac{7}{2} (b_i + \mu_i)^2 \eta^2{}_{i,1}) \upsilon_{i,1}^2 - \sum_{i=1}^{N} [\sum_{m=2}^{n} (g_{i,m} - 1) \upsilon_{i,m}^2] \\ & + \sum_{i=1}^{N} [-\tfrac{\gamma_{i,1}}{2} \left\| \tilde{\theta}_{i,1} \right\|^2 - \sum_{m=2}^{n-1} \tfrac{\gamma_{i,m}}{2} \|\tilde{\theta}_{i,m}\|^2 - \tfrac{1}{2} \sum_{j \in N_i} \sigma_{j,1} a_{i,j} \left\| \tilde{\theta}_{j,1} \right\|^2] \\ & + \sum_{i=1}^{N} [\sum_{m=1}^{n-1} \tfrac{\gamma_{i,m}}{2} \left\| \theta_{i,m}^* \right\|^2 + \tfrac{1}{2} \sum_{j \in N_i} \sigma_{j,1} a_{i,j} \left\| \theta_{j,1}^* \right\|^2] \\ & + \sum_{i=1}^{N} \{ \left\| d_{*,1}^* \right\|^2 + \left\| \varepsilon_{*,1}^* \right\|^2 \} + \sum_{i=1}^{N} \sum_{m=1}^{n} \tfrac{1}{2} r_{i,m}^2 \\ & + \sum_{i=1}^{N} \sum_{m=2}^{n-1} \tfrac{1}{2} \tilde{\theta}_{i,m}^2 + \sum_{i=1}^{N} \tilde{\theta}_{i,n}^T \gamma_{i,n} \theta_{i,n} + \sum_{i=1}^{N} 0.557 \underline{\rho}_i \varsigma_i \end{aligned} \tag{84}$$

With the assistant of Young's inequality, we can deduce

$$\tilde{\theta}_{i,n}^T \gamma_{i,n} \theta_{i,n} \leq \frac{\gamma_{i,n}}{2} \|\tilde{\theta}_{i,n}\| + \frac{\gamma_{i,n}}{2} \left\| \theta_{i,n}^* \right\|. \tag{85}$$

Combining (84) and (85) yields

$$\begin{aligned} \dot{V}_n \leq & \sum_{i=1}^{N} \big\{ -\big[ \lambda_{\min}(Q_i) - \tfrac{5}{2} - \tfrac{1}{2} \|P_i\|^2 - \tfrac{1}{2} \|P_i\|^2 \sum_{p=2}^{n} H_{i,p}^2 \big] \left\| \underline{\tilde{x}}_i \right\|^2 \\ & + \tfrac{1}{2} \|P_i\|^2 \big( \|\varepsilon_i^*\|^2 + \|d_i^*\|^2 \big) + \tfrac{1}{2} \sum_{p=1}^{n} \tilde{\theta}_{i,p}^T \tilde{\theta}_{i,p} \} \\ & - \sum_{i=1}^{N} (g_{i,1} - \tfrac{7}{2} (b_i + \mu_i)^2 \eta^2{}_{i,1}) \upsilon_{i,1}^2 - \sum_{i=1}^{N} \big[ \sum_{m=2}^{n} (g_{i,m} - 1) \upsilon_{i,m}^2 \big] \\ & + \sum_{i=1}^{N} \big[ -\tfrac{\gamma_{i,1}}{2} \left\| \tilde{\theta}_{i,1} \right\|^2 - \sum_{m=2}^{n} \tfrac{\gamma_{i,m}}{2} \|\tilde{\theta}_{i,m}\|^2 - \tfrac{1}{2} \sum_{j \in N_i} \sigma_{j,1} a_{i,j} \left\| \tilde{\theta}_{j,1} \right\|^2 \big] \\ & + \sum_{i=1}^{N} \big[ \sum_{m=1}^{n} \tfrac{\gamma_{i,m}}{2} \left\| \theta_{i,m}^* \right\|^2 + \tfrac{1}{2} \sum_{j \in N_i} \sigma_{j,1} a_{i,j} \left\| \theta_{j,1}^* \right\|^2 \big] \\ & + \sum_{i=1}^{N} \big\{ \left\| d_{*,1}^* \right\|^2 + \left\| \varepsilon_{*,1}^* \right\|^2 \big\} + \sum_{i=1}^{N} \sum_{m=1}^{n} \tfrac{1}{2} r_{i,m}^2 + \sum_{i=1}^{N} \sum_{m=2}^{n} \tfrac{1}{2} \tilde{\theta}_{i,m}^2 + \sum_{i=1}^{N} 0.557 \underline{\rho}_i \varsigma_i \end{aligned} \tag{86}$$

