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September 25, 2024

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# Resistance theory of asymmetric 2×*n* circuit network

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( 2024-09-01 )

## Abstract

This article proposes a class of asymmetric 2×*n* resistor network model which contains multiple independent resistors, the study of asymmetric resistor network models is a challenge. We conducted in-depth research on this issue using the RT-V theory (Chinese Physics B, 2017, **26**(9): 090503) and achieved new theoretical breakthroughs. This paper derived five original equivalent resistance formulae for this asymmetric 2×*n* circuit network, and also discusses the analytical expressions for equivalent resistance in different special cases, and several interesting results have been derived, indicating that the original resistance formula has powerful functionality. Finally, visual graphics of two types of equivalent resistances are provided using MATLAB drawing tools, which reveals the law of equivalent resistance changing with resistance variables. The research theory and technology in this article will provide a new theoretical basis for related scientific, engineering, and simulation research.

**Key words:** asymmetric network, RT-V method, matrix equation, equivalent resistance

## 1. Introduction

With the rapid development of electronic technology, circuit systems are becoming increasingly complex. As a basic component of circuits, the complexity and diversity of resistor networks (RNs) have also increased. Many abstract and complex problems need to be solved by establishing intuitive models. The establishment and research of resistor network models (RNMs) not only contribute to a deeper understanding of the working principles of circuits, but also have significant implications for integrated circuit design [1], circuit optimization [2], and fault diagnosis [3]. The research on RNMs are based on the theoretical foundations of multiple disciplines such as circuit theory, mathematical graph theory, matrix theory, and optimization algorithms [4-6]. As a fundamental research method, it has gradually penetrated into many disciplinary fields, promoting their development to a certain extent.

The establishment and research of RNMs have a history of more than 170 years, dating back to 1845 when Kirchhoff, who had just graduated from university, proposed two laws for complex circuit calculations in his academic paper [7], namely the famous Kirchhoff laws (loop voltage law and node current law). This law establishes the foundation of circuit analysis theory and solves a series of complex scientific problems. With the development of the electrification era, the definition of RNs has begun to generalize, including infinite networks composed of the same resistors or capacitors. At this time, the new theories are needed to study the electrical characteristics of infinite networks. Scholars in references [8-10] have proposed research on the resistance between two points in an infinite grid, as well as some new ideas and methods. In 2000, Professor Cserti from Roland University in Hungary established the Green's function technique (LGF) for calculating the equivalent resistance of infinite rectangular networks [9], which applied the LGF to infinite resistance networks and provided a method for calculating the equivalent resistance of infinite resistance networks. In addition, Asad [11,12] and Owaidat [13-16] used LGF to study complex infinite circuit networks. Their research works have promoted the research and development of infinite networks and made significant contributions. In 2004, Professor Wu FY from Northeastern University proposed the Laplacian matrix method [17] (Wu's LM method) for computing large-scale finite RNs, and used the eigenvalues and eigenvectors of the Laplacian matrix to calculate the resistance between any two nodes in the RN. Wu's LM method has also been well developed and applied in the study of equivalent resistance. For example, references [18-22] applied Wu's LM method to study the equivalent resistance between any two points in a resistance network under special circumstances, but this method is not applicable to networks on any boundary. In 2011, Professor Tan Z-Z from Nantong University in China proposed a new original theory called the recursion-transform (RT) theory [1] to solve RN problems with arbitrary boundaries. By applying Tan's RT method, the equivalent resistance of finite $m\times n$ RNs with different boundaries as shown in Refs. [23-31] can be calculated. This methodology only requires the establishment of matrix equations along one direction, simplifying the expression of the results and suitable for studying infinite and finite RNMs with arbitrary boundaries.

At present, Professor Tan has elevated the problem of RNs to RNMs, opening up a new research field [1]. The establishment of RT theory has promoted the study of RNMs for various complex structures. For example, in reference [32], for the first time, the RT-V method was used to obtain accurate potential formulas for arbitrary $m\times n$ cobweb and Fan networks, and potential formulas for infinite and semi infinite networks were derived. Reference [33] studied an arbitrary m×n apple surface network with a pair of non-uniform boundary resistances based on RT-V theory. Paper [34] studied the resistance between any node in two n-order periodic resistance networks using the RT-V method. Reference [35] used the RT-V method to establish two new sets of equations to determine the electrical characteristics of an irregular $2\times n$ hammock network. This paper proposes a class of asymmetric $2\times n$ RNMs containing multiple independent resistors, as shown in Fig.1. The model is very complex and consists of five independent resistor elements. We conducted in-depth research on this issue by using Professor Tan's RT-V method established in reference [32]. We derived five original equivalent resistance formulas for asymmetric $2\times n$ circuit networks and discussed analytical expressions for equivalent resistance in different special cases. Finally, we used MATLAB drawing tools such as those used in references [36, 37] to provide visual representations of two types of equivalent resistance, revealing the variation of equivalent resistance with resistance variables.

Recently, reference [38] studied a class of periodic and asymmetric $2\times n$ circuit networks embedded with T-shaped structures, and this article intends to study another type of non-periodic and asymmetric $2\times n$ circuit networks. Although their partial structures are similar, their essential problems are different because their boundary conditions are different (one is periodic structure, the other is non-periodic structure), and their equivalent resistance results are different. People understand that even a slight change in the conditions of the RNM will alter their electrical characteristics. Therefore, it is necessary to study non-periodic and asymmetric $2\times n$ circuit networks.

## 2. Main conclusions and formulae

The $2\times n$ RNM of the embedded inverted T-shaped circuit in Figure 1 has two characteristics : firstly, it is a very complex $2\times$n order RNM; Secondly, it can be inferred from its sub model that the model does not have symmetry vertically. Therefore, the $n$-order RNM embedded in an inverted T-shaped circuit is an asymmetric $2\times n$-order RNM. In this RNM, the lower nodes are defined from left to right as $A_0$ to $A_n$, the middle nodes corresponding to the lower nodes are defined as $B_0$ to $B_n$, and the remaining intermediate nodes are $C_1$ to $C_n$ in sequence. The top of the model is the horizontal axis $D_0$ to $D_n$ with the same potential, and $D_k$ in all expressions in the text is represented by $O$ (because they are located on the same wire). Refer to Figure 1, there are the five independent arbitrary resistance elements are arranged in this model. The resistance element $r$ is arranged between any two nodes on the horizontal axis $A_0A_n$, the $r_2$ is arranged between any two nodes on the horizontal axis $B_0B_n$. In the vertical direction, the $r_0$ is placed between $A_k$ and $B_k$, $r_3$ is placed between $B_k$ and $D_k$, the resistance element $r_1$ is arranged between $C_k$ and $D_k$. In this paper, the RT-V method is also used to study and derive the analytical formula for the equivalent resistance between any two nodes $A_{x_1}$ (and $B_{x_1}$) and $P_{x_2}(=A_{x_2},B_{x_2},O)$ in an $2\times n$ RN.

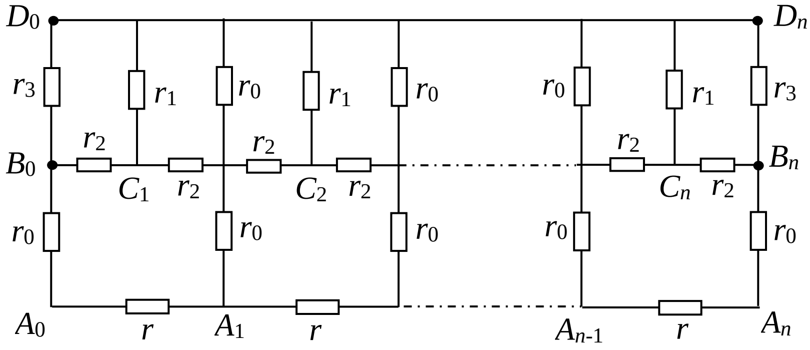


Fig.1. An asymmetric $2\times$n resistor network model embedded with T circuit, containing five different resistor elements, and $r_3=r_0//(2r_1+r_2)$.

## 2.1 Necessary parameter definitions

The RN studied in this article is very complex. In order to make the expression results more concise, the parameters in the article are defined as follows.

$$p_1 = \frac{1}{2}h_{r0} + (h_{21}+2)(h_{20}+1) - \sqrt{\Delta},$$

$$p_2 = \frac{1}{2}h_{r0} + (h_{21}+2)(h_{20}+1) + \sqrt{\Delta}, \tag{1}$$

$$\Delta = (\frac{1}{2}h_{r0} + 1 - d)^2 + (h_{21}+2)h_{20}h_{r0}, \tag{2}$$

In the above equation, $d = (h_{20}+1)(h_{21}+2) - 1$, and

$$h_{sk} = r_s / r_k, \quad h_{r0} = r / r_0, \quad r_3 = r_0 // (2r_1 + r_2), \tag{3}$$

and define

$$B_k^{(i)} = \frac{\lambda_i^k - \bar{\lambda}_i^k}{\lambda_i - \bar{\lambda}_i}, \quad \Delta B_k^{(i)} = B_{k+1}^{(i)} - B_k^{(i)}, \quad (i = 1, 2) \tag{4}$$

$$\lambda_i = \frac{1}{2}\left(p_i + \sqrt{p_i^2 - 4}\right), \quad \bar{\lambda}_i = \frac{1}{2}\left(p_i - \sqrt{p_i^2 - 4}\right). \tag{5}$$

$$F_{k,v}^{(i)} = \Delta B_{x_k}^{(i)} \Delta B_{n-x_v}^{(i)}, \tag{6}$$

The above parameters expressions (1)-(6) are applicable to the entire text and facilitate further research in the following sections.

