A comprehensive estimation method of phase error in the frequency-interleaving digital-to-analog converter

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This letter studied the phase error in the frequency-interleaving digital-to-analog converter (FI-DAC) and proposed a comprehensive phase estimation method. Firstly, the model of FI-DAC system was established, and the phase error were considered that consists of three parts: time delay error, initial phase error and nonlinear phase error. By analyzing the phase function characteristics of the system, the least squares (LS) was used to linearly fit the phase functions of the non-overlapping bands to estimate both the time delay and initial phase of each sub-band channel. Then, the nonlinear phase error of the system was estimated by the difference between the system phase function and the estimated linear phase function. Finally, the effectiveness of the proposed estimation method was verified by the built FI-DAC test bench.

Introduction: Digital waveform synthesis technology is widely used in various fields because of its advantages of flexible generation of waveform signals [1–3]. With the continuous development of electronic technology, higher demand for bandwidth of generated signals is put forward. The performance of the digital-to-analog (DAC) limits the bandwidth of the generated signal. Therefore, FI-DAC is proposed to break through the limitation of DAC's performance to generate higher bandwidth signal by means of parallel structure of multiple DACs. However, the linearity of the phase response of the FI-DAC system is affected by the phase error between the sub-band channels, which affects the quality of output signal. The error include the time delay, the initial phase error of the mixers local oscillator (LO), and the nonlinear characteristics of the analog filter. In [4], the researchers proposed a "Three-point" method to simultaneously estimate the delay time and phase offset according to the auto-power spectrum of the output of FI-DAC's overlapping band. The premise of this method is that the phase response of each sub-band is linear, that is, the nonlinear phase error of the analog filter is not considered. In real circuit application, the analog filter has nonlinear phase, and the non-linearity is the most serious in the transition band, which will cause the phase response in the overlapping band to be nonlinear, which cannot be estimated by the "Three-point" method.

In this letter, a comprehensive estimation method of phase error based on linear fitting was proposed to achieve phase estimation for FI-DAC system with nonlinear phase characteristics. Firstly, the system model of FI-DAC was established to analyze the frequency response, and then the LS was used to linearly fit the segments of the phase function that are less affected by the non-linearity of the analog filter. According to the fitted phase lines, both delay time and LO's phase error of each sub-band can be estimated. Nonlinear phase error can be estimated by subtracting the approximated linear phase responses from the system phase response. Finally, the effectiveness of the proposed estimation method was verified by the built FI-DAC test bench.

System model: To generate a wideband signal far beyond the DAC's bandwidth, multiple DACs with the same performance are used to synthesize a part of the spectrum band of the wideband signal respectively. Therefore, it is necessary to partition the wideband signal into multiple narrow-band signals and down-convert them to baseband that can be synthesized by the single DAC. After digital-to-analog conversion, up-conversion is required to restore the sub-band signal to its original frequency site. The FI-DAC structure as shown in Figure 1, and the frequency response of a single sub-band channel can be expressed as:

$$\begin{aligned} H_m(j\Omega) = \sum_{k=-\infty}^{+\infty} & H_{partm}\left(e^{j\left(\Omega T_s - \frac{2k\pi}{M}\right)}\right) H_{dlm}\left(e^{j\left(\Omega T_s - \frac{2k\pi}{M} - \frac{(m-1)\pi}{M}\right)}\right) \\ & \times H_{DAC}\left(j\left(\Omega - \frac{m-1}{M}\cdot\frac{\Omega_s}{2}\right)\right) \\ & \times H_{abm}\left(j\left(\Omega - \frac{m-1}{M}\cdot\frac{\Omega_s}{2}\right)\right) H_{abm}(j\Omega) \end{aligned} \tag{1}$$

Where, $H_m(j\Omega)$ denotes the frequency response of the m-th sub-band channel, $H_{partm}(j\Omega)$, $H_{dlm}(j\Omega)$, $H_{alm}(j\Omega)$,$H_{abm}(j\Omega)$ denote the frequency response of the spectrum partitioning module, the digital low-pass filter, the analog low-pass filter, the analog band-pass filter of the m-th sub-band channel. Ideally, the frequency response of the m-th sub-band channel should satisfy:

$$H_m(j\Omega) = \begin{cases} C\cdot e^{j\Omega t} & \Omega \in \left[\frac{m-1}{M}\cdot\frac{\Omega_s}{2}, \frac{m}{M}\cdot\frac{\Omega_s}{2}\right] \\ 0 & \text{else} \end{cases} \tag{2}$$

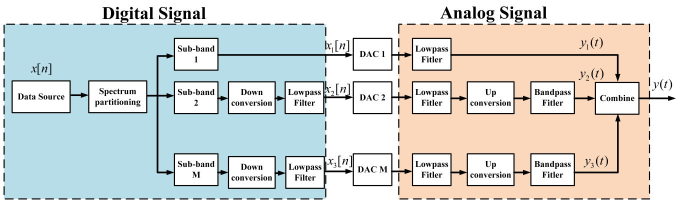

Fig 1 *The structure of FI-DAC system.*

However, the both time delay and the initial phase of each sub-band are different, and the analog filter also causes nonlinear error. Denote that t_m , δ_m , $\delta'_m(\Omega)$ as the time delay, initial phase and the nonlinear error of m-th sub-band channel, respectively. The filter cannot achieve the ideal filtering effect of "brickwork", so there is a transition band on both sides of the sub-band passband (the first sub-band does not have a left transition band). Therefore, the frequency response of m-th sub-band channel can be rewritten as:

$$H_m(j\Omega) = \begin{cases} C\cdot e^{j\varphi_m(\Omega)} & \Omega \in \left[\Omega_m^-, \Omega_m^-\right] \\ 0 & \text{else} \end{cases} \tag{3}$$

Where,

$$\varphi_m(\Omega) = \Omega t_m + \delta'_m(\Omega) + \delta_m \tag{4}$$

$$\Omega_m^- = \frac{m-1}{M}\cdot\frac{\Omega_s}{2} - \Omega_{\text{tranm}}^- \tag{5}$$

$$\Omega_m^+ = \frac{m}{M}\cdot\frac{\Omega_s}{2} + \Omega_{\text{tranm}}^+ \tag{6}$$

Ω_{tranm}^- and Ω_{tranm}^+ denote the left transition band and right transition band, respectively. Therefore, the frequency response of the FI-DAC system is

$$\begin{aligned} H(j\Omega) &= \sum_{m=1}^{M} H_m(j\Omega) \\ &= \begin{cases} C_1(\Omega)\cdot e^{j\varphi_1(\Omega)} & \Omega \in \left[0, \Omega_2^-\right] \\ C_m(\Omega)\cdot e^{j\varphi_m(\Omega)} & \Omega \in \left[\Omega_{m-1}^+, \Omega_{m+1}^-\right] \\ C_M(\Omega)\cdot e^{j\varphi_M(\Omega)} & \Omega \in \left[\Omega_{M-1}^+, \Omega_M^+\right] \\ C_m(\Omega)\cdot e^{j\Omega\varphi_m(\Omega)} + C_{m-1}(\Omega)\cdot e^{j\varphi_{m-1}(\Omega)} & \Omega \in \left[\Omega_m^-, \Omega