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Dynamic models for CoVID-19 and data analysis †

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Summary

In this letter, two time delay dynamic models, TDD-NCP model and Fudan-CCDC model, are introduced to track the data of COVID-19. The TDD-NCP model is developed recently by Cheng's group group in Fudan and SUFE. The TDD-NCP model introduced the time delay process into the differential equations to describe the latent period of the epidemic. The Fudan-CDCC model is established when Wenbin Chen suggested to determine the kernel functions in the TDD-NCP model by the public data from CDCC. By the public data of the cumulative confirmed cases in different regions in China and different countries, these models can clearly illustrate that the containment of the epidemic highly depends on early and effective isolations.

KEYWORDS:
COVID-19; (Statistical) time delay dynamical system; TDD-NCP model; Fudan-CCDC model

1 | INTRODUCTION

In Dec. 2019, a pneumonia of unknown cases of unknown cause emerged in Wuhan, Hubei province in China. The Wuhan Municipal Health Commission reported 27 cases of viral pneumonia including 7 severely ill cases on Dec. 12, 2019. The causative agent identified by the Chinese authorities was designated as Novel Coronavirus Pneumonia (COVID-19) by the World Health Organization (WHO). As of Feb. 20, 2020, there have been 74675 confirmed cases in mainland of China and more than 2000 people died. The virus have been spread widely to 26 countries, and the situations in Japan and South Korea are becoming serious as well, as of Feb. 22, 2020, there have been already 756 infected cases in Japan (including the Diamond Princess Cruise).

COVID-19 raised intense attention not only within China but internationally. People concerned about the spread of epidemic and its development trend. Many mathematical researches focused on the modelling of the spread and development of the COVID-19. For instance, considering the epidemic's feature of spreading during the latent period, Cheng's group applied the time delay process to describe the typical feature and proposed a novel dynamical system to predict the outbreak and evolution of COVID-19. This model was called TDD-NCP model [1,2]. Based on this work, Chen et al. also proposed a time delay dynamic system with external source to describe the trend of local outbreak for COVID-19 [3]. Then, considering the fractional order derivatives, Chen et al. proposed a novel time delay dynamic system with fractional order [4] . In their system, the Riemann-Liouville derivative is added which can describe the confirmed and cured people's growth process. Later, based on Chinese Centers for Disease Control (CCDC) statistical data, Shao et al. proposed a series time delay dynamic system (called Fudan-CCDC model) [5], and they estimated the reproductive number R_0 of COVID-19 in [6] based on Wallinga and Lipsitch frame work [8]. The conclusion is, the R_0 estimated is in [3.25, 3.4] of COVID-19 which is bigger than that of SARS. Some paralleled results we refer for readers [9,10,11,12,13,14,15]. Specially, in Feb. 23, Fudan-CCDC model is used to warn that there could be a rapid outbreak in Japan if no effective quarantine measures are carried out immediately [7].

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[0]**Abbreviations:** COVID-19, Corona Virus Disease 2019; TDD-NCP model, Time Delay Dynamical-Novel Coronavirus Pneumonia model; CCDC,Chinese Center for Disease Control and Prevention.

In this paper, we provide a brief introduction of TDD-NCP model[1,2], and Fudan-CCDC model[5]. We use Fudan-CCDC model to reconstruct some important parameters (including growth rate, isolation rage, initial date) and predict the cumulative number of confirmed cases in some of cities in China. In addition, due to the serious concern of possible severe outbreak in Japan, we also analyse the different evolutions of COVID-19 in Japan with different isolation rates in future days. The future circumstances of COVID-19 in Singapore will also be predicted.

It is worth to emphasize that, the data employed in this paper are acquired from WIND, which are provided by China Health and Health Commission, but the data also can be easily found at every Chinese news websites. The code is running on Matlab, where some optimization packages are used.

2 | TDD-NCP MODEL

The following notations will be utilized in TDD-NCP model:

- $I(t)$: accumulated number of infected people at time t,
- $J(t)$: accumulated number of confirmed people at time t,
- $G(t)$: the number of infected and isolated but yet diagnosed cases at time t, they are infected in fact, but are not confirmed by the hospital,
- $R(t)$: accumulated number of cured people at time t.

Assumptions:

- At time t, the exposed people may transmit to others are $I_0(t) = I(t) - G(t) - J(t)$, with the spread rate β, which is a fixed but unknown constant.
- No matter having been isolated or not, the cumulative diagnosed people $J(t)$ some from the previous infected ones. The infected people averagely experience of latent period of τ_1 days. We use γ to represent the morbidity.
- Due to the quarantine strategy of government, some infected people have been isolated during τ_1 latent days, the averagely experience of isolated period is of τ_1' days, then they confirmed. The isolated rate is assumed be ℓ, larger value of ℓ suggests that the government implement tougher controlling.
- It is τ_2 days averagely for the diagnosed people become cured with rate κ or dead with rate $1 - \kappa$.