## 2.2 Five original results

In the RNM shown in Fig. 1, for the horizontal axis $A_0A_n$ , from left to right, the first node on the left side is $A_0$, and the $k$-th node is $A_k$ . alike, assuming that the $k$-th node on the central axis $B_0B_n$ is $B_k$. The parameters of each resistor involved in the article are labeled in Fig. 1, in this asymmetric $2\times n$ RNM, the analytical expressions for the equivalent resistance between any node $A_{x_1}$ and node $P_{x_2}(=A_{x_2}, B_{x_2}, O)$ , as well as between node $B_{x_1}$ and node $P'_{x_2}(=B_{x_2}, O)$ are the following five conclusions.

### *2.2.1 Analytical formula of equivalent resistance* $R(A_{x_1}, B_{x_2})$

We have derived an accurate equivalent resistance analytical formula $R_n(A_{x_1}, B_{x_2})$ for the study of RNM in Figure 1

$$\frac{R_n\left(A_{x_1}, B_{x_2}\right)}{r} = (2d - p_1)\frac{F_{1,1}^{(1)} - 2u_1F_{1,2}^{(1)} + u_1^2F_{2,2}^{(1)}}{(p_2 - p_1)(p_1 - 2)B_{n+1}^{(1)}} - (2d - p_2)\frac{F_{1,1}^{(2)} - 2u_2F_{1,2}^{(2)} + u_2^2F_{2,2}^{(2)}}{(p_2 - p_1)(p_2 - 2)B_{n+1}^{(2)}}. \tag{7}$$

where $d = (h_{20}+1)(h_{21}+2) - 1$, and

$$u_i = h_{20}(2 + h_{21}) / (2d - p_i), \tag{8}$$

other parameters $B_k^{(i)}$ , $h_{sk}$ , $F_{k,v}^{(i)}$ etc. are defined in equations (1)-(6).

*2.2.2 Analytical formula of equivalent resistance* $R(A_{x_1}, O)$

We then derived the second equivalent resistance formula $R_n(A_{x_1}, O)$ for the complex 2 × *n* RN of Fig.1,

$$\frac{R_n\left(A_{x_1}, O\right)}{r} = \frac{(2d-p_1)F_{1,1}^{(1)}}{(p_2-p_1)(p_1-2)B_{n+1}^{(1)}} - \frac{(2d-p_2)F_{1,1}^{(2)}}{(p_2-p_1)(p_2-2)B_{n+1}^{(2)}}, \tag{9}$$

where $d = (h_{20}+1)(h_{21}+2)-1$, and $t_1, t_2$ are defined in equation (1).

*2.2.3 Analytical formula of equivalent resistance* $R(A_{x_1}, A_{x_2})$

Furthermore, we derived the third equivalent resistance formula $R_n(A_{x_1}, A_{x_2})$ for the complex 2 × *n* RN of Fig.1,

$$\frac{R_n(A_{x_1}, A_{x_2})}{r} = (2d-p_1)\frac{F_{1,1}^{(1)} - 2F_{1,2}^{(1)} + F_{2,2}^{(1)}}{(p_2-p_1)(p_1-2)B_{n+1}^{(1)}} - (2d-p_2)\frac{F_{1,1}^{(2)} - 2F_{1,2}^{(2)} + F_{2,2}^{(2)}}{(p_2-p_1)(p_2-2)B_{n+1}^{(2)}}. \tag{10}$$

where all parameters $B_k^{(i)}$, $h_{sk}$, $d$, $F_{k,v}^{(i)}$ etc. are defined in equations (1)-(6).

*2.2.4 Analytical formula of equivalent resistance* $R(B_{x_1}, B_{x_2})$

The fourth equivalent resistance formula $R_n(B_{x_1}, B_{x_2})$ we derived in the complex 2 × *n* RN of Fig.1 is

$$\frac{R_n\left(B_{x_1}, B_{x_2}\right)}{r_2} = \frac{2+h_{21}}{p_2-p_1}\left( (\frac{h_{r0}}{p_1-2}-1)\frac{F_{1,1}^{(1)} - 2F_{1,2}^{(1)} + F_{2,2}^{(1)}}{B_{n+1}^{(1)}} - (\frac{h_{r0}}{p_2-2}-1)\frac{F_{1,1}^{(2)} - 2F_{1,2}^{(2)} + F_{2,2}^{(2)}}{B_{n+1}^{(2)}} \right) \tag{11}$$

where all parameters $B_k^{(i)}$, $h_{sk}$, $p_1, p_2$, $F_{k,v}^{(i)}$ etc. are defined in equations (1)-(6).

*2.2.5 Analytical formula of equivalent resistance* $R(B_{x_1}, O)$

The fifth equivalent resistance formula $R_n(B_{x_1}, O)$ we derived in the complex 2 × *n* RN of Fig.1 is

$$\frac{R_n\left(B_{x_1}, O\right)}{r_2} = \frac{2+h_{21}}{p_2-p_1}\left( (\frac{h_{r0}}{p_1-2}-1)\frac{F_{1,1}^{(1)}}{B_{n+1}^{(1)}} - (\frac{h_{r0}}{p_2-2}-1)\frac{F_{1,1}^{(2)}}{B_{n+1}^{(2)}} \right), \tag{12}$$

where all parameters $B_k^{(i)}$, $h_{sk}$, $p_1, p_2$, $F_{k,v}^{(i)}$ etc. are defined in equations (1)-(6).

From the above formulas (7), (9), (10), (11), and (12), we can see that although their expression are succinct, their contents are very abundant, and they are the research conclusions obtained for the first time in this article. The following text will provide the specific calculation process and draw the above conclusions.

## 3 Basic methods and theories

This article uses the most cutting edge theory, the RT-V theory established by Professor Tan [32], to study the complex circuit network in Figure 1. This theory has five essential research process. The RT-V theory will be used for calculation and derivation in the following text.

### 3.1 Construction of Matrix Equations

According to the RT-V theory, the Kirchhoff node current law ($\sum r_i^{-1}V_k = 0$) is first applied to establish a differential equation model for node voltage (including a general equation model and a boundary constrained equation model) to study the complex $2\times n$ RN in Figure 1. According to the structural characteristics of the RN in Figure 1, assuming that the potential of node O be zero $U_O = 0$, the current $I$ flows into the network from node $A_{x_1}$ and exits at node $B_{x_2}$ (or $A_{x_2}$ ,$O$). In order to establish the equation model, the sub network diagram shown in Figure 2, as well as the resistance and voltage parameters, were provided.

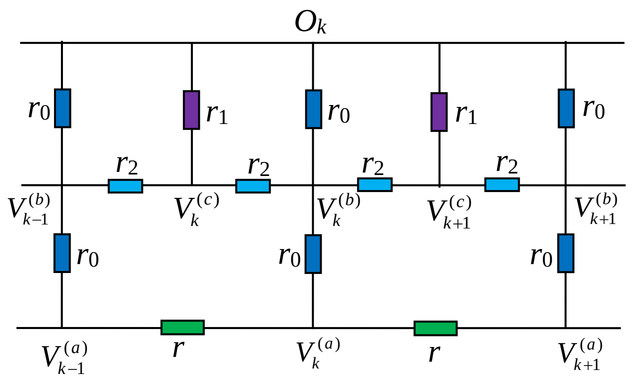


Fig.2. A sub network diagram with resistors and voltage parameters used for analyzing circuits

#### *3.1.1 Voltage equation for any node*

When the current is input from node $A_{x_1}$ to node $A_{x_2}$ output, analyze Figure 2 and apply Kirchhoff's node current law ($\sum r_i^{-1}V_k = 0$) to obtain the node voltage equation system as follows

$$(h_{21}+2)V_k^{(c)} = V_{k-1}^{(b)} + V_k^{(b)}, \tag{13}$$

$$(h_{21}+2)V_{k+1}^{(c)} = V_{k+1}^{(b)} + V_k^{(b)}. \tag{14}$$

$$V_{k+1}^{(a)} = (h_{r0}+2)V_k^{(a)} - h_{r0}V_k^{(b)} - V_{k-1}^{(a)} + rI(\delta_{kx_2} - \delta_{kx_1}), \tag{15}$$

$$V_{k+1}^{(c)} + V_k^{(c)} + h_{20}V_k^{(a)} - 2(h_{20}+1)V_k^{(b)} = 0, \tag{16}$$

In equation (15), $\delta_{k,x}$ is the Dirac function, and $h_{sk} = r_s / r_k$, $h_{r0} = r / r_0$.

To eliminate $V_k^{(c)} + V_{k+1}^{(c)}$, substitute equations (13) and (14) into equation (16) and simplify it

$$V_{k+1}^{(b)} = -(h_{21}+2)h_{20}V_k^{(a)} + 2[(h_{21}+2)(h_{20}+1)-1]V_k^{(b)} - V_{k-1}^{(b)} = 0. \tag{17}$$

Represent equations (15) and (17) in matrix form

$$\begin{bmatrix} V_{k+1}^{(a)} \\ V_{k+1}^{(b)} \end{bmatrix} = D_{2\times 2}\begin{bmatrix} V_k^{(a)} \\ V_k^{(b)} \end{bmatrix} - \begin{bmatrix} V_{k-1}^{(a)} \\ V_{k-1}^{(b)} \end{bmatrix} + r(\delta_{kx_2} - \delta_{kx_1})\begin{bmatrix} 1 \\ 0 \end{bmatrix} I, \tag{18}$$

from the previous text, it is known that $a = (2+h_{21})(1+h_{20}) - 1$, so in the above equation

$$D_{2\times 2}=\begin{pmatrix}(2+h_{r0}) & -h_{r0}\\ -(h_{21}+2)h_{20} & 2d\end{pmatrix}. \tag{19}$$

Equation (18) is the recursive equation of the key matrix required to solve the analytical formula for equivalent resistance.