By defining $\psi = \sum_{i=1}^{N}\{\left\|d_{*,1}^{*}\right\|^{2} + \left\|\varepsilon_{*,1}^{*}\right\|^{2}\} + \sum_{i=1}^{N}\sum_{m=1}^{n}\frac{1}{2}r_{i,m}^{2} + \sum_{i=1}^{N}[\sum_{m=1}^{n}\frac{\gamma_{i,m}}{2}\left\|\theta_{i,m}^{*}\right\|^{2} + \frac{1}{2}\sum_{j\in N_i}\sigma_{j,1}a_{i,j}\left\|\theta_{j,1}^{*}\right\|^{2}] + \sum_{i=1}^{N}0.557\underline{\rho}_i\varsigma_i + \sum_{i=1}^{N}\frac{1}{2}\|P_i\|^{2}(\|\varepsilon_i^{*}\|^{2} + \|d_i^{*}\|^{2})$, the following $\dot{V}_n$ can be obtained

$$
\begin{aligned}
\dot{V}_n \leq & \sum_{i=1}^{N}\{-[\lambda_{\min}(Q_i) - \tfrac{5}{2} - \tfrac{1}{2}\|P_i\|^2 - \tfrac{1}{2}\|P_i\|^2\sum_{p=2}^{n}H_{i,p}^2]\left\|\tilde{\underline{x}}_i\right\|^2 + \tfrac{1}{2}\sum_{m=1}^{n}\tilde{\theta}_{i,m}^{T}\tilde{\theta}_{i,m}\} \\
& - \sum_{i=1}^{N}(g_{i,1} - \tfrac{7}{2}(b_i+\mu_i)^2\eta^2{}_{i,1})v_{i,1}^2 - \sum_{i=1}^{N}[\sum_{m=2}^{n}(g_{i,m}-1)v_{i,m}^2] \\
& + \sum_{i=1}^{N}[-\tfrac{\gamma_{i,1}}{2}\|\tilde{\theta}_{i,1}\|^2 - \sum_{m=2}^{n}\tfrac{\gamma_{i,m}}{2}\|\tilde{\theta}_{i,m}\|^2 - \tfrac{1}{2}\sum_{j\in N_i}\sigma_{j,1}a_{i,j}\left\|\tilde{\theta}_{j,1}\right\|^2] \\
& + \sum_{i=1}^{N}\sum_{m=2}^{n}\tfrac{1}{2}\tilde{\theta}_{i,m}^2 + \psi \\
\leq & \sum_{i=1}^{N}\{-[\lambda_{\min}(Q_i) - \tfrac{5}{2} - \tfrac{1}{2}\|P_i\|^2 - \tfrac{1}{2}\|P_i\|^2\sum_{p=2}^{n}H_{i,p}^2]\left\|\tilde{\underline{x}}_i\right\|^2\} \\
& - \sum_{i=1}^{N}(g_{i,1} - \tfrac{7}{2}(b_i+\mu_i)^2\eta^2{}_{i,1})v_{i,1}^2 - \sum_{i=1}^{N}[\sum_{m=2}^{n}(g_{i,m}-1)v_{i,m}^2] \\
& - \sum_{i=1}^{N}(\tfrac{\gamma_{i,1}}{2} - \tfrac{1}{2})\|\tilde{\theta}_{i,1}\|^2 - \sum_{i=1}^{N}\sum_{m=1}^{n}(\tfrac{\gamma_{i,m}}{2} - 1)\tilde{\theta}_{i,m}^{T}\tilde{\theta}_{i,m} \\
& - \sum_{i=1}^{N}\sum_{j\in N_i}\tfrac{1}{2}\sigma_{j,1}a_{i,j}\left\|\tilde{\theta}_{j,1}\right\|^2 + \psi
\end{aligned}
\tag{87}
$$

The inequality (87) can be simplified as

$$
\begin{aligned}
\dot{V}_n \leq & -\sum_{i=1}^{N}r_0\left\|\tilde{\underline{x}}_i\right\|^2 - \sum_{i=1}^{N}\sum_{m=1}^{n}hv_{i,m}^2 - \sum_{i=1}^{N}\sum_{m=1}^{n}\varpi\|\tilde{\theta}_{i,1}\|^2 \\
& -\tfrac{1}{2}\sum_{i=1}^{N}\sum_{j\in N_i}c\tfrac{a_{i,j}}{\beta_{j,1}}\left\|\tilde{\theta}_{j,1}\right\|^2 + \psi
\end{aligned}
\tag{88}
$$

where $r_0 = \lambda_{\min}(Q_i) - \frac{5}{2} - \frac{1}{2}\|P_i\|^2 - \frac{1}{2}\|P_i\|^2\sum_{p=2}^{n}H_{i,p}^2$, $h = \min\{g_{i,1} - \frac{7}{2}(b_i+\mu_i)^2\eta^2{}_{i,1}, (g_{i,2}-1), ...., (g_{i,m}-1)\}$, $\varpi = \min\{(\frac{\gamma_{i,1}}{2} - \frac{1}{2}), ...., (\frac{\gamma_{i,m}}{2} - 1)\}$, $c = \min\{\sigma_{j,1}\beta_{j,1}\}$, and $2 \leq p \leq n$, $1 \leq i \leq M$, $j \in N_i$.

*Remark 2.* In the backstepping design framework, the virtual controller at each step contains the derivatives of the virtual controller at the previous step, which denoting that the derivatives of the virtual control signal need to be computed. As the order of the system increases, the number of differentiation terms will become more numerous and the order of differentiation will increase, which will make the structure of the controller at the next step more complex and more difficult to design. To overcome such difficulties, a first-order filter is introduced in[21] to avoid the the detailed calculation of the above differentiation terms. Nevertheless, the filter causes the filtering errors, which impairs system performances. Following work[22], in this paper, the compensation signals will be designed to counteract such errors for obtaining better system performances.

