An acceleration method employing sparse sensing matrix for fast analysis of the wide-angle electromagnetic problems based on compressive sensing

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The electromagnetic scattering problem over a wide incident angle can be rapidly solved by introducing the compressive sensing theory into the method of moments, whose main computational complexity is comprised of two parts: a few calculations of matrix equations and the recovery of original induced currents. To further improve the method, a novel construction scheme of measurement matrix is proposed in this paper. With help of the measurement matrix, one can obtain a sparse sensing matrix, and consequently the computational cost for recovery can be reduced by at least half. The scheme is described in detail, the analysis of computational complexity and numerical experiments are provided to demonstrate the effectiveness.

Introduction: Method of moments (MoM) possesses the advantage of high accuracy in solving the electromagnetic (EM) scattering problems [1]. But it will cost a huge computational amount when the incident wave is from a wide-angle range, since the procedure needs to be repeatedly implemented at every angle increment. Recently, a fast method based on MoM conjunction with compressive sensing (CS) theory for solving the wide-angle EM scattering problems has been proposed [2]. In this method, a kind of new excitation sources containing abundant information of different incident angles is built firstly. Then, the induced currents over a wide-angle can be solved by means of the sparse transform and the recovery algorithm from the measurement results, which are obtained by the calculations of traditional MoM with the new sources. The computational complexity of the fast method mainly consists of two parts: one is the measurement, i.e., a few calculations of MoM; the other is to acquire the projections of induced currents in sparse domain.

In order to further improve the fast method, much effort has been devoted to the research on these two parts, and many effective schemes, such as efficient basis function [3] and two-dimensional CS [4], have been devised. In this paper, a novel scheme for designing the measurement matrix is raised, by which the sensing matrix shows remarkable sparsity when the orthogonal basis is selected as the sparse transform. Accordingly, the computational complexity for acquire the projections can be sharply decreased. The principle and the complexity analysis are presented, and the effectiveness is validated by numerical experiments, in which five typical orthogonal bases [5], such as fast Fourier transform (FFT) basis and Hermite basis, are taken as the sparse transforms respectively.

Fast method based on CS: The wide-angle EM scattering problem solving by the traditional MoM can be described as a matrix equation with multiple right-hand sides

$$\mathbf{Z}\left[\mathbf{I}_1\,\mathbf{I}_2\cdots\mathbf{I}_n\right]=\left[\mathbf{V}_1\,\mathbf{V}_2\cdots\mathbf{V}_n\right], \tag{1}$$

in which, $\mathbf{Z}$ is the impedance matrix, $\mathbf{V}_1$ to $\mathbf{V}_n$ are the excitation vectors at n different incident angles, and $\mathbf{I}_1$ to $\mathbf{I}_n$ represent the n corresponding induced current vectors.
In the fast method, M new excitation vectors based on CS theory are constructed as

$$\mathbf{V}_i^{\mathrm{CS}}=c_{i1}\mathbf{V}_1+c_{i2}\mathbf{V}_2+\cdots+c_{in}\mathbf{V}_n\quad(i=1,2,\cdots,M), \tag{2}$$

where c_{ij} is the element in the measurement matrix $\mathbf{\Phi}$. In CS theory, the measurement matrix must satisfy the restricted isometry property (RIP) [6], which ensures the accurate reconstruction of original signal. In general, Gaussian random matrix is often used as $\mathbf{\Phi}$.

Substituting (2) to (1), one can obtain M current vectors under the new excitations by

$$\mathbf{Z}\left[\mathbf{I}_1^{\mathrm{CS}}\,\mathbf{I}_2^{\mathrm{CS}}\cdots\mathbf{I}_M^{\mathrm{CS}}\right]=\left[\mathbf{V}_1^{\mathrm{CS}}\,\mathbf{V}_2^{\mathrm{CS}}\cdots\mathbf{V}_M^{\mathrm{CS}}\right]. \tag{3}$$

Due to the linearity of the problem, $\mathbf{I}_1^{\mathrm{CS}}$ to $\mathbf{I}_M^{\mathrm{CS}}$ can be written as

$$\mathbf{I}_i^{\mathrm{CS}}=c_{i1}\mathbf{I}_1+c_{i2}\mathbf{I}_2+\cdots+c_{in}\mathbf{I}_n\quad(i=1,2,\cdots,M). \tag{4}$$