Consider the boundary condition equation of associated node $A_x$, when the current $I$ is input from node $A_{x_1}$, it is obtained from equation (18)

$$\begin{bmatrix}V_{x_1+1}^{(a)}\\ V_{x_1+1}^{(b)}\end{bmatrix}=D_{2\times 2}\begin{bmatrix}V_{x_1}^{(a)}\\ V_{x_1}^{(b)}\end{bmatrix}-\begin{bmatrix}V_{x_1-1}^{(a)}\\ V_{x_1-1}^{(b)}\end{bmatrix}-r\begin{bmatrix}1\\ 0\end{bmatrix}I, \tag{20}$$

When current $I$ flows into the network from node $A_{x_2}$,equation (18) yields

$$\begin{bmatrix}V_{x_2+1}^{(a)}\\ V_{x_2+1}^{(b)}\end{bmatrix}=D_{2\times 2}\begin{bmatrix}V_{x_2}^{(a)}\\ V_{x_2}^{(b)}\end{bmatrix}-\begin{bmatrix}V_{x_2-1}^{(a)}\\ V_{x_2-1}^{(b)}\end{bmatrix}+r\begin{bmatrix}1\\ 0\end{bmatrix}I. \tag{21}$$

Moreover, when current $I$ flows into the network from node $B_{x_1}$ on the $B_0B_n$ axis and flows out of the network from node $B_{x_2}$, based on the above analysis, the matrix equation is obtained using the same method above

$$\begin{bmatrix}V_{k+1}^{(a)}\\ V_{k+1}^{(b)}\end{bmatrix}=D_{2\times 2}\begin{bmatrix}V_{k}^{(a)}\\ V_{k}^{(b)}\end{bmatrix}-\begin{bmatrix}V_{k-1}^{(a)}\\ V_{k-1}^{(b)}\end{bmatrix}+r_2(h_{21}+2)(\delta_{kx_2}-\delta_{kx_1})\begin{bmatrix}0\\ 1\end{bmatrix}I, \tag{22}$$

the matrix $D_{2\times 2}$ in the above equation comes from equation (19).

### *3.1.2 Boundary condition equation*

The above equation is not sufficient to solve the node voltage, and it is necessary to establish conditional equations for the left and right boundaries. Establish boundary condition equations using Kirchhoff's law for the circuit shown in Figure 3.

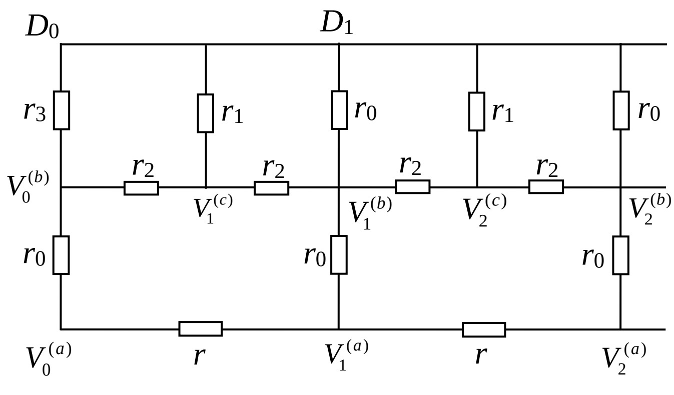


Fig.3. The network model and its parameters for the left boundary, where $r_3=r_0//(2r_1+r_2)$.

According to the circuit on the left boundary shown in Figure 3, a matrix equation can be established using a technique similar to equation (18)

$$\begin{bmatrix} V_1^{(a)} \\ V_1^{(b)} \end{bmatrix} = \begin{pmatrix} (h_{r0}+1) & -h_{r0} \\ -h_{20}(2+h_{21}) & [(2+h_{21})(1+h_{20}+h_{23})-1] \end{pmatrix} \begin{bmatrix} V_0^{(a)} \\ V_0^{(b)} \end{bmatrix}. \tag{23}$$

Research has found that only when $(2+h_{21})(1+h_{20}+h_{23}) = 2d = 2[(2+h_{21})(1+h_{20})-1]$ at that time can the network model have a concise analytical expression, which leads to $h_{23} = h_{20} + h_{21}/(2+h_{21})$ and further simplifies to $r_3 = r_0 //(2r_1 + r_2)$. This is why we choose this value, otherwise it is impossible to obtain an accurate equivalent resistance analytical expression. So equation (23) can be abbreviated as

$$\begin{bmatrix} V_1^{(a)} \\ V_1^{(b)} \end{bmatrix} = (D_{2\times2} - E_{2\times2}) \begin{bmatrix} V_0^{(a)} \\ V_0^{(b)} \end{bmatrix}, \tag{24}$$

Where E is the identity matrix.

Similarly, the conditional equation for the right boundary can be written

$$\begin{bmatrix} V_{n-1}^{(a)} \\ V_{n-1}^{(b)} \end{bmatrix} = (D_{2\times2} - E_{2\times2}) \begin{bmatrix} V_n^{(a)} \\ V_n^{(b)} \end{bmatrix}, \tag{25}$$

The above equations established using the RT-V method are all equations required for calculating the analytical expression of equivalent resistance.

## 3.2 Transformation of Matrix Equations

### *3.2.1 Main matrix equation transformation*

Use the matrix transformation method in RT theory to indirectly calculate the solution of the above equation. According to the matrix transformation method established in references [39-42], perform matrix transformation on equation (22) (ignoring the input and output of current). Assuming the existence of an undetermined second-order matrix $Q_{2\times2}$, multiply both ends of equation (22) by $Q_{2\times2}$ simultaneously and perform matrix transformation to obtain

$$Q_{2\times2} \begin{bmatrix} V_{k+1}^{(a)} \\ V_{k+1}^{(b)} \end{bmatrix} = Q_{2\times2} D_{2\times2} \begin{bmatrix} V_k^{(a)} \\ V_k^{(b)} \end{bmatrix} - Q_{2\times2} \begin{bmatrix} V_{k-1}^{(a)} \\ V_{k-1}^{(b)} \end{bmatrix}. \tag{26}$$

Find the eigenvalue of $D_{2\times2}$ from formula $\det[D_{2\times2} - pE] = 0$ below, that is

$$\begin{vmatrix} (2+h_{r0}) - p & -h_{r0} \\ -(h_{21}+2)h_{20} & 2d - p \end{vmatrix} = 0, \tag{27}$$

therefore

$$p^2 - (h_{r0} + 2d + 2)p + 2d(2+h_{r0}) - (h_{21}+2)h_{20}h_{r0} = 0, \tag{28}$$

the solution of $p$ in the above equation is

$$p_1 = \frac{1}{2}h_{r0} + (h_{21}+2)(h_{20}+1) - \sqrt{\Delta},$$

$$p_2 = \frac{1}{2}h_{r0} + (h_{21}+2)(h_{20}+1) + \sqrt{\Delta}, \tag{29}$$

where $\Delta = (\frac{1}{2}h_{r0} + 1 - d)^2 + (h_{21}+2)h_{20}h_{r0}$, and $d = (h_{20}+1)(h_{21}+2) - 1$.

Next, calculate the eigenvector $Q_{2\times2}$ through diagonalization matrix transformation

$$Q_{2\times2}\begin{pmatrix}(2+h_{r0}) & -h_{r0}\\ -(h_{21}+2)h_{20} & 2d\end{pmatrix}=\begin{pmatrix}p_1 & 0\\ 0 & p_2\end{pmatrix}Q_{2\times2}\,, \tag{30}$$

where we assume

$$Q_{2\times2}=\begin{pmatrix}a_1 & b_1\\ a_2 & b_2\end{pmatrix}, \tag{31}$$

whereupon, according to the matrix identity, from equations (30) and (31), it can be obtained that

$$Q_{2\times2}=\begin{pmatrix}(h_{21}+2)h_{20} & 2+h_{r0}-p_1\\ (h_{21}+2)h_{20} & 2+h_{r0}-p_2\end{pmatrix}. \tag{32}$$

Next, use the following transformation matrix to simplify the matrix equation

$$\begin{bmatrix}Y_k^{(1)}\\ Y_k^{(2)}\end{bmatrix}=Q_{2\times2}\begin{bmatrix}V_k^{(a)}\\ V_k^{(b)}\end{bmatrix}. \tag{33}$$

Therefore, without considering boundary flow conditions, the main matrix equation (22) is transformed into

$$\begin{bmatrix}Y_{k+1}^{(1)}\\ Y_{k+1}^{(2)}\end{bmatrix}=\begin{pmatrix}p_1 & 0\\ 0 & p_2\end{pmatrix}\begin{bmatrix}Y_k^{(1)}\\ Y_k^{(2)}\end{bmatrix}-\begin{bmatrix}Y_{k-1}^{(1)}\\ Y_{k-1}^{(2)}\end{bmatrix}, \tag{34}$$

then, the above equation can be simplified as

$$Y_{k+1}^{(s)}=p_sY_k^{(s)}-Y_{k-1}^{(s)}.\quad (s=1,2) \tag{35}$$

Through observation, it was found that the main matrix equation (22) is a relatively complex two-dimensional equation, while the transformed equation (35) is a relatively simple one-dimensional equation. It is obvious that the transformed equation is easier to solve.

*3.2.2 Transform boundary condition equation*

Next, combining the transformation of the boundary condition matrix equation, equations (24) and (25) are transformed using the method of equation (26) to obtain

$$Y_1^{(i)}=(p_i-1)Y_0^{(i)}.\quad (i=1,2) \tag{36}$$

$$Y_{n-1}^{(i)}=(p_i-1)Y_n^{(i)}.\quad (i=1,2) \tag{37}$$

*3.2.3 Transformation of Current Excitation Matrix Equation*

Considering the transformation of boundary condition equations for current flowing in and out of node $A_x$ and out of node $B_x$ in the network, using the above method, when current $I$ flows into the network from node $A_{x_1}$, equation (20) is transformed into

$$Y_{x_1+1}^{(i)}=p_iY_{x_1}^{(i)}-Y_{x_1-1}^{(i)}-rI(h_{21}+2)h_{20}\,. \tag{38}$$

When the current flows out from node $A_{x_2}$, equation (21) yields

$$Y_{x_2+1}^{(i)}=p_iY_{x_2}^{(i)}-Y_{x_2-1}^{(i)}+rI(h_{21}+2)h_{20}\,. \tag{39}$$

Besides, when the current flows in from node $B_{x_1}$ and flows out from node $B_{x_2}$, by transforming equation (22), we can obtain

$$Y_{x_1+1}^{(i)} = p_i Y_{x_1}^{(i)} - Y_{x_1-1}^{(i)} - r_2 I(h_{21}+2)(2+h_{r0}-p_i). \tag{40}$$

$$Y_{x_2+1}^{(i)} = p_i Y_{x_2}^{(i)} - Y_{x_2-1}^{(i)} + r_2 I(h_{21}+2)(2+h_{r0}-p_i). \tag{41}$$

The above equation provides us with preliminary preparation for promoting the main results.