The model will be described as follows:

$$\frac{dI}{dt} = \beta I_0(t)\,, \tag{1}$$

$$\frac{dJ}{dt} = \gamma \int_0^t h_1(t - \tau_1, t')\beta I_0(t')dt'\,, \tag{2}$$

$$\frac{dG}{dt} = \ell I_0(t) - \int_0^t h_2(t - \tau_1', t')G(t')dt'\,, \tag{3}$$

$$\frac{dR}{dt} = \kappa \int_0^t h_3(t - \tau_1 - \tau_2, t')\beta I_0(t')dt'\,, \tag{4}$$

in which the $h_i(\hat{t}, t')$ with $i = 1, 2, 3$ are distribution which should be normalized as $\int_0^t h_i(\hat{t}, t')dt' = 1$. The h_i can be chosen as normal distribution with $h_i = c_{i1}e^{-c_{i2}(\hat{t}-t')^2}$ or chose be $\delta-$function $h_i(\hat{t}, t') = \delta(\hat{t} - t')$ for simplicity, which means every infected individual experienced the same latent period and treatment period.

The model can be utilized to predict the tendency of outbreak for COVID-19. Suppose we know the initial conditions $\{I, J, G, R\}|_{t=t_0}$, where t_0 is the initial time, knowing the infected number J_{data} and dead number R_{data}. Setting the morbidity

$\gamma = 0.99$, the latent period $\tau_1 = 7$, $\tau_1' = 14$ and $\tau_2 = 12$, we can identify the spread rate β, the isolated rate ℓ and cured rate κ via following two optimization problem:

$$(\beta^*, \ell^*) = \text{argmin}_{\beta,\ell} \|J(\beta, \ell, t) - J_{\text{data}}\|\,,$$
$$\kappa^* = \text{argmin}_{\kappa} \|R(\beta^*, \ell^*, \kappa, t) - R_{\text{data}}\|\,.$$

With the reconstructed $(\beta^*, \ell^*, \kappa^*)$, one can put them into system (1) and solve it numerically.

3 | FUDAN-CCDC MODEL

The Fudan-CCDC model was established when Cheng, one of the author for TDD-NCP model, suggested to use the time delay model to fit the real data. The Fudan-CCDC model is described as

$$\frac{dI}{dt} = rI_0(t)\,, \tag{5}$$

$$\frac{dJ}{dt} = r\int_{-\infty}^{t} f_4(t-s)I_0(s)ds\,, \tag{6}$$

$$\frac{dG}{dt} = \ell(t)\int_{-\infty}^{t} f_2(t-s)I_0(s)ds - \ell(t)\int_{-\infty}^{t} f_4(t-s)I_0(s)ds\,. \tag{7}$$

One can also use the discrete system with each step representing one day just as we have implemented in the code:

$$I(t+1) = I(t) + rI_0(t)\,, \tag{8}$$

$$J(t+1) = J(t) + r\sum_{s\le t} f_4(t-s)I_0(s)\,, \tag{9}$$

$$G(t+1) = G(t) + \ell(t)\sum_{s\le t} f_2(t-s)I_0(s) - \ell(t)\sum_{s\le t} f_4(t-s)I_0(s)\,. \tag{10}$$

We further make some assumptions on the following transition probability.

- $f_2(t)$: the transition probability from infection to illness onset,
- $f_3(t)$: the transition probability from illness onset to hospitalization,
- $f_4(t)$: the transition probability from infection to hospitalization, which can be calculated via the convolution of $f_2(t)$ and $f_3(t)$,

We assume the log-normal distribution for $f_2(t)$ and the Weibull distrubution for $f_3(t)$, and the distribution parameters can be estimated from CCDC by fitting the figures[16]. In addition, we denote the constant r be growth rate which assumed to be equal to β in TDD-NCP model. Another important improvement in Fudan-CCDC model is taking the consideration of isolated rate ℓ, which is distinct for different time stages at different regions. So we may assume

$$\ell = \begin{cases} \ell_1,\ t < t_\ell; \\ \ell_2,\ \text{else}. \end{cases}$$

This means the government adopt different quarantine strategy at time t_ℓ.

The parameters $r, \ell_1, \ell_2, t_\ell$ are all be reconstructed numerically via the following optimization problem

$$\min_{r,\ell_1,\ell_2,t_\ell,t_0} \|J(r, \ell_1, \ell_2, t_\ell, t) - J_{\text{data}}\|\,.$$

The model can also track the initial date of the epidemic when provided $I(t_0)$. In the following, we list these important parameters in some cities in China, the Diamond Princes Cruise Ship, Japan without cruise, and Singapore are also contained in the table.