The M current vectors are regarded as the results of M measurements of the original induced current vectors $\mathbf{I}_1$ to $\mathbf{I}_n$. If the original induced current vectors have sparse representations, (4) can be described as

$$\mathbf{\Phi}\left[\mathbf{I}_1\,\mathbf{I}_2\cdots\mathbf{I}_n\right]^{\mathrm{T}}=\mathbf{\Phi\Psi}\left[\boldsymbol{\alpha}_1\,\boldsymbol{\alpha}_2\cdots\boldsymbol{\alpha}_N\right]=\left[\mathbf{I}_1^{\mathrm{CS}}\,\mathbf{I}_2^{\mathrm{CS}}\cdots\mathbf{I}_M^{\mathrm{CS}}\right]^{\mathrm{T}}, \tag{5}$$

in which $\mathbf{\Psi}$ is the sparse transform, N is the number of the basis functions, and $\boldsymbol{\alpha}_1$ to $\boldsymbol{\alpha}_N$ are the projections of each column of $[\mathbf{I}_1\,\mathbf{I}_2\ldots\mathbf{I}_n]^{\mathrm{T}}$ in sparse domain. In the wide-angle EM scattering problems, FFT basis, Hermite basis or other orthogonal bases are often selected as $\mathbf{\Psi}$ [5].

By utilizing the recovery algorithm (e.g., orthogonal matching pursuit (OMP) [7]), the projections are solved by

$$\begin{aligned}&\left[\hat{\boldsymbol{\alpha}}_1\,\hat{\boldsymbol{\alpha}}_2\cdots\hat{\boldsymbol{\alpha}}_N\right]=\mathrm{argmin}\left\|\left[\hat{\boldsymbol{\alpha}}_1\,\hat{\boldsymbol{\alpha}}_2\cdots\hat{\boldsymbol{\alpha}}_N\right]\right\|_L\\&s.t.\mathbf{\Theta}\left[\boldsymbol{\alpha}_1\,\boldsymbol{\alpha}_2\cdots\boldsymbol{\alpha}_N\right]=\left[\mathbf{I}_1^{\mathrm{CS}}\,\mathbf{I}_2^{\mathrm{CS}}\cdots\mathbf{I}_M^{\mathrm{CS}}\right]^{\mathrm{T}},\end{aligned} \tag{6}$$

where $\mathbf{\Theta}$ is the sensing matrix and $\mathbf{\Theta}=\mathbf{\Phi\Psi}$. Then, the original induced current vectors can be reconstructed by

$$\left[\hat{\mathbf{I}}_1\,\hat{\mathbf{I}}_2\cdots\hat{\mathbf{I}}_n\right]^{\mathrm{T}}=\mathbf{\Psi}\left[\hat{\boldsymbol{\alpha}}_1\,\hat{\boldsymbol{\alpha}}_2\cdots\hat{\boldsymbol{\alpha}}_N\right]. \tag{7}$$

The computational complexity for solving (6) by OMP is $O(nKMN)$, where K is the sparsity of original induced current vectors in sparse domain, and the inner products of the columns in sensing matrix and the measurement results are the dominant computational cost.

Proposed scheme: To reduce the complexity of recovery, one can make the sensing matrix sparse to decrease the cost of inner products. For the purpose, a novel construction scheme of measurement matrix is proposed as follows:

First, by randomly extracting P columns from $\mathbf{\Psi}$, one can obtain $\boldsymbol{\psi}_1$ to $\boldsymbol{\psi}_P$, where $\boldsymbol{\psi}$ represents the column of $\mathbf{\Psi}$.

Afterwards, to better satisfy RIP, a linear superposition of these P vectors is implemented as

$$\boldsymbol{\varphi}_1 = d_{11}\boldsymbol{\psi}_1 + d_{12}\boldsymbol{\psi}_2 + \cdots + d_{1P}\boldsymbol{\psi}_P. \quad (8)$$

Finally, repeating the above two steps M times and a new measurement matrix $[\boldsymbol{\varphi}_1\ \boldsymbol{\varphi}_2 \ldots \boldsymbol{\varphi}_M]^{\mathrm{T}}$ is established, in which

$$\boldsymbol{\varphi}_i = d_{i1}\boldsymbol{\psi}_1 + d_{i2}\boldsymbol{\psi}_2 + \cdots + d_{iP}\boldsymbol{\psi}_P \quad (i = 1, 2, \cdots, M). \quad (9)$$

Obviously, when an orthogonal basis is taken as $\mathbf{\Psi}$, the inner products of $\boldsymbol{\varphi}_i$ and the $(n-P)$ columns in $\mathbf{\Psi}$ that are not extracted in the first step are zero respectively. In other words, there are only P non-zero elements in the i-th row of $\mathbf{\Theta}$. A sparse $\mathbf{\Theta}$ with MP non-zero elements is obtained by using the proposed measurement matrix, and the operations of inner products in recovery algorithm are significantly accelerated. Accordingly, the computational complexity of solution to (6) is decreased to $O(\eta nKMN)$, where η is the proportion of nonzero elements in $\mathbf{\Theta}$ and $\eta = P/n$. Generally, P is much less than n.