## 4. Deduction of Main Conclusions

Analysis of the circuit shown in Figure 1 reveals two possible scenarios: $x_2 \ge x_1$ or $x_2 \le x_1$. Since all the calculation processes of situation $x_2 \le x_1$ are completely similar to situation $x_2 \ge x_1$, this article only considers the case of $x_2 \ge x_1$.

### 4.1 Calculation of equivalent resistance $R_n(A_{x_1}, B_{x_2})$

*4.1.1 Special solutions of matrix equations under constraint conditions*

Firstly, considering the condition that current $I$ is input from node $A_{x_1}$ to node $B_{x_2}$, the required matrix equation is solved as equation (35), and its corresponding constraint equations are (38) and (41), respectively. Equations (36) and (37) are common boundary conditions.

Equation $\lambda^2 = p_i\lambda - 1$ is the eigenvalue of equation (35), let its two solutions be $\lambda_i$ and $\bar{\lambda}_i$ respectively, and solve them as follows

$$\lambda_i = \frac{1}{2}\left(p_i + \sqrt{p_i^2-4}\right), \quad \bar{\lambda}_i = \frac{1}{2}\left(p_i - \sqrt{p_i^2-4}\right). \tag{42}$$

According to the method for solving the difference equation established in references [31-33], equation (35) was solved and its piecewise function solution was obtained as follows

$$Y_k^{(i)} = Y_1^{(i)} B_k^{(i)} - Y_0^{(i)} B_{k-1}^{(i)}, \ (0 \le k \le x_1) \tag{43}$$

$$Y_k^{(i)} = Y_{x_1+1}^{(i)} B_{k-x_1}^{(i)} - Y_{x_1}^{(i)} B_{k-x_1-1}^{(i)}, \ (x_1 \le k \le x_2) \tag{44}$$

$$Y_k^{(i)} = Y_{x_2+1}^{(i)} B_{k-x_2}^{(i)} - Y_{x_2}^{(i)} B_{k-x_2-1}^{(i)}, \ (x_2 \le k \le n) \tag{45}$$

in the above equation $B_k^{(i)} = (\lambda_i^k - \bar{\lambda}_i^k)/(\lambda_i - \bar{\lambda}_i)$ is defined by equation (4).

Let's start calculating the analytical expression for $Y_k^{(i)}$, by (36) and (43) to derive

$$Y_k^{(i)} = \Delta B_k^{(i)} Y_0^{(i)}, \ (0 \le k \le x_1), \tag{46}$$

to solve the equation, we need to eliminate $Y_{x_1+1}^{(i)}$ and $Y_{x_2+1}^{(i)}$ in equations (44) and (45). So, substituting equation (38) into equation (44) together with (46), yielding ($x_1 \le k \le x_2$)

$$Y_k^{(i)} = \Delta B_k^{(i)} Y_0^{(i)} - Ir(h_{21}+2)h_{20} B_{k-x_1}^{(i)}, \ (x_1 \le k \le x_2) \tag{47}$$

by substituting both equation (41) and equation (47) into equation (45), yielding ($x_2 \le k \le n$)

$$Y_k^{(i)} = \Delta B_k^{(i)} Y_0^{(i)} + r_2 I(h_{21}+2)(2+h_{r0}-p_s) B_{k-x_2}^{(i)} - Ir(h_{21}+2)h_{20} B_{k-x_1}^{(i)}, \tag{48}$$

to find out $Y_0^{(i)}$, take equation (48) with $k = \{n, n-1\}$ into (37), we have

$$Y_0^{(i)} = I(h_{21}+2)\frac{rh_{20}\Delta B_{n-x_1}^{(i)} - r_2(2+h_{r0}-p_i)\Delta B_{n-x_2}^{(i)}}{(p_i-2)B_{n+1}^{(i)}}. \tag{49}$$

Substitute (49) into (47) and export ( $x_1 \le k \le x_2$ , $i$=1,2)

$$Y_k^{(i)} = I(h_{21}+2)\frac{rh_{20}\Delta B_{x_1}^{(i)}\Delta F_{n-k}^{(i)} - r_2(h_{r0}+2-p_i)\Delta B_k^{(i)}\Delta B_{n-x_2}^{(i)}}{(p_i-2)B_{n+1}^{(i)}}, \tag{50}$$

The calculation of equivalent resistance $R_n(A_{x_1}, B_{x_2})$ will mainly depend on equation (50).

*4.1.2 Calculate the equivalent resistance* $R_n(A_{x_1}, B_{x_2})$

Based on the RT-V method for calculating equivalent resistance in reference[32], we can first invert (33) and apply the matrix equation (32) of $Q_{2\times 2}$ to obtain it

$$V_k^{(a)} = \frac{(2+h_{r0}-p_1)Y_k^{(2)} - (2+h_{r0}-p_2)Y_k^{(1)}}{(2+h_{21})h_{20}(p_2-p_1)}, \tag{51}$$

$$V_k^{(b)} = \frac{Y_k^{(1)} - Y_k^{(2)}}{p_2-p_1}. \tag{52}$$

Next, we can derive the following formula from Ohm's law

$$R_n\left(A_{x_1}, B_{x_2}\right) = (V_{x_1}^{(a)} - V_{x_2}^{(b)})/I. \tag{53}$$

Simplify the equations of $V_k^{(a)}$ and $V_k^{(\mathrm{b})}$ by substituting them into the above equation

$$R_n\left(A_{x_1}, B_{x_2}\right) = \frac{1}{I}\left(\frac{(h_{r0}+2-p_1)Y_{x_1}^{(2)} + (2+h_{21})h_{20}Y_{x_2}^{(2)}}{(2+h_{21})h_{20}(p_2-p_1)} - \frac{(h_{r0}+2-p_2)Y_{x_1}^{(1)} + (2+h_{21})h_{20}Y_{x_2}^{(1)}}{(2+h_{21})h_{20}(p_2-p_1)}\right). \tag{54}$$

Because of $p_1 + p_2 = h_{r0} + 2 + 2d$, therefore $h_{r0} + 2 - p_1 = p_2 - 2d$, substituting (50) with $k = x_1, x_2$ into (54) yields

$$R_n\left(A_{x_1}, B_{x_2}\right) = r\left(\begin{array}{c}\dfrac{(2d-p_1)F_{1,1}^{(1)} - 2h_{20}(2+h_{21})F_{1,2}^{(1)} - h_{2r}(2+h_{21})(2d-p_2)F_{2,2}^{(1)}}{(p_2-p_1)(p_1-2)B_{n+1}^{(1)}} \\ -\dfrac{(2d-p_2)F_{1,1}^{(2)} - 2h_{20}(2+h_{21})F_{1,2}^{(2)} - h_{2r}(2+h_{21})(2d-p_1)F_{2,2}^{(2)}}{(p_2-p_1)(p_2-2)B_{n+1}^{(2)}}\end{array}\right). \tag{55}$$

Because of

$$(p_1-2d)(p_2-2d) = (h_{r0}+2-p_1)(h_{r0}+2-p_2) = -(2+h_{21})h_{20}h_{r0}, \tag{56}$$

using Eq.(56) to obtain $p_2 - 2d = h_{20}h_{r0}(2+h_{21})/(2d-p_1)$ and defining $u_i = h_{20}(2+h_{21})/(2d-p_i)$, then simplifying using Eq. (55) yields

$$\frac{R_n\left(A_{x_1}, B_{x_2}\right)}{r} = (2d-p_1)\frac{F_{1,1}^{(1)} - 2u_1F_{1,2}^{(1)} + u_1^2F_{2,2}^{(1)}}{(p_2-p_1)(p_1-2)B_{n+1}^{(1)}} - (2d-p_2)\frac{F_{1,1}^{(2)} - 2u_1F_{1,2}^{(2)} + u_1^2F_{2,2}^{(2)}}{(p_2-p_1)(p_2-2)B_{n+1}^{(2)}}. \tag{57}$$

In summary, the equivalent resistance $R_n(A_{x_1}, B_{x_2})$ between nodes $A_{x_1}$ and $B_{x_2}$ was obtained, and parameter (6) was proved.