## 3.3 | Stability Analysis

With the above derivation, the following results can be acquired.

**Theorem 1.** For the nonlinear multi-agents systems (1) with output constraints (8) meeting **Assumptions** 1-4, by introducing the nonlinear mapping approach (28), the event-triggered mechanism (74)-(76), the combining backstepping technique and the fuzzy state estimator (18) to construct the virtual controller (44) and (59), the actual controller (75) and the adaptive laws (46), (47), (60) and (83), the following results can be gained

**(1)** The system output has not overstepped the output constraints during system operation.

**(2)** All signals of closed-loop systems are ultimately bounded.

**Proof :** By designing $\kappa = \min\{\frac{2r_0}{\lambda_{\max}(P_i)}, 2h, 2\varpi, v\}$, $\dot{V}_n$ can be further simplified as

$$\dot{V}_n \leq -\kappa V_n + \psi \tag{89}$$

Multiplying both sides of the inequality by $e^{\kappa t}$ leads to

$$e^{\kappa t}\dot{V}_n \leq -\kappa V_n e^{\kappa t} + \psi e^{\kappa t} \tag{90}$$

Considering the integral of (90) over the interval $\in [0,t]$, one has the following inequality

$$V_n e^{\kappa t}|_0^t - \int_0^t V_n d e^{\kappa t} \leq -\kappa \int_0^t V_n e^{\kappa t} dt + \int_0^t \psi e^{\kappa t} dt \tag{91}$$

By computing (91), one has

$$V_n(t) \leq V_n(0)e^{-\kappa t} + \frac{\psi}{\kappa}(1 - e^{-\kappa t}) \tag{92}$$

From (89), we can obtain some information that if set $\kappa > \frac{\psi}{w}$, then $\dot{V}_n(t) \leq 0$ on $V_n(t) = w$. Thus, $V_n(t) \leq w$ is an invariant set. It can be deduced that if $V_n(0) \leq w$, the inequality holds $V_n(t) \leq w$ for $\forall t \geq 0$. Hence, all signals in closed-loop systems are ultimately bounded. In addition, it can be gained from (92) that $\|s_1\|^2 \leq 2V_n(0)e^{-\kappa t} + 2\frac{\psi}{\kappa}(1 - e^{-\kappa t})$ ($s_1 = [s_{1,1}, s_{2,1}, ..., s_{M,1}]$). Since $\lim_{t\to\infty} e^{-\kappa t} = 0$, we can further acquire that $\lim_{t\to\infty} \|s_{i,1}\| \leq \sqrt{\frac{2\psi}{\kappa}}$, denoting that $s_{i,1}$ can converges to an ideal areas by selecting appropriate parameters $\psi$, $\kappa$. According to (31) and **Assumptions** 2 and 3, the boundedness of auxiliary variable $\xi_{i,1}$ can be guaranteed. Thus, for systems initial condition meeting $x_{i,1}(0) \in \Omega_{x_{i,1}}$, the system output has not violated its constraints. The proof is complete.

*Remark 3.* Following (69), by defining $E_i(t) = \omega_i - u_i, \forall t \in [t_k, t_{k+1})$, it can be deduced that $\frac{d|E_i(t)|}{dt} = \frac{d\sqrt{E_i^2(t)}}{dt} \leq sgn(E_i(t))\dot{E}_i(t) \leq |\dot{E}_i(t)| \leq o_i$ with $o_i > 0$ being a constant. Due to $E_i(t_k) = 0$ and $\lim_{t\to t_{k+1}} E_i(t_{k+1}) = q_i|u_i| + z_i$, which mean that the interexecution time interval $T = t_{k+1} - t_k \geq \frac{q_i|u_i|+z_i}{o_i}$. Meanwhile, we can further infer that $T$ has a lower bound $T^* = \frac{z_i}{o_i} > 0$. Thus, the Zeno phenomenon is avoided.

## 4 | SIMULATION EXAMPLE

In this section, a numerical simulation example would be used to confirm the validity and feasibility of the proposed method. The dynamic of each agent are modeled as follows

$$\begin{cases} \dot{x}_{i,1} = x_{i,2} + \phi_{i,1}(\underline{x}_{i,1}) + d_{i,1} \\ \dot{x}_{i,2} = m_i(u_i) + \phi_{i,2}(\underline{x}_{i,2}) + d_{i,2} \\ y_i = x_{i,1}, \qquad i = 1, \ldots, 4, \end{cases} \tag{93}$$

where the system functions are set as $\phi_{i,1}(\underline{x}_{i,1}) = -\sin(x_{i,1})$ and $\phi_{i,2}(\underline{x}_{i,2}) = -\cos(x_{i,2})$; the external disturbances are selected as $d_{i,1} = -0.1\sin(10t)$ and $d_{i,2} = -0.1\cos(10t)$; the output constrains functions are chosen as $L_{i,1} = 3 + 0.5\sin(t)$ and $L_{i,2} = 3 + 3\sin(t)$. The ideal trajectories provided by $y_d = \sin(t)$. The undirected communication graph of the whole multi-agents systems is exhibited in **Fig** 1