Using the proposed measurement matrix and an orthogonal basis as $\mathbf{\Phi}$ and $\mathbf{\Psi}$ respectively, (5) can be transformed as

$$\begin{aligned}&\left[\boldsymbol{\varphi}_1\ \boldsymbol{\varphi}_2 \cdots \boldsymbol{\varphi}_M\right]^{\mathrm{T}} \mathbf{\Psi}\left[\boldsymbol{\alpha}_1\ \boldsymbol{\alpha}_2 \cdots \boldsymbol{\alpha}_N\right] = \mathbf{S}\mathbf{\Psi}^{\mathrm{T}}\mathbf{\Psi}\left[\boldsymbol{\alpha}_1\ \boldsymbol{\alpha}_2 \cdots \boldsymbol{\alpha}_N\right] \\ &= \mathbf{S}\left[\boldsymbol{\alpha}_1\ \boldsymbol{\alpha}_2 \cdots \boldsymbol{\alpha}_N\right] = \left[\mathbf{I}_1^{\mathrm{CS}}\ \mathbf{I}_2^{\mathrm{CS}} \cdots \mathbf{I}_M^{\mathrm{CS}}\right]^{\mathrm{T}},\end{aligned} \quad (10)$$

in which, the proposed measurement matrix is expressed as the multiplication of a sparse random matrix $\mathbf{S}$ and the transposed sparse transform. The i-th row of $\mathbf{S}$ has P random coefficients d_{i1} to d_{iP} in the columns corresponding to the randomly extracted columns from $\mathbf{\Psi}$ in the i-th measurement. Other entries in $\mathbf{S}$ are all zero.

So, (10) can be considered as measuring the sparse signals $[\boldsymbol{\alpha}_1\ \boldsymbol{\alpha}_2 \cdots \boldsymbol{\alpha}_N]$ directly with the sparse random matrix, which is proved to satisfy a different form of RIP, so-called RIP(p) for p equal (or very close) to 1 [8,9]. Therefore, (6) can produce an accurate solution with high probability by using the proposed measurement matrix. It is interesting to note that, if the random coefficients are all set to 1 in (10), the sparse random matrix is simplified to a sparse binary matrix (SBM), which consists of only 0 and 1. SBM is often applied as the measurement matrix in wireless sensor networks, since it is easy to be implemented on hardware and has low complexity [10].

Numerical results: Two numerical experiments with perfect electrical conductor (PEC) objects of different shape are presented in this section to validate the proposed scheme, in which the electric field integral equation (EFIE) is established to solve the problems, and OMP is taken as the recovery algorithm. For the convenience of comparison, we define the recovery error as

$$\Delta = \frac{\left\|\left[\hat{\mathbf{I}}_1\ \hat{\mathbf{I}}_2 \cdots \hat{\mathbf{I}}_n\right] - \left[\mathbf{I}_1\ \mathbf{I}_2 \cdots \mathbf{I}_n\right]\right\|_2}{\left\|\left[\mathbf{I}_1\ \mathbf{I}_2 \cdots \mathbf{I}_n\right]\right\|_2} \quad (11)$$

A sphere with the radius of 0.1m illuminated by the plane waves of 300MHz is considered firstly, who contains 480 RWG basis functions. The incident waves are set in the *xoy* plane, and the incident angle is divided into 1°,2°, ..., 360°.

FFT basis is used as the sparse transform. As is shown in Figure 1, the similar precision can be achieved at the same number (70) of measurements by applying the Gaussian random matrix and the proposed measurement matrix respectively, while the number of extracting columns P is larger than 100. It means that one can get a sensing matrix with nonzero elements accounting for 100/360 (η) in the best case. Thus, the computational cost for acquiring the projections of original induced currents in sparse domain is cut by about two-thirds ($1-\eta$) by using the proposed measurement matrix rather than Gaussian random matrix, meanwhile the computational complexity of measurement for both is the same. The comparison of the computing time for acquiring the projections is presented in Table 1, which further proves the high efficiency.

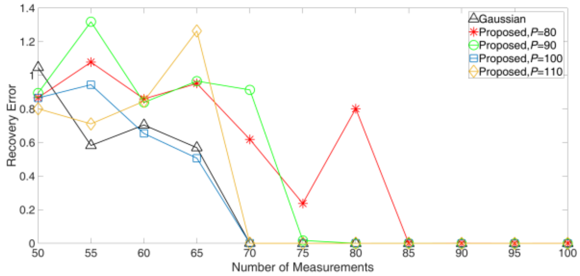

Fig.1 *Relationship between the recovery error and the number of measurements in the case of FFT basis*

Table 1. Computing time of recovery for sphere.