## 4.2 Calculation of equivalent resistance $R_n(A_{x_1}, O)$

*4.2.1 Special solutions of matrix equations under constraint conditions*

To calculate the equivalent resistance $R_n(A_{x_1}, O)$, the current can be input from node $A_{x_1}$ and output from node O. At this point, equations (43) - (45) given earlier are applicable to this situation, and equations (46) and (47) are also applicable to this situation. However, the scope of equation (47) needs to be modified to $x_1 \le k \le n$, so equation (47) needs to be rewrite.

$$Y_k^{(i)} = \Delta B_k^{(i)} Y_0^{(s)} - Ir(2+h_{21})h_{20} B_{k-x_1}^{(i)} \text{ , } (x_1 \le k \le n) \tag{58}$$

To find out $Y_0^{(s)}$, take equation (58) with $k = \{n, n-1\}$ into (37), we therefore have after algebraic operations

$$Y_0^{(i)} = Ir(2+h_{21})h_{20} \frac{\Delta B_{n-x_1}^{(i)}}{(t_i - 2)B_{n+1}^{(i)}} . \tag{59}$$

Since equation (46) also applies to this situation, substituting (59) into (46) yields

$$Y_k^{(i)} = Ir(2+h_{21})h_{20} \frac{\Delta B_k^{(i)} \Delta B_{n-x_1}^{(i)}}{(p_s - 2)B_{n+1}^{(i)}} \text{ , } (0 \le k \le x_1) \tag{60}$$

*4.2.2 Calculation of equivalent resistance* $R_n(A_{x_1}, O)$

Equation (51) is a universal inverse transformation equation, it also applies to the situation here. Similarly, the analytical formula $R_n(A_{x_1}, O) = (V_{x_1}^{(a)} - 0)/I$ for calculating equivalent resistance through Ohm's law, where $V_{x_1}^{(a)}$ is expressed by equation (51), therefore

$$R_n\left(A_{x_1}, O\right) = \frac{1}{I}\left(\frac{(h_{r0}+2-p_1)Y_{x_1}^{(2)} - (h_{r0}+2-p_2)Y_{x_1}^{(1)}}{(2+h_{21})h_{20}(p_2 - p_1)}\right). \tag{61}$$

Use equation (60) to eliminate $Y_k^{(i)}$ in the above equation and simplify it to obtain

$$R_n\left(A_{x_1}, O\right) = r\frac{(h_{r0}+2-p_2)F_{1,1}^{(1)}}{(p_1-p_2)(p_1-2)B_{n+1}^{(1)}} - r\frac{(h_{r0}+2-p_1)F_{1,1}^{(2)}}{(p_1-p_2)(p_2-2)B_{n+1}^{(2)}} . \tag{62}$$

Because of $p_1 + p_2 = h_{r0} + 2d + 2$, Substituting $h_{r0} + 2 - p_2 = p_1 - 2d$ into (62) yields conclusion (9). At present, the equivalent resistance $R_n(A_{x_1}, O)$ between nodes $A_{x_1}$ and $O$ is solved, and formula (9) is proven.

## 4.3 Calculation of equivalent resistance $R_n(A_{x_1}, A_{x_2})$

Consider the special solutions of matrix equations under constraint conditions. Equations (38) and (39) provide the constraint equations for the input current from node $A_{x_1}$ and the output current from node $A_{x_2}$, while equations (43) - (45) provide the piecewise functional solutions for the main matrix equation (35), respectively. Combining equation (47) ($x_1 \le k \le x_2$), solve the system of equations to obtain

$$Y_k^{(i)} = \Delta B_k^{(i)} Y_0^{(i)} - Ir(2+h_{21})h_{20} B_{k-x_1}^{(i)} \text{ , } (x_1 \le k \le x_2) \tag{63}$$

$$Y_k^{(i)} = \Delta B_k^{(i)} Y_0^{(i)} + Ir(2+h_{21})h_{20} (B_{k-x_2}^{(i)} - B_{k-x_1}^{(i)}) \text{ , } (x_2 \le k \le n) \tag{64}$$

To find out $Y_0^{(i)}$, take equation (64) with $k = \{n, n-1\}$ into (37), we have

$$Y_0^{(i)} = Ir(2+h_{21})h_{20}\frac{\Delta B_{n-x_1}^{(i)} - \Delta B_{n-x_2}^{(i)}}{(p_i-2)B_{n+1}^{(i)}}. \tag{65}$$

Substitute (65) into (63) and export ( $x_1 \le k \le x_2$ )

$$\frac{Y_k^{(i)}}{Ir} = (2+h_{21})h_{20}\frac{\Delta B_{x_1}^{(i)}\Delta B_{n-k}^{(i)} - \Delta B_k^{(i)}\Delta B_{n-x_2}^{(i)}}{(p_s-2)B_{n+1}^{(i)}}, \ (x_1 \le k \le x_2) \tag{66}$$

Using a method similar to the calculation process mentioned earlier, and then applying Ohm's law to calculate the analytical formula for equivalent resistance, we obtain

$$R_n(A_{x_1}, A_{x_2}) = (V_{x_1}^{(a)} - V_{x_2}^{(a)})/I. \tag{67}$$

where $V_{x_1}^{(a)}$ is expressed by equation (51).

Substitute equation (51) into equation (67) to obtain

$$R_n(A_{x_1}, A_{x_2}) = \frac{1}{I}\left(\frac{(p_2-2d)(Y_{x_1}^{(b)} - Y_{x_2}^{(b)})}{(2+h_{21})h_{20}(p_2-p_1)} - \frac{(p_1-2d)(Y_{x_1}^{(a)} - Y_{x_2}^{(a)})}{(2+h_{21})h_{20}(p_2-p_1)}\right). \tag{68}$$

Substituting Eq.(66) with $k=\{x_1,x_2\}$ into (68) yields

$$R_n(A_{x_1}, A_{x_2}) = r(2d-p_1)\frac{F_{1,1}^{(1)} - 2F_{1,2}^{(1)} + F_{2,2}^{(1)}}{(p_2-p_1)(p_1-2)B_{n+1}^{(1)}} - r(2d-p_2)\frac{F_{1,1}^{(2)} - 2F_{1,2}^{(2)} + F_{2,2}^{(2)}}{(p_2-p_1)(p_2-2)B_{n+1}^{(2)}}. \tag{69}$$

At this point, the equivalent resistance $R_n(A_{x_1}, A_{x_2})$ between nodes $A_{x_1}$ and $A_{x_2}$ is solved, and formula (10) is proven.

### 4.4 Calculation of equivalent resistance $R_n(B_{x_1}, B_{x_2})$

When the current $I$ is input from node $B_{x_1}$ to output from node $B_{x_2}$, We obtain constraint equations (40) and (41). Except for the different input and output condition equations of current $I$, the general solutions (43) - (46) given above still apply to this situation. So the general solution was combined with the constraint equations (40) and (41) to obtain the solution

$$Y_k^{(i)} = \Delta B_k^{(i)} Y_0^{(i)} - Ic_i B_{k-x_1}^{(i)}, \ (x_1 \le k \le x_2) \tag{70}$$

$$Y_k^{(i)} = \Delta B_k^{(i)} Y_0^{(i)} + Ic_i(B_{k-x_2}^{(i)} - B_{k-x_1}^{(i)}), \ (x_2 \le k \le n) \tag{71}$$

where $c_i = r_2(h_{21}+2)(h_{r0}+2-p_i)$.

To understand $X_0^{(s)}$, take equation (71) with $k=\{n,n-1\}$ into (37), we have

$$\Delta B_{n-1}^{(i)} Y_0^{(i)} + Ic_i(B_{n-1-x_2}^{(i)} - B_{n-1-x_1}^{(i)}) = (p_i-1)[\Delta B_n^{(i)} Y_0^{(i)} + Ic_i(B_{n-x_2}^{(i)} B_{n-x_1}^{(i)})]. \tag{72}$$

Simplify to obtain

$$Y_0^{(i)} = Ic_i \frac{\Delta B_{n-x_1}^{(i)} - \Delta B_{n-x_2}^{(i)}}{(p_i-2)B_{n+1}^{(i)}}. \tag{73}$$

Substitute (73) into (70) and export ( $x_1 \le k \le x_2$ )

$$Y_k^{(i)} = Ic_i \frac{\Delta B_{x_1}^{(i)}\Delta B_{n-k}^{(i)} - \Delta B_k^{(i)}\Delta B_{n-x_2}^{(i)}}{(p_i-2)B_{n+1}^{(i)}}, \tag{74}$$

the calculation of equivalent resistance $R_n(B_{x_1}, B_{x_2})$ will mainly depend on equation (74).

Thus, Ohm's law can be used to derive

$$R_n\left(B_{x_1},B_{x_2}\right)=(V_{x_1}^{(b)}-V_{x_2}^{(b)})/I\,. \tag{75}$$

The expression of $V_k^{(b)}$ is given by equation (52). Substitute equation (52) into equation (75) to derive

$$R_n\left(B_{x_1},B_{x_2}\right)=\frac{1}{I}\left(\frac{Y_{x_1}^{(1)}-Y_{x_2}^{(1)}}{p_2-p_1}-\frac{Y_{x_1}^{(2)}-Y_{x_2}^{(2)}}{p_2-p_1}\right). \tag{76}$$

So substituting (74) with $k=\{x_1,x_2\}$ into (76) and simplifies it to obtain

$$R_n\left(B_{x_1},B_{x_2}\right)=\frac{r_2(2+h_{21})}{p_2-p_1}\left((\frac{h_{r0}}{p_1-2}-1)\frac{F_{1,1}^{(1)}-2F_{1,2}^{(1)}+F_{2,2}^{(1)}}{B_{n+1}^{(1)}}-(\frac{h_{r0}}{p_2-2}-1)\frac{F_{1,1}^{(2)}-2F_{1,2}^{(2)}+F_{2,2}^{(2)}}{B_{n+1}^{(2)}}\right) \tag{77}$$

Obviously, the equation (11) proposed earlier has been proven.