It can be analyzed from table that the growth rate r is approximately around 0.31. The Chinese government adopted very strong quarantine strategy from Jan.17,2020. Actually, it is reported that there have bee already 70548 infected cases and 1875 dead

TABLE 1 The growth rate r, quarantine strategy, and initial date reconstructed from Fudan-CCDC model

	r	ℓ_1	ℓ_2	t_0	t_ℓ
Wuhan	0.3269	0.1928	0.4695	Dec.15	Jan.15
Hubei without Wuhan	0.3079	0.1773	0.4712	Dec.16	Jan.16
Shanghai	0.3001	0.1244	0.5748	Dec.27	Jan.16
Beijing	0.3057	0.1557	0.5778	Dec.27	Jan.17
Diamond Princess Cruise Ship	0.3085	0.2092	/	Jan.7	/
Japan without cruise	0.3271	0.2872	/	Jan.20	/
Singapore	0.3057	0.2461	0.4399	Jan.7	Jan.18

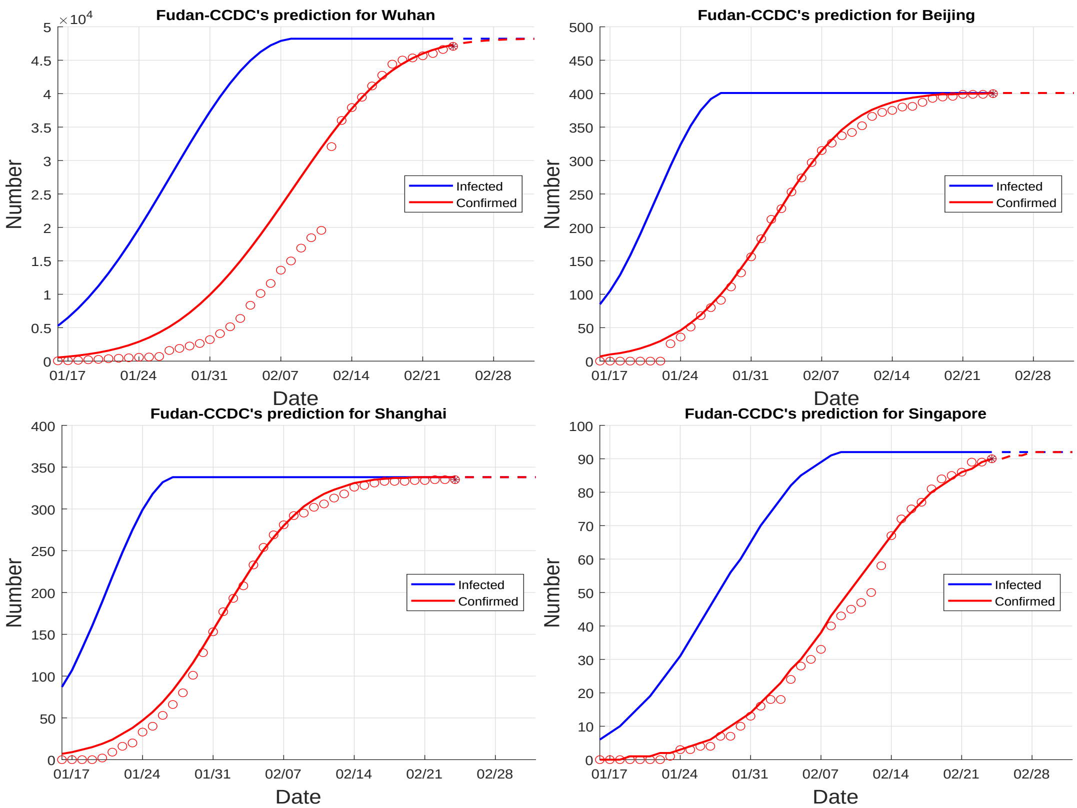

FIGURE 1 The tendency of outbreak for the COVID-19 in some cities in China and in Singapore.

cases. The following figures show the forecast the tendency of outbreak for the COVID-19 in some cities in China and in Singapore.

We also present the simulation results for COVID-19 in the Diamond Princess Cruise, everyone on board get off the cruise, passengers on board began to disembark in batches on Feb. 19, 2020.

Finally, we present the estimated results of possible future scenarios for COVID-19 in Japan (without cruise) under different choices of ℓ_2. We conclude from the figure as follows:

- $\ell_2 = 0.4$ which is a little bigger than $\ell_1 = 0.2872$ but not enough. It means the quarantine strategy taken by the Japanese government are insufficient, the number of infected people will remain increasing exponentially.
- $\ell_2 = 0.43$, the measures taken by the Japanese government are not sufficient and the number of infected people will rise at a slower rate.