Measurement matrix	FFT (M=70)	Hermite (M=35)	Laguerre (M=40)
Gaussian	26.83 s	10.97 s	14.23 s
Proposed	8.96 s (P=100)	4.59 s (P=120)	6.92 s (P=130)

To show the universality of the proposed technique for orthogonal bases, FFT basis is respectively replaced by Hermite and Laguerre basis, and the corresponding results are shown in Figure 2 and Figure 3. It is evident that, for both Hermite basis and Laguerre basis, much less computational complexity for acquiring the projections is available with the proposed measurement matrix than with Gaussian random matrix under the same condition of measurement. The comparisons of the computing time in the case of two orthogonal bases are provided in Table 1.

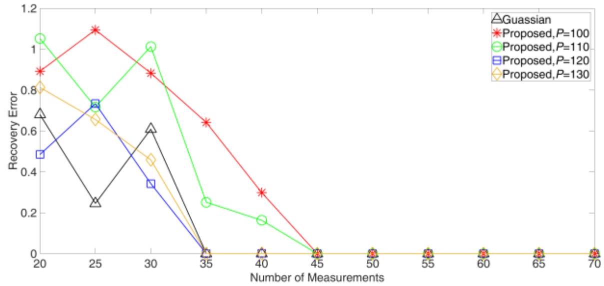

Fig.2 *Relationship between the recovery error and the number of measurements in the case of Hermite basis*

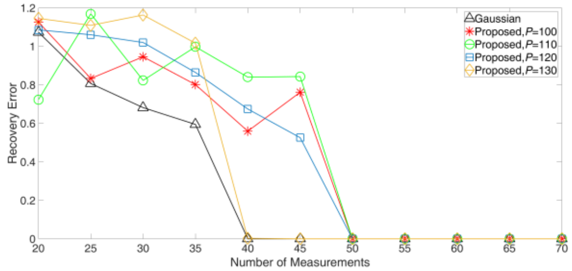

Fig.3 *Relationship between the recovery error and the number of measurements in the case of Laguerre basis*

Then, consider a missile model who contains 1963 RWG basis functions, and the second kind of Chebyshev basis and Legendre basis are chosen as the sparse transforms respectively. Other experimental parameters are the same with the previous one. Both Figure 4 and Figure 5 indicate that the proposed construction scheme of measurement matrix is also validity for the complex shaped objects.

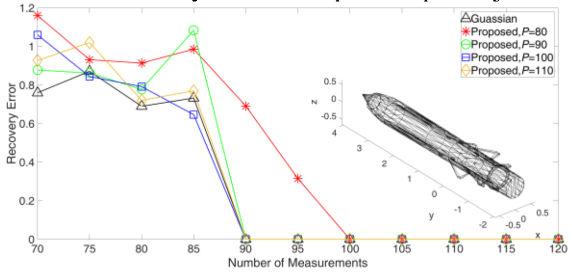

Fig.4 *Relationship between the recovery error and the number of measurements in the case of the 2nd kind of Chebyshev basis.*

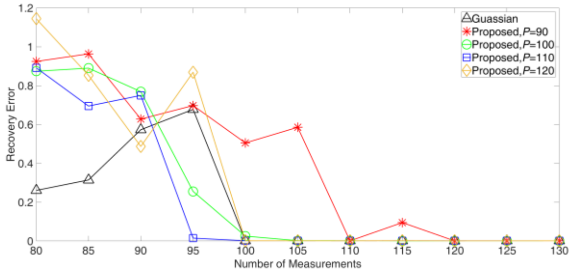

Fig.5 *Relationship between the recovery error and the number of measurements in the case of Legendre basis.*

By using the second kind of Chebyshev basis and the proposed measurement matrix (M=90, P=100), and setting the elements less than 10^{-14} in the sensing matrix to zero, there are only 9000 non-zero elements in the sensing matrix, which is consistent with the expected number MP. Hence, the solution to (6) with the help of OMP is accelerated, which is demonstrated in Table 2.

Table 2. Computing time of recovery for missile model.

Measurement matrix	2nd kind of Chebyshev (M=90)	Legendre (M=100)
Gaussian	120.78 s	181.20 s
Proposed	38.72 s (P=90)	59.91 s (P=110)

Conclusion: A novel scheme for constructing the measurement matrix has been developed. One can get a sparse sensing matrix by adopting the proposed measurement matrix in the solution to wide-angle EM scattering problem based on MoM conjunction with CS. In addition, the number of measurements required for both Gaussian random matrix and the proposed one is the same. Consequently, the computational complexity for acquiring the projections by using recovery algorithm can be significantly reduced under the condition that the computational cost for measurement remains unchanged.

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Data availability statement: The data that support the findings of this study are available from the corresponding author upon reasonable request.

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