

# Supporting Information for “Hysteretic behavior of flow recession dynamics: Application of machine learning and learning from the machine”

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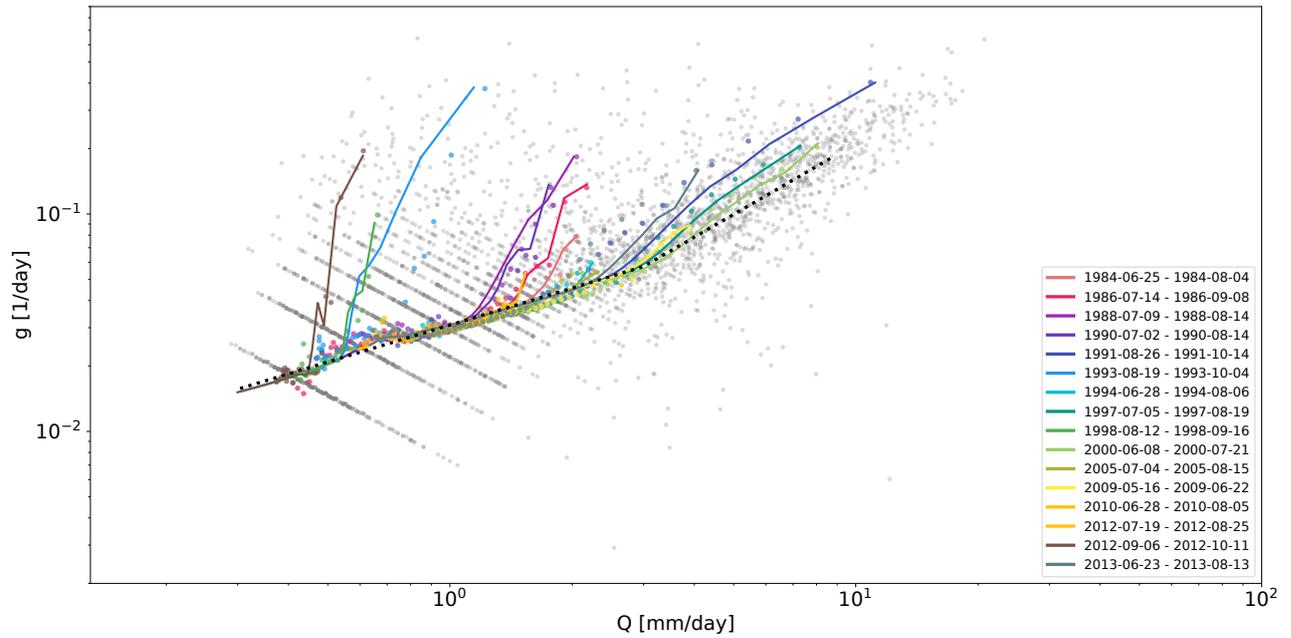
1. Figures S1 to S2

**Introduction** This supporting information is to provide additional figures.

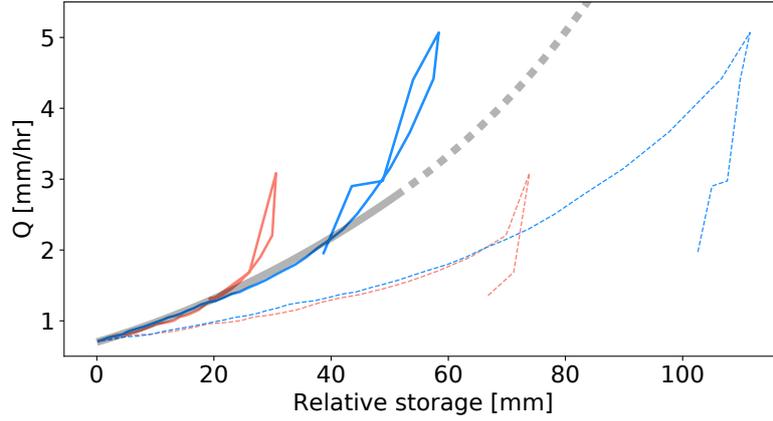
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## References

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- Dralle, D. N., Hahm, W. J., Rempe, D. M., Karst, N. J., Thompson, S. E., & Dietrich, W. E. (2018). Quantification of the seasonal hillslope water storage that does not drive streamflow. *Hydrological Processes*, *32*(13), 1978–1992. doi: [10.1002/hyp.11627](https://doi.org/10.1002/hyp.11627)



**Figure S1.** The event trajectory of the 16 events and the parameterized dense area. The solid lines illustrate the event trajectories estimated using the LSTM model in the forward simulation mode (colored differently for each event), and the correspondingly colored dots illustrate the half-step ahead estimation for each event. The grey dots are the CTS method-based estimation. The black dotted line illustrates the parameterized dense area.



**Figure S2.** Hysteretic storage-discharge (S-Q) relationships for the two events that are used for Figure 1b in the main text. The solid red and blue lines are the same as the relationships shown in Figure 1b, and those are estimated without considering  $ET$ . The estimated storage is the (relative) “active” storage which is set to zero at  $Q = 0.7$  mm/day. The solid gray line is the S-Q relationship estimated using the parameterized attractor  $g(Q)$  that is discussed in the main text; i.e., the relative storage  $S(Q) = \int_{Q_0}^Q (1/g(Q))dQ = \frac{1}{a(2-b)}(Q^{2-b} - Q_0^{2-b})$ , where  $g(Q) = aQ^{b-1}$ ,  $a = 0.031$ ,  $b = 1.57$ , and  $Q_0 = 0.7$  mm/day. The dashed gray line is the S-Q relationship estimated using the parameterized upper dense area. The red and blue dashed lines are the relationships taking into account  $ET$  (by using  $dS/dt = -Q - ET$ ). In this case, the estimated storage is the (relative) total storage which is set to zero at  $Q = 0.7$  mm/day. The difference between the active storage and the total storage is “inactive” storage (which sometimes refers to as “indirect” storage (Dralle et al., 2018) or “hydraulically-disconnected” storage (Carrer et al., 2019)), which is set to zero at  $Q = 0.7$  mm/day in this example. The overlap of the total S-Q relationship for the two events under low flow conditions implies that  $g = (dQ/dt)/(-Q - ET)$  may be used to find another attractor that can be utilized to estimate a hysteretic (relative) total S-Q relationship.