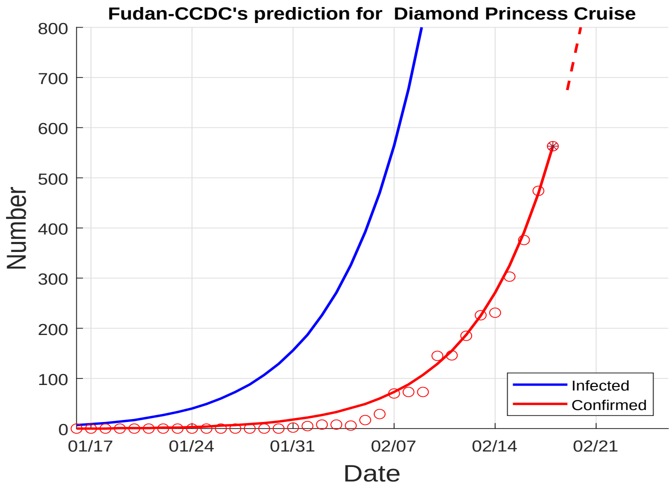

FIGURE 2 The simulation results for COVID-19 in the Diamond Princess Cruise.

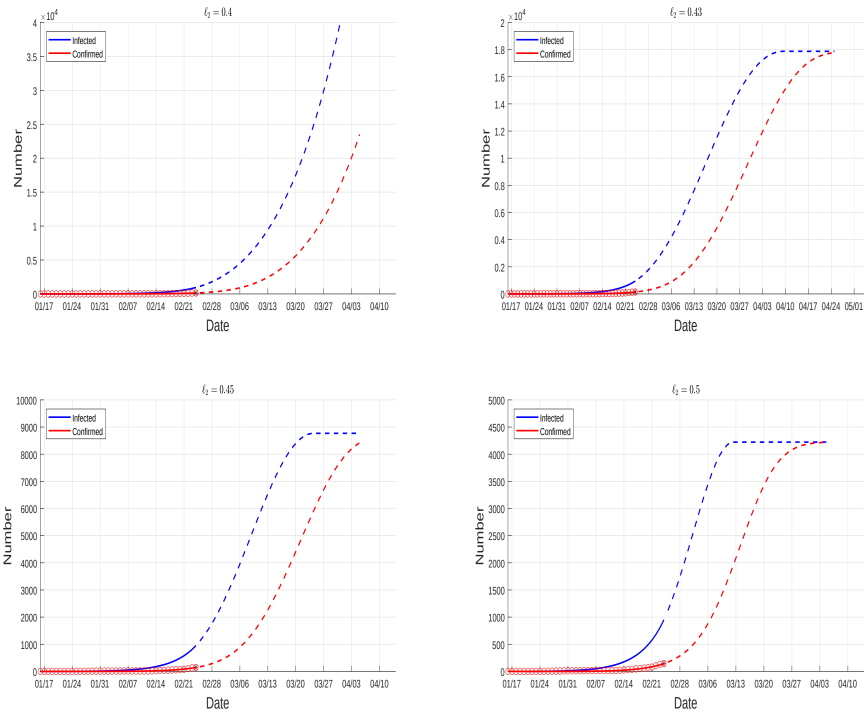

FIGURE 3 Possible future scenarios for COVID-19 in Japan (without cruise)

- $\ell_2 = 0.45$, the stabilization period will come a little earlier. The accumulative number of infected cases will decrease notable .
- $\ell_2 = 0.5$, which means the quarantine strategy is almost the same strength as those in Shanghai, the epidemic will soon be under control, and the cumulative number of infected cases will be approximately 4000.

4 | FURTHER DISCUSSION

During the following research, we will focus on the several questions:

- Parameter Identification Problems: From the observed data, based on our model, we would like to identify the sources terms, which indicate when and how the patients infected. Actually, when applying our Fudan-CCDC model to analyze the tendency of COVID-19 in Korea, we successfully track down a super spreader on Feb. 7,2020. The following analysis related to the influence of such super spreaders will be our concentration.
- Stability problems with respect to the growth rate r: When we applied TDD-NCP and Fudan-CCDC models to the different regions in China and different nations, a very interesting observation is that, even we take almost same parameter r and although the kernels in these two models are different, we can have similar results. This leads to our further consideration on the stability of this parameter.
- The observability and controllability theory for two dynamical systems with respect to the isolated parameter ℓ: This parameter plays a significant role in our models. Estimating this parameter can help the government making the decision whether the government should increase the quarantine strategy. It would be interesting to study the observability and controllability of two models with respect to ℓ. The optimal control problem will be very useful.
- What is the relation between our models and the classical SEIR models.

Other versions of our models can also been developed, such as random input and random parameters. Moreover, the methods here can generalized in other fields, such as finance, risk management and social networks, we will discuss these topics in the future and also welcome other groups to join us.

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Conflict of interest

The authors declare no potential conflict of interests.

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