To gain the expected control objective, the relative parameters are configured as $l_{i,1} = 7$, $l_{i,2} = 9$, $\lambda_{i,1} = 10$, $\lambda_{i,2} = 10$, $\gamma_{i,1} = 0.2$, $\gamma_{i,2} = 2$, $g_{i,1} = 15$, $g_{i,2} = 30$, $r_{i,1} = 5$, $r_{i,2} = 0.2$, $\partial_{i,2} = 0.05$, $q_i = 0.5$, $z_i = 0.2$, $\varsigma_i = 1$, $\rho_{ir} = 1$, $\rho_{ir} = 1.1$, $h_{ir} = 0.45$, $h_{il} = -0.45$. Meanwhile, the initial values of control systems are assigned as $x_{1,1} = 1$, $x_{1,2} = 0$, $x_{2,1} = 0.7$, $x_{2,2} = 0$, $x_{3,1} = 0.4$,$x_{3,2} = 0$, $x_{4,1} = -0.2$, $x_{4,2} = 0$, and $\hat{x}_{i,1} = \hat{x}_{i,2} = 0$, $\theta_{i,1} = \theta_{i,2} = [0,0,0,0,0,0,0]^T$.

Through applying the presented approach in this paper, the following simulation results can be obtained.

The tracking performances of system are shown in **Figs** 2 and 3 , in which the tracking errors can converge into a small interval, and the consensus objective can be obtained. In **Figs** 4 and 5 , the system states and the estimates are exhibited. In **Figs** 6 and 7 , the event triggered-based controller and the input signals are displayed, and the inter-execution internal of triggering are expressed. The adaptive laws of each agent are exhibited in **Fig** 8 .

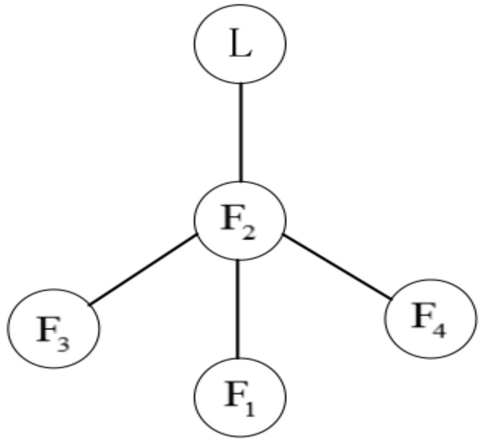


**FIGURE 1** The undirected communication protocol of multi-agents.

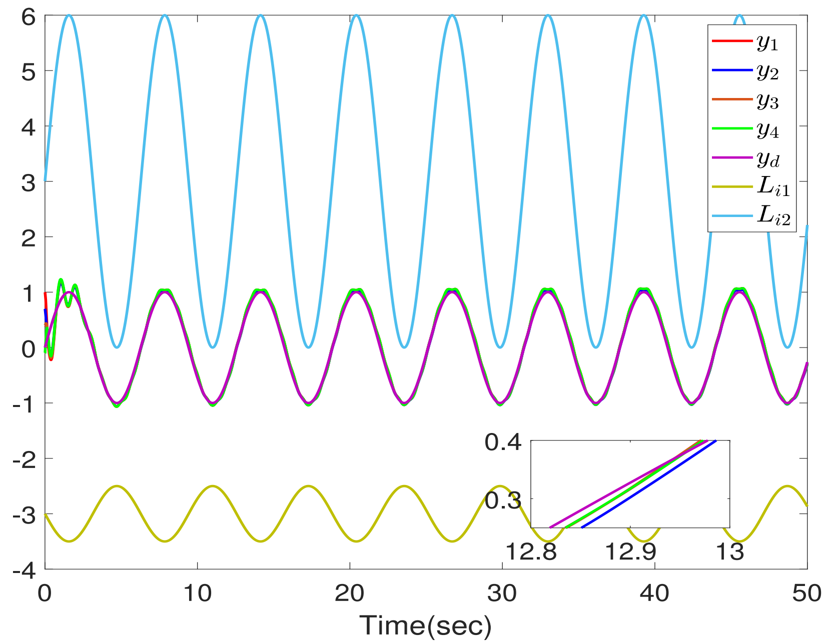


**FIGURE 2** The output $y_i$ of each agent and the ideal signal $y_d$.

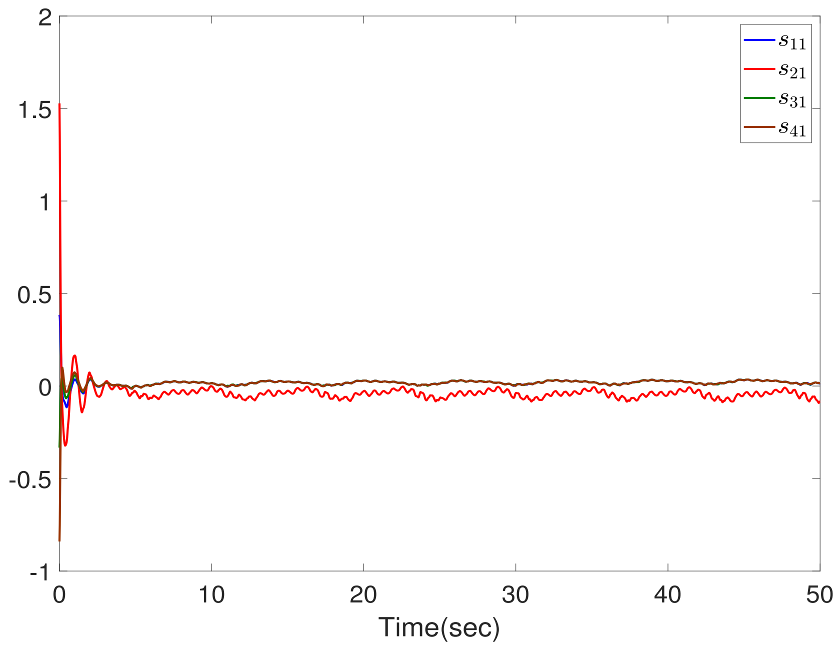


**FIGURE 3** The tracking errors $s_{i,1}$ of each agent.

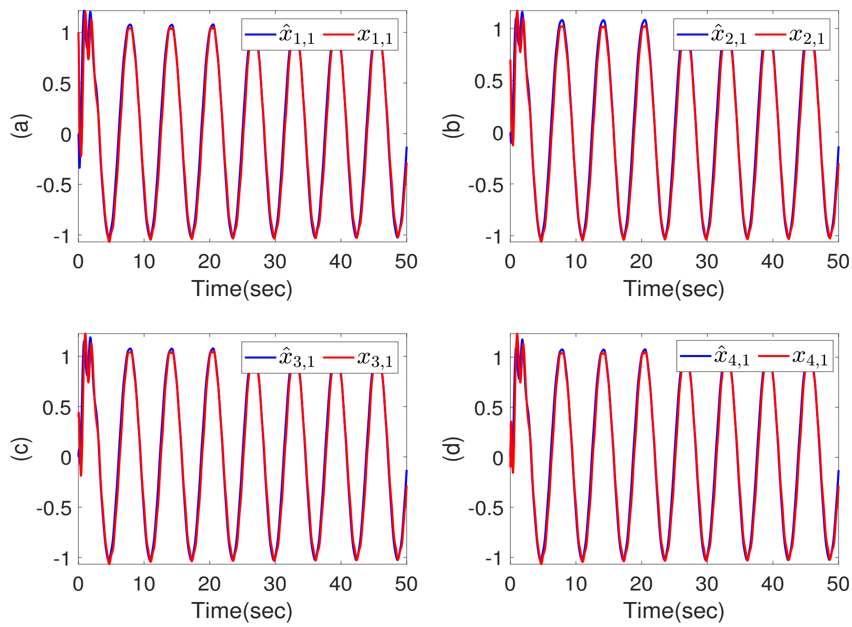


**FIGURE 4** The state of each agent and the corresponding estimate.

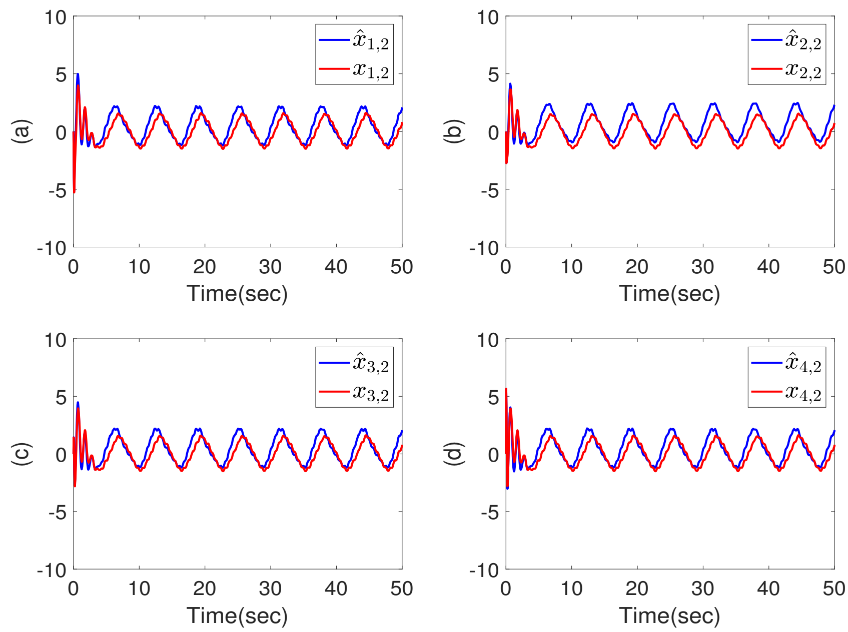


**FIGURE 5** The state of each agent and the corresponding estimate.

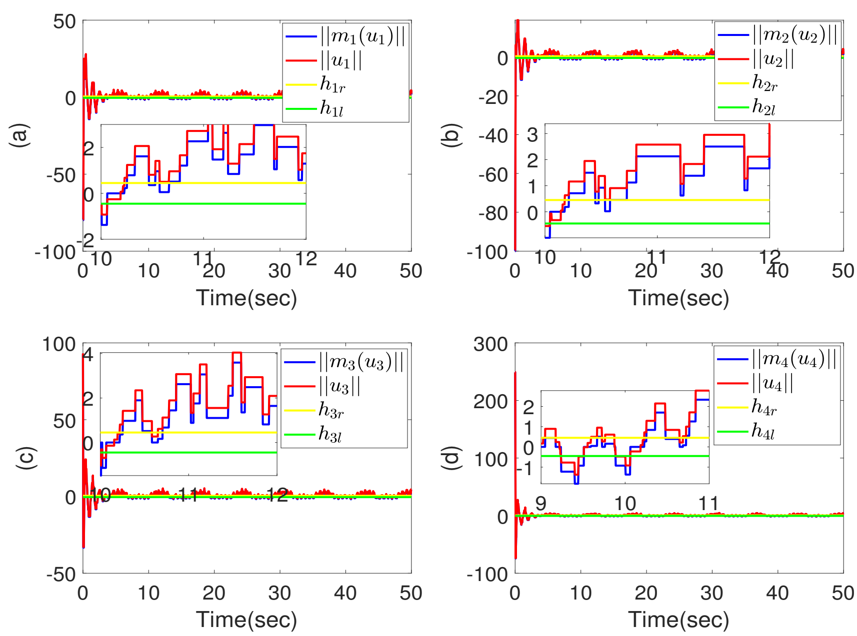


**FIGURE 6** The input signals $u_i$ and the control signal $m_i(u_i)$ subject to input dead zone.

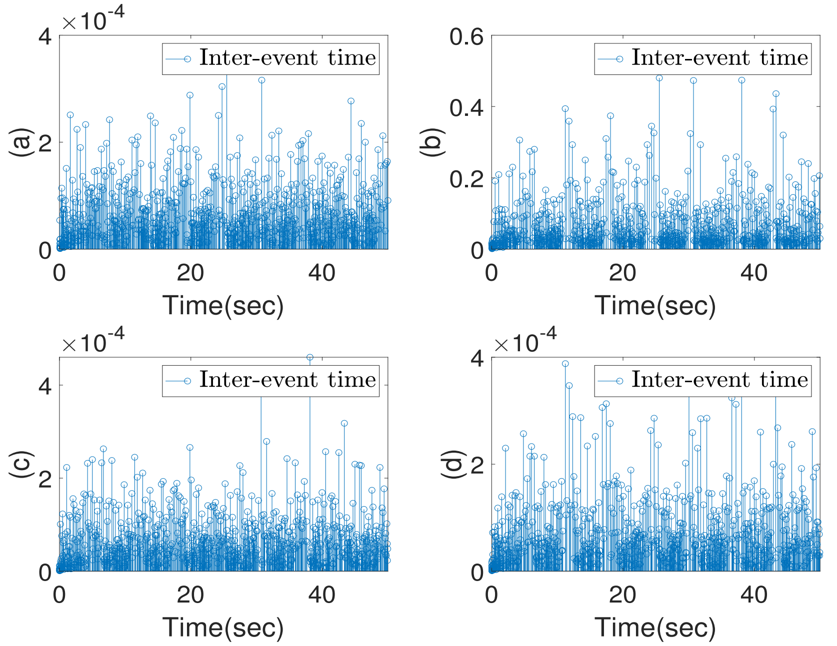


**FIGURE 7** The inter-execution internal of event triggering.

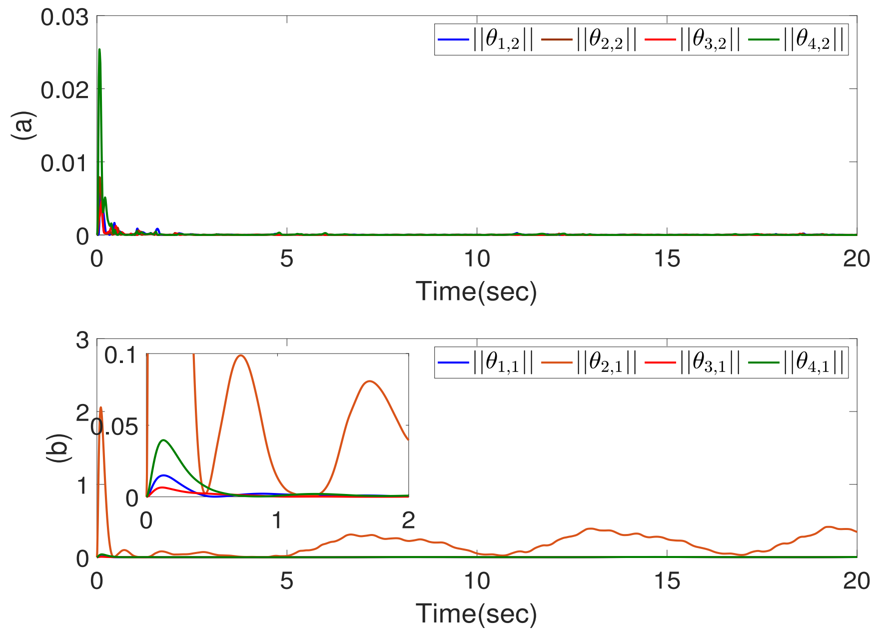


**FIGURE 8** The adaptive laws $\theta_{i,1}$ and $\theta_{i,2}$.

## 5 | CONCLUSION

In this work, an adaptive event triggered-based leader-follower consensus control strategy has been reported for nonlinear multi-agents systems with output constraints. A fuzzy state estimator has been constructed by utilizing the system outputs to reconstruct the unavailable state. By introducing an advanced nonlinear mapping approach, the two cases with and without constraints can be handled uniformly without no needing to change the control structure. In view of the restrictions of communication channels from controller to actuator, an event triggered mechanism with relative threshold has been used to reduce the information transmission. Meanwhile, a hyperbolic tangent function has been employed to reduce the chattering issue. In addition, it has been demonstrated through Lyapunov function that all signals in closed-loop systems are ultimately bounded and the time-varying output constraints have not been overstepped. At last, a numerical simulation example has been applied to confirm the effectiveness and feasibility of the reported strategy.

## ACKNOWLEDGMENTS

This work was partially supported by the National Natural Science Foundation of China, under Grant 62203064.

## CONFLICT OF INTEREST STATEMENT

No potential conflict of interest was reported by the authors.

## DATA AVAILABILITY STATEMENT

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

## ORCID

Zixuan Huang https://orcid.org/0009-0000-5508-1475

## References

1. Guo Qinglin, Zhang Ming. A novel approach for multi-agent-based Intelligent Manufacturing System. *Information Sciences.* 2009;179(18):3079-3090.

2. Le Duong, Plaku Erion. Multi-Robot Motion Planning With Dynamics via Coordinated Sampling-Based Expansion Guided by Multi-Agent Search. *IEEE Robotics and Automation Letters.* 2019;4(2):1868-1875.

3. Ren Wei, Beard Randal W, Atkins Ella M. Information consensus in multivehicle cooperative control. *IEEE Control systems magazine.* 2007;27(2):71–82.

4. Zhao Heng, Wang Huanqing, Niu Ben, Zhao Xudong, Xu Ning. Adaptive fuzzy decentralized optimal control for interconnected nonlinear systems with unmodeled dynamics via mixed data and event driven method. *Fuzzy Sets and Systems.* 2024;474:108735.

5. Liu Shihui, Niu Ben, Xu Ning, Zhao Xudong. Zero-Sum Game-Based Decentralized Optimal Control for Saturated Nonlinear Interconnected Systems via a Data and Event Driven Approach. *IEEE Systems Journal.* 2024;.

6. Dong Xiwang, Yu Bocheng, Shi Zongying, Zhong Yisheng. Time-varying formation control for unmanned aerial vehicles: Theories and applications. *IEEE Transactions on Control Systems Technology.* 2014;23(1):340–348.

7. Battistelli Giorgio, Benavoli Alessio, Chisci Luigi. Data-driven communication for state estimation with sensor networks. *Automatica.* 2012;48(5):926–935.

8. Deng Fang, Guo Su, Zhou Rui, Chen Jie. Sensor multifault diagnosis with improved support vector machines. *IEEE transactions on automation science and engineering.* 2015;14(2):1053–1063.

9. Hong Yiguang, Hu Jiangping, Gao Linxin. Tracking control for multi-agent consensus with an active leader and variable topology. *Automatica.* 2006;42(7):1177–1182.

10. Yu Wenwu, Chen Guanrong, Cao Ming. Some necessary and sufficient conditions for second-order consensus in multi-agent dynamical systems. *Automatica.* 2010;46(6):1089–1095.

11. Cao Weijun, Zhang Jinhui, Ren Wei. Leader–follower consensus of linear multi-agent systems with unknown external disturbances. *Systems & Control Letters.* 2015;82:64–70.

12. Ghasemzadeh Seyyed Vahid, Safarinejadian Behrouz. Leader–Follower Tracking in Nonlinear Multi-agent Systems via Different Velocity and Position Graph Topologies with External Disturbance. *Iranian Journal of Science and Technology, Transactions of Electrical Engineering.* 2023;:1–12.

13. Tabuada Paulo. Event-triggered real-time scheduling of stabilizing control tasks. *IEEE Transactions on Automatic control.* 2007;52(9):1680–1685.

14. Liu Shihui, Wang Huanqing, Liu Yunfeng, Xu Ning, Zhao Xudong. Sliding-mode surface-based adaptive optimal nonzero-sum games for saturated nonlinear multi-player systems with identifier-critic networks. *Neurocomputing.* 2024;584:127575.

15. Cao Yumeng, Niu Ben, Wang Huanqing, Zhao Xudong. Event-based adaptive resilient control for networked nonlinear systems against unknown deception attacks and actuator saturation. *International Journal of Robust and Nonlinear Control.* 2024;34(7):4769–4786.

16. Girard Antoine. Dynamic triggering mechanisms for event-triggered control. *IEEE Transactions on Automatic Control.* 2014;60(7):1992–1997.

17. Hu Xiaoyan, Li Yuan-Xin, Hou Zhongsheng. Event-triggered fuzzy adaptive fixed-time tracking control for nonlinear systems. *IEEE Transactions on Cybernetics.* 2020;52(7):7206–7217.

18. Wu Xiangjun, Ding Shuo, Xu Ning, Niu Ben, Zhao Xudong. Periodic event-triggered bipartite containment control for nonlinear multi-agent systems with input delay. *International Journal of Systems Science.* 2023;.

19. Huang Sai, Zong Guangdeng, Xu Ning, Wang Huanqing, Zhao Xudong. Adaptive dynamic surface control of MIMO nonlinear systems: A hybrid event triggering mechanism. *International Journal of Adaptive Control and Signal Processing.* 2024;38(2):437–454.

20. Wu Xiangjun, Zhao Ning, Ding Shuo, Wang Huanqing, Zhao Xudong. Distributed Event-Triggered Output-Feedback Time-Varying Formation Fault-Tolerant Control for Nonlinear Multi-Agent Systems. *IEEE Transactions on Automation Science and Engineering.* 2024;.

21. Wang Liqi, Dong Jiuxiang. Adaptive Fuzzy Consensus Tracking Control for Uncertain Fractional-Order Multiagent Systems With Event-Triggered Input. *IEEE Transactions on Fuzzy Systems.* 2022;30(2):310-320.

22. Hao Ruolan, Wang Hongbin, Zheng Wei. Dynamic event-triggered adaptive command filtered control for nonlinear multi-agent systems with input saturation and disturbances. *ISA Transactions.* 2022;.

23. Huang Sai, Zong Guangdeng, Niu Ben, Xu Ning, Zhao Xudong. Dynamic Self-Triggered Fuzzy Bipartite Time-Varying Formation Tracking for Nonlinear Multi-Agent Systems With Deferred Asymmetric Output Constraints. *IEEE Transactions on Fuzzy Systems.* 2024;.

24. Wei Fansen, Zhang Liang, Niu Ben, Zong Guangdegn. Adaptive decentralized fixed-time neural control for constrained strong interconnected nonlinear systems with input quantization. *International Journal of Robust and Nonlinear Control.* 2024;.

25. Tang Fanghua, Niu Ben, Wang Huanqing, Zhang Liang, Zhao Xudong. Adaptive fuzzy tracking control of switched MIMO nonlinear systems with full state constraints and unknown control directions. *IEEE Transactions on Circuits and Systems II: Express Briefs.* 2022;69(6):2912–2916.

26. Zhu Boyan, Xu Ning, Zong Guangdeng, Zhao Xudong. Adaptive optimized backstepping tracking control for full state constrained nonlinear strict-feedback systems without using barrier Lyapunov function method. *Optimal Control Applications and Methods.* 2024;.

27. Liu Minggang, Xu Ning. Adaptive Neural Predefined-Time Hierarchical Sliding Mode Control of Switched Under-Actuated Nonlinear Systems Subject to Bouc-Wen Hysteresis. *International Journal of Systems Science.* 2024;.

28. Li Yulin, Niu Ben, Zong Guangdeng, Zhao Jinfeng, Zhao Xudong. Command filter-based adaptive neural finite-time control for stochastic nonlinear systems with time-varying full-state constraints and asymmetric input saturation. *International Journal of Systems Science.* 2022;53(1):199–221.

29. He Youqian, Chang Xiao-Heng, Wang Huanqing, Zhao Xudong. Command-filtered adaptive fuzzy control for switched MIMO nonlinear systems with unknown dead zones and full state constraints. *International Journal of Fuzzy Systems.* 2023;25(2):544–560.

30. Ji Qiutong, Chen Gang, He Qiurui. Neural network-based distributed finite-time tracking control of uncertain multi-agent systems with full state constraints. *IEEE Access.* 2020;.

31. Xu Jiahong, Wang Lijie, Liu Yang, Sun Jize, Pan Yingnan. Finite-time adaptive optimal consensus control for multi-agent systems subject to time-varying output constraints. *Applied Mathematics and Computation.* 2022;.

32. Cao Ye, Wen Changyun, Song Yongduan. A unified event-triggered control approach for uncertain pure-feedback systems with or without state constraints. *IEEE Transactions on Cybernetics.* 2019;51(3):1262–1271.

33. Cao Zhongwen, Niu Ben, Zong Guangdeng, Xu Ning. Small-gain technique-based adaptive output constrained control design of switched networked nonlinear systems via event-triggered communications. *Nonlinear Analysis: Hybrid Systems.* 2023;.

34. Li Jian, Xu Wenqing, Wu Zhaojing, Liu Yungang. Practical Tracking Control via Switching for Uncertain Nonlinear Systems With Both Dead-Zone Input and Output Constraint. *IEEE Transactions on Circuits and Systems II: Express Briefs.* 2023;70(6):2031-2035.

35. Li Yulin, Xu Ning, Niu Ben, Chang Yi, Zhao Jinfeng, Zhao Xudong. Small-gain technique-based adaptive fuzzy command filtered control for uncertain nonlinear systems with unmodeled dynamics and disturbances. *International Journal of Adaptive Control and Signal Processing.* 2021;35(9):1664–1684.

36. Zhao Lin, Yu Jinpeng, Lin Chong. Distributed adaptive output consensus tracking of nonlinear multi-agent systems via state observer and command filtered backstepping. *Information Sciences.* 2019;.