## 4.5 Calculation of equivalent resistance $R_n(B_{x_1},O)$

*4.5.1 Special solutions of matrix equations under constraint conditions*

To calculate the equivalent resistance $R_n(B_{x_1},O)$, the current can be input from $B_{x_1}$ and output from node O. At this point, Eqs. (43) - (46) given earlier are applicable to the situation here, Eq. (70) also applies to this situation. But the range of Eq. (70) needs to be modified to $x_1\le k\le n$, so equation (70) needs to be rewritten

$$Y_k^{(i)}=\Delta B_k^{(i)}Y_0^{(s)}-r_2I(h_{21}+2)(h_{r0}+2-p_i)B_{k-x_1}^{(i)}\,,\ (x_1\le k\le n) \tag{78}$$

To find out $Y_0^{(s)}$, take Eq. (78) with $k=\{n,n-1\}$ into (37), we have

$$Y_0^{(s)}=r_2I(2+h_{21})(h_{r0}+2-p_i)\frac{\Delta B_{n-x_1}^{(i)}}{(p_i-2)B_{n+1}^{(i)}}\,. \tag{79}$$

Substitute (79) into (46) to calculate

$$Y_k^{(i)}=r_2I(2+h_{21})(h_{r0}+2-p_i)\frac{\Delta B_k^{(i)}\Delta B_{n-x_1}^{(i)}}{(p_i-2)B_{n+1}^{(i)}}\,,\ (0\le k\le x_1) \tag{80}$$

*4.5.2 Calculation of equivalent resistance* $R_n(B_{x_1},O)$

Equation (52) also applies to this situation. Similarly, using Ohm's law to calculate the analytical formula $R_n(B_{x_1},O)=(V_{x_1}^{(b)}-0)/I$ for equivalent resistance, where $V_{x_1}^{(b)}$ is represented by equation (52), therefore

$$R_n\left(B_{x_1},O\right)=\frac{1}{I}\left(\frac{Y_{x_1}^{(1)}-Y_{x_1}^{(2)}}{p_2-p_1}\right), \tag{81}$$

substituting (80) with $k=x_1$ into (81) yields

$$R_n\left(B_{x_1},O\right)=r_2(\frac{2+h_{21}}{p_2-p_1})\left((\frac{h_{r0}}{p_1-2}-1)\frac{\Delta B_{x_1}^{(1)}\Delta B_{n-x_1}^{(1)}}{B_{n+1}^{(1)}}-(\frac{h_{r0}}{p_2-2}-1)\frac{\Delta B_{x_1}^{(2)}\Delta B_{n-x_1}^{(2)}}{B_{n+1}^{(2)}}\right), \tag{82}$$

At this point, the equivalent resistance $R_n(B_{x_1},O)$ between nodes $B_{x_1}$ and $O$ is solved, and formula (12) is proven.

## 5. Special Situations and Discussions

The asymmetric $2\times n$ RN has already been calculated in the above text, which inclouding the analytical formulae for the equivalent resistances between five pairs of nodes: $(A_{x_1},B_{x_2})$, $(A_{x_1},O)$, $(A_{x_1},A_{x_2})$, $(B_{x_1},B_{x_2})$, $(B_{x_1},O)$. Due to the abstract analytical formula for equivalent resistance given in the previous text, in order to better understand the physical meaning of the conclusion and compare and verify its correctness, we will discuss some special cases below.

To facilitate the verification of the equivalent resistance of the RN, we can use the structural transformation between $\mathrm{T}\Leftrightarrow\nabla$ in the network model of Fig.1 to obtain the $2\times n$-order RN of the structure shown in Fig. 4.

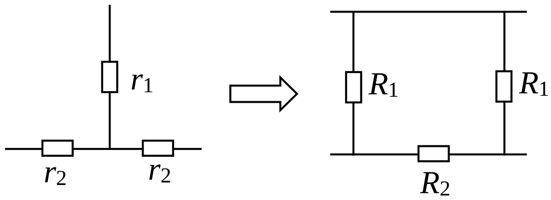


Fig.4. Circuit transformation: T↔▽

In the transformation shown in Fig. 4, the equivalent resistance can be solved

$$R_1 = 2r_1 + r_2,\quad R_2 = \frac{r_2}{r_1}(2r_1 + r_2)\,. \tag{83}$$

So, we can equate the circuit diagram to the structure of Fig. 5. In Fig. 5, there be

$$r_4 = r_0 \,//\, R_1 \,//\, R_1 = \frac{r_0(2r_1 + r_2)}{2r_0 + 2r_1 + r_2}\,. \tag{84}$$

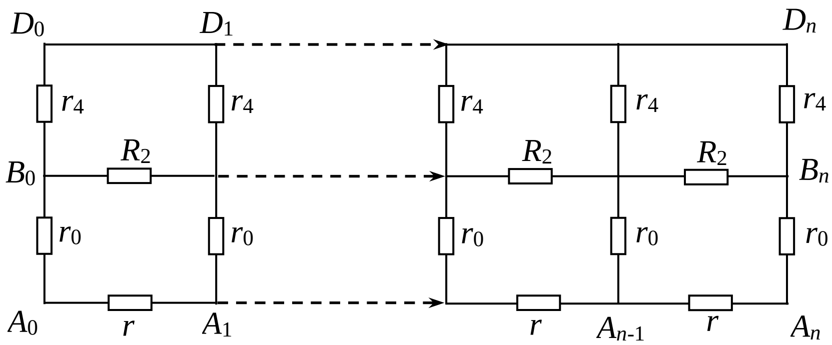


Fig.5. A equivalent circuit network of asymmetric $2\times n$ resistance network

### 5.1 The case of $n=0$

When $n=0$, the $2\times n$-order RNM shown in Fig.1 can be degraded into the $2\times 0$-order RNM shown in Fig.6 (rotate from vertical to horizontal direction).

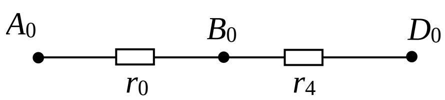


Fig.6. Expressed a $2\times 0$ circuit network with $n$=0 (rotated $90^0$)

When $n=0$, there be

$$B_0^{(i)}=0\,,\ \ B_1^{(i)}=1\,,\ \ \Delta B_0^{(i)}=1\,,\ \ F_{0,0}^{(i)}=1\,. \tag{85}$$

From Eq. (7) or (55), we have

$$R_0\left(A_0,B_0\right)=\frac{r(h_{r0}+2-p_1)+2h_{20}r(2+h_{21})-r_2(2+h_{21})(h_{r0}+2-p_2)}{(p_2-p_1)(p_2-2)}$$

$$-\frac{r(h_{r0}+2-p_2)+2h_{20}r(2+h_{21})-r_2(2+h_{21})(h_{r0}+2-p_1)}{(p_2-p_1)(p_1-2)}\,. \tag{86}$$

Simplify (86) then export

$$R_n\left(A_0,B_0\right)=r\,\frac{p_1+p_2-4-h_{r0}-(2+h_{21})h_{20}}{(p_2-2)(p_1-2)}\,. \tag{87}$$

Because $p_1$ and $p_2$ are the two roots of equation (28), therefore

$$p_1+p_2=h_{r0}+2(2+h_{21})(1+h_{20})$$

$$p_1p_2=2a(h_{r0}+2)-(2+h_{21})h_{20}h_{r0} \tag{88}$$

So the molecule of equation (87) can be simplified as

$$(p_1+p_2-4)-h_{r0}-(2+h_{21})h_{20}=2h_{21}+(2+h_{21})h_{20} \tag{89}$$

So, by substituting (88) into the denominator of equation (87), we obtain

$$\begin{aligned}(p_1-2)(p_2-2)&=p_2p_1-2(p_1+p_2)+4\\&=2a(h_{r0}+2)-(2+h_{21})h_{20}h_{r0}-2h_{r0}-4(2+h_{21})(1+h_{20})+4\end{aligned} \tag{90}$$

Because of $d=(2+h_{21})(1+h_{20})-1$, Substitute the value of *d* into equation (90) to calculate

$$(p_1-2)(p_2-2)=h_{r0}[2h_{21}+(2+h_{21})h_{20})] \tag{91}$$

Substitute equations (89) and (91) into (87) to calculate

$$R_n\left(A_0,B_0\right)=r/h_{r0}=r_0\,. \tag{92}$$

Equation (92) is identical to the actual circuit result shown in Fig.6, which verifies the correctness of the equation at $n$=0.

Next, verify the situation of $R_0\left(A_0,0\right)$. When $n$=0, substitute equation (85) into equation (9) and export

$$R_0\left(A_0,O\right)=r\,\frac{(2a-p_1)}{(p_2-p_1)(p_1-2)}-r\,\frac{(2d-p_2)}{(p_2-p_1)(p_2-2)}\,, \tag{93}$$

simplify and obtain

$$R_0\left(A_0,O\right)=r\,\frac{2(d-1)}{(p_1-2)(p_2-2)}\,, \tag{94}$$

because of $d=(h_{20}+1)(2+h_{21})-1$, therefore $d-1=(2+h_{21})(1+h_{20})-2=h_{21}+(2+h_{21})h_{20}$, substituting *a* and (91) it into equation (94), we have

$$R_0\left(A_0,O\right)=2r_0\,\frac{h_{21}+(2+h_{21})h_{20}}{2h_{21}+(2+h_{21})h_{20}}=r_0\left(1+\frac{(2+h_{21})h_{20}}{2h_{21}+(2+h_{21})h_{20}}\right), \tag{95}$$

simplify by substituting $h_{2k}=r_2/r_k$ into the above equation

$$R_0\left(A_0,O\right)=r_0\left(1+\frac{2r_1+r_2}{2r_0+2r_1+r_2}\right). \tag{96}$$

According to the actual circuit calculation shown in Fig.6, it is obtained that

$$R_0\left(A_0,O\right)=r_0+r_4=r_0+r_0\frac{2r_1+r_2}{2r_0+2r_1+r_2}. \tag{97}$$

Obviously, the theoretical result (96) is identical to the actual circuit result shown in Fig. 6, which verifies the correctness of $R_n(A_{x_1},O)$ at $n$=0.

Next, verify the situation of $R_0\left(B_0,0\right)$. When $n$=0, substitute equation (85) into equation (12) and export

$$R_0\left(B_0,O\right)=r_2\left(\frac{2+h_{21}}{p_2-p_1}\right)\left(\frac{h_{r0}}{p_1-2}-\frac{h_{r0}}{p_2-2}\right)=r_2\frac{h_{r0}(2+h_{21})}{(p_1-2)(p_2-2)}. \tag{98}$$

Substituting equation (91) into equation (98) yields

$$R_0\left(B_0,O\right)=r_2\frac{2+h_{21}}{2h_{21}+(2+h_{21})h_{20}}, \tag{99}$$

simplify by substituting $h_{2k}=r_2/r_k$ into the above equation

$$R_0\left(B_0,O\right)=r_0\frac{2r_1+r_2}{2r_0+2r_1+r_2}. \tag{100}$$

From the actual circuit calculation shown in Fig. 6, $R_0\left(B_0,O\right)=r_4$ is obtained. Obviously, the theoretical result (100) is completely identical to the actual circuit result shown in Fig. 6, which verifies the correctness of $R_n(B_{x_1},O)$ at $n$=0.

From Eqs.(10) and (11), the analytical formula for the equivalent resistance between nodes $A_{x_1}$ ($B_{x_1}$) and $A_{x_2}$ ($B_{x_2}$) in the complex $2\times n$ RN of Fig.1 is

$$R_0(A_0,A_0)=0,\quad R_0(B_0,B_0)=0 \tag{101}$$

Obviously, the equivalent resistance calculated by the theoretical formula when $n=0$ is completely consistent with the actual circuit results, verifying the correctness of the theoretical formula when $n=0$.

In addition, when $n=1$, a similar verification method can be used for verification, and the verification results are completely correct. The calculation process will not be given here.

### 5.2 The case of $r_1\to\infty$ and $r_0=r, r_2=0.5r$

When $r_1\to\infty$ and $r_0=r, r_2=0.5r$, the complex $2\times n$ RNM of Fig.1 is simplified to the RNM shown in Fig. 5 with $R_2$=$r$, $r_4$=$r_0$.

Because $\lim_{r_1\to\infty}(h_{21})=\lim_{r_1\to\infty}(r_2/r_1)=0$ and $h_{20}$=$r_2/r_0=1/2$, $h_{r0}$=$r/r_0=1$, $d=(2+h_{21})(1+h_{20})-1$ =2, from equations (1), (2), (5), (7) and (8), we can obtain

$$p_1=4-2\cos\theta_1,\ p_2=4-2\cos\theta_2,\ \theta_i=(2i-1)\pi/5 \tag{102}$$

$$\lambda_i=2-\cos\theta_i+\sqrt{(2-\cos\theta_i)^2-1},$$

$$\bar{\lambda}_i = 2-\cos\theta_i - \sqrt{(2-\cos\theta_i)^2 - 1}, \tag{103}$$

Substituting Eqs. (102)-(103) into analytical equations (6), (9) and (10) can be simplified to obtain

$$\frac{R_n\left(A_{x_1}, B_{x_2}\right)}{r} = \frac{q_2 F_{1,1}^{(1)} + 2F_{1,2}^{(1)} - q_1 F_{2,2}^{(1)}}{(p_1-p_2)(p_1-2)B_{n+1}^{(1)}} - \frac{q_1 F_{1,1}^{(2)} + 2F_{1,2}^{(2)} - q_2 F_{2,2}^{(2)}}{(p_1-p_2)(p_2-2)B_{n+1}^{(2)}}. \tag{104}$$

Inside $q_s = h_{r0} + 2 - p_s$.

Using (102) to obtain

$$q_i = h_{r0} + 2 - p_i = 2\cos\theta_i - 1, \quad \cos\theta_1 = \frac{1+\sqrt{5}}{4}, \quad \cos\theta_2 = \frac{1-\sqrt{5}}{4}, \tag{105}$$

so the calculation shows that

$$\frac{q_1}{p_1-p_2} = \frac{2\cos\theta_1 - 1}{2\cos\theta_2 - 2\cos\theta_1} = -\frac{4}{5}\left(\frac{5-\sqrt{5}}{8}\right) = -\frac{4}{5}\sin^2(\theta_1) = -\frac{4}{5}\sin^2(2\theta_2) \tag{106}$$

$$\frac{q_2}{p_1-p_2} = \frac{2\cos\theta_2 - 1}{2\cos\theta_2 - 2\cos\theta_1} = \frac{4}{5}\left(\frac{5+\sqrt{5}}{8}\right) = \frac{4}{5}\sin^2(\theta_2) = \frac{4}{5}\sin^2(2\theta_1). \tag{107}$$

Substituting equations (106) and (107) into (104) yields

$$R_n\left(A_{x_1}, B_{x_2}\right) = \frac{2}{5} r \left( \frac{F_{1,1}^{(1)} \sin^2(2\theta_1) + 2F_{1,2}^{(1)} \sin(\theta_1)\sin(2\theta_1) + F_{2,2}^{(1)} \sin^2(\theta_1)}{(1-\cos\theta_1)B_{n+1}^{(1)}} + \frac{F_{1,1}^{(2)} \sin^2(2\theta_2) + 2F_{1,2}^{(2)} \sin(\theta_2)\sin(2\theta_2) + F_{2,2}^{(2)} \sin^2(\theta_2)}{(1-\cos\theta_2)B_{n+1}^{(2)}} \right), \tag{108}$$

the remaining parameters $B_k^{(i)}, \Delta, h_{sk}$, $F_{k,v}^{(i)}$ etc. are defined in equations (1)-(6).

The analytical formula for the equivalent resistance between nodes $A_{x_1}$ and $O$ in the $2\times n$ RN of Fig.1 is

$$R_n\left(A_{x_1}, O\right) = \frac{2r}{5}\left( \frac{F_{1,1}^{(1)} \sin^2(2\theta_1)}{(1-\cos\theta_1)B_{n+1}^{(1)}} + \frac{F_{1,1}^{(2)} \sin^2(2\theta_2)}{(1-\cos\theta_2)B_{n+1}^{(2)}} \right), \tag{109}$$

Because using (105) to derive $\frac{2}{5}\left(\frac{\sin^2(2\theta_i)}{1-\cos\theta_i}\right) = \cot^2(\theta_i)$, equation (109) can be rewritten as

$$\frac{R_n\left(A_{x_1}, O\right)}{r} = \frac{\Delta B_{x_1}^{(1)} \Delta B_{n-x_1}^{(1)}}{B_{n+1}^{(1)}} \cot^2(\theta_1) + \frac{\Delta B_{x_1}^{(2)} \Delta B_{n-x_1}^{(2)}}{B_{n+1}^{(2)}} \cot^2(\theta_2), \tag{110}$$

The analytical formula for the equivalent resistance between nodes $B_{x_1}$ and $O$ in the $2\times n$ RN of Fig.1 is

$$R_n\left(B_{x_1}, O\right) = \frac{2r}{5}\left( \frac{F_{1,1}^{(1)} \sin^2(\theta_1)}{(1-\cos\theta_1)B_{n+1}^{(1)}} + \frac{F_{1,1}^{(2)} \sin^2(\theta_2)}{(1-\cos\theta_2)B_{n+1}^{(2)}} \right), \tag{111}$$

because $\sin^2(\theta_i) = 4\sin^2(\frac{1}{2}\theta_i)\cos^2(\frac{1}{2}\theta_i) = 2(1-\cos\theta_i)\cos^2(\frac{1}{2}\theta_i)$, equation (111) can be rewritten as

$$R_n\left(B_{x_1}, O\right) = \frac{4r_0}{5}\left( \frac{F_{1,1}^{(1)}}{B_{n+1}^{(1)}} \cos^2(\frac{1}{2}\theta_1) + \frac{F_{1,1}^{(2)}}{B_{n+1}^{(2)}} \cos^2(\frac{1}{2}\theta_2) \right), \tag{112}$$

the analytical formula for the equivalent resistance between nodes $A_{x_1}$ and $A_{x_2}$ in the 2 × $n$ RN of Fig.1 is

$$\frac{R_n(A_{x_1},A_{x_2})}{r}=\frac{F_{1,1}^{(1)}-2F_{1,2}^{(1)}+F_{2,2}^{(1)}}{B_{n+1}^{(1)}}\cot^2(\theta_1)+\frac{F_{1,1}^{(2)}-2F_{1,2}^{(2)}+F_{2,2}^{(2)}}{B_{n+1}^{(2)}}\cot^2(\theta_2). \tag{113}$$

The analytical formula for the equivalent resistance between nodes $B_{x_1}$ and $B_{x_2}$ in the 2 × $n$ RN of Fig.1 is

$$\frac{R_n(B_{x_1},B_{x_2})}{r}=\frac{4}{5}\left(\frac{F_{1,1}^{(1)}-2F_{1,2}^{(1)}+F_{2,2}^{(1)}}{B_{n+1}^{(1)}}\cos^2(\frac{1}{2}\theta_1)+\frac{F_{1,1}^{(2)}-2F_{1,2}^{(2)}+F_{2,2}^{(2)}}{B_{n+1}^{(2)}}\cos^2(\frac{1}{2}\theta_2)\right). \tag{114}$$

The equivalent resistance $R_n(A_{x_1},O)$ of the $2\times n$ fan network model was once a research topic in Refs. [41, 42], and Ref. [39] investigated more general issues. Comparing these conclusions, it was found that the conclusions drawn in this paper (108) - (114) are consistent with those given in references [41,42], which indirectly proves the correctness of the calculation results in this paper.

### 5.3 The case of $x_2=x_1$

Here we mainly discusses the problem of equation (7). When $x_2=x_1$, there be $F_{1,2}^{(i)}=F_{2,2}^{(i)}=F_{1,1}^{(i)}$, equation (7) can be simplified,

$$\frac{R_n\left(A_{x_1},B_{x_1}\right)}{r}=(2d-p_1)\frac{(1-u_1)^2\Delta B_{x_1}^{(1)}\Delta B_{n-x_1}^{(1)}}{(p_2-p_1)(p_1-2)B_{n+1}^{(1)}}-(2d-p_2)\frac{(1-u_2)^2\Delta B_{x_1}^{(2)}\Delta B_{n-x_1}^{(2)}}{(p_2-p_1)(p_2-2)B_{n+1}^{(2)}}. \tag{115}$$

Among them $F_{1,1}^{(i)}=\Delta B_{x_1}^{(i)}\Delta B_{n-x_1}^{(i)}$ , $u_i=h_{20}(2+h_{21})/(2d-p_i)$.

Specifically, when $x_2=x_1=0$ and $F_{1,1}^{(i)}=\Delta B_0^{(i)}\Delta B_{n-0}^{(i)}=\Delta B_n^{(i)}$, it is derived from (115)

$$\frac{R_n\left(A_0,B_0\right)}{r}=(2d-p_1)\frac{(1-u_1)^2\Delta B_n^{(1)}}{(p_2-p_1)(p_1-2)B_{n+1}^{(1)}}-(2d-p_2)\frac{(1-u_2)^2\Delta B_n^{(2)}}{(p_2-p_1)(p_2-2)B_{n+1}^{(2)}}. \tag{116}$$

Obviously, when $x_2=x_1$, the analytical formula for equivalent resistance degenerates into a relatively simple result, and they are all original results derived from this article.

### 5.4 Visualize resistance relationship

In order to further reveal the relationship between equivalent resistance with $r_1$ and node position, we have used the formulae given in the previous text to draw their visualization graphs, which can clearly show the relationship between equivalent resistance with $r_1$ and node position. Here we consider the visualization of equivalent resistances $R_n(A_0,B_x)$ and $R_n(A_x,0)$. The parameter design in the graph is $r_0=r_2=r=1$ (bΩ), where unit b is any value, such as b=1, b=1000, and so on. Let $r_1$ be a variable, $A_x$ be a movable point on the $A_0A_n$ axis, and $B_x$ be a movable point on the $B_0B_n$ axis.

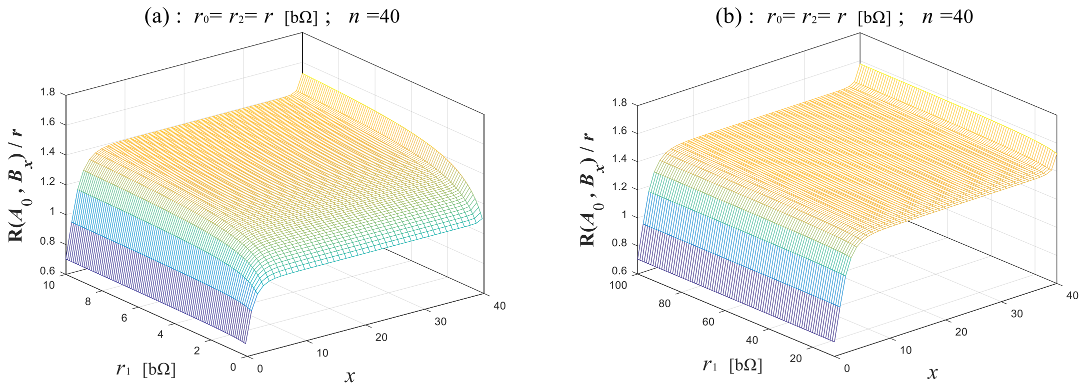


Fig. 6. Visual characteristic of equivalent resistance $R_n(A_0,B_x)/r$ varying with $r_1$. In Fig. (a), $r_1 \in [0,10]$, and in Fig. (b), $r_1 \in [10,100]$

### *5.4.1 Visual representation of equivalent resistance* $R_n(A_0,B_x)$

Draw a graph of the change in equivalent resistance between $A_0$ and $B_x$. Since $R_n(A_0,B_x)/r$ is a dimensionless value, regardless of the unit [bΩ] of resistance, the unit [bΩ] does not affect the structure of the graph. We take the maximum order of the network as *n*=40. In order to facilitate the investigation of the variation characteristics of equivalent resistance $R_n(A_0,B_x)/r$ with $r_1$, we considered two segmented intervals $r_1 \in [0,10]$ and $r_1 \in [10,100]$.

From Fig. 6 (a), it can be seen that when *x* is determined (e.g. *x*=20, etc.), the equivalent resistance $R_n(A_0,B_x)/r$ for $r_1 \in [0,10]$ gradually increases with the increase of $r_1$; When $r_1$ is determined (e.g. $r_1$=4, etc.), $R_n(A_0,B_x)/r$ within the range of $x \in [0,10]$ significantly increases with the increase of *x*, and $R_n(A_0,B_x)/r$ within the range of $x \in [10,35]$ hardly changes with the increase of *x*, but $R_n(A_0,B_x)/r$ within the range of $x \in [35,40]$ significantly increases with the increase of *x*. From Fig. 6 (b), it can be seen that when *x* is determined (such as x=20, etc.), the equivalent resistance $R_n(A_0,B_x)/r$ of $r_1 \in [10,100]$ hardly changes with the increase of $r_1$, indicating that the magnitude of the equivalent resistance $R_n(A_0,B_x)/r$ when $r_1 > 10$ is not significantly affected by $r_1$.

### *5.4.2 Visual representation of equivalent resistance* $R_n(A_x,O)$

Draw a graph of the change in equivalent resistance between $A_x$ and *O*. Since $R_n(A_x,O)/r$ is a dimensionless value, regardless of the unit (bΩ) of resistance, it does not affect the structure of the graph. We take the maximum order of the network as *n*=40. In order to facilitate the investigation of the variation characteristics of equivalent resistance $R_n(A_x,O)/r$ with respect to $r_1$, we consider two segmented intervals $r_1 \in [0,10]$ and $r_1 \in [10,100]$.

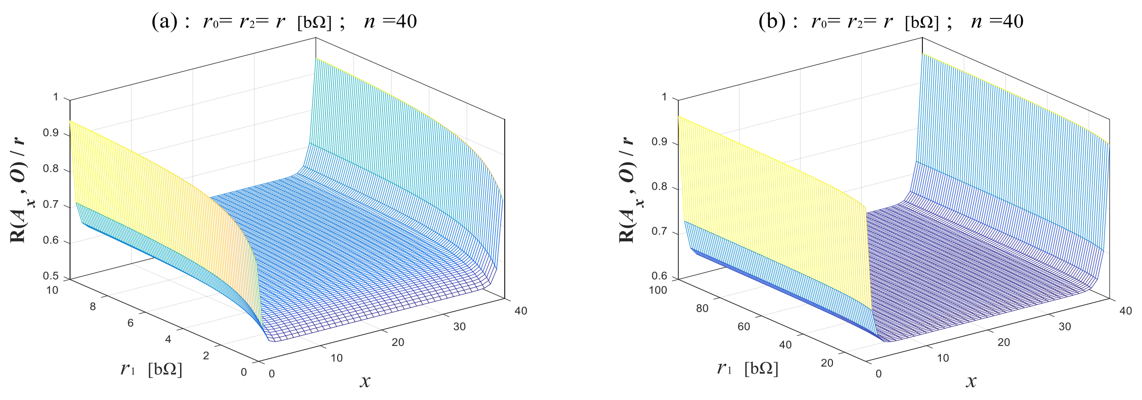


Fig.7. Characteristics of equivalent resistance $R_n(A_x,O)/r$ as a function of $r_1$, where $r_1\in[0,10]$ in Fig. (a), $r_1\in[10,100]$ in Fig. (b)

From Fig.7 (a), it can be seen that when $x$ is determined (e.g. $x$=10, etc.), and $r_1\in[0,10]$, the equivalent resistance $R_n(A_x,O)/r$ gradually increases with the increase of $r_1$; When $r_1$ is determined (such as $r_1$=4, etc.), the equivalent resistance $R_n(A_x,O)/r$ within the range of $x\in[0,40]$ shows a U-shaped change with the increase of $x$, showing a significant increase in the equivalent resistance at both ends. The equivalent resistance $R_n(A_x,O)/r$ within the range of $x\in[5,35]$ exhibits a relatively small resistance value. From Fig. 7 (b), it can be seen that when $x$ is determined (such as $x$=20, etc.), when $r_1\in[10,100]$, the equivalent resistance $R_n(A_x,O)/r$ remains almost unchanged as $r_1$ increases, this indicates that the magnitude of the equivalent resistance $R_n(A_x,O)/r$ when $r_1>10$ is not significantly affected by $r_1$.

## 6 Summary and comment

This article proposes an asymmetric $2\times n$ RNM that is a challenging issue. The research here indicates that the RT-V method established in reference [32] has broad application value and can solve complex circuit networks. The five original analytical formulas for equivalent resistance proposed in the article are all derived using the RT-V method. Due to the presence of five independent resistive elements in the model, such as $r_0, r$, $r_1, r_2$, $r_3$ all of which are random values, this model has generality and can generate a series of special network models. The article validates and compares the results of this article when discussing special cases, indicating that formulas (7) - (12) hold for everything, especially when discussing $r_1\to\infty$ and $r_0=r, r_2=0.5r$, equivalent resistance analytical equations (108) - (114) expressed in trigonometric functions are derived.

Solving the analytical formula for equivalent resistance is a fundamental problem, as once the equivalent resistance formula is obtained, many complex circuits can be solved through variable substitution techniques. For example, taking the circuit network in this article as an example, consider a class of RLC complex impedance circuits as shown in Fig.1, one can assume $r_0=-j/\omega C$, $r=j\omega L$, $r_2=j\omega L_2$, $r_1=-j/\omega C_1$, by substituting these relationships into the equivalent resistance formulae, the equivalent complex impedance analytical formula can be derived. For example, in a fractional order complex impedance circuit, let $r_0 = \frac{1}{\omega^{\lambda_i} C_0} \mathrm{e}^{j(-\lambda_i \pi/2)}$, , $r_1 = \frac{1}{\omega^{\lambda_i} C_1} \mathrm{e}^{j(-\lambda_i \pi/2)}$, $r = \omega^{\bar{\lambda}_i} L \mathrm{e}^{j(\bar{\lambda}_i \pi/2)}$, $r_2 = \omega^{\bar{\lambda}_i} L_2 \mathrm{e}^{j(\bar{\lambda}_i \pi/2)}$, where $0 \le \lambda_i \le 1$, $0 \le \bar{\lambda}_i \le 1$. By substituting these relationships into the equivalent resistance formulae, the equivalent complex impedance analytical formula for fractional order circuits can be derived. The research work in this article can promote the research and development of complex circuit network models; The analytical formula for equivalent resistance we have derived can provide a theoretical basis for related research and simulation studies in engineering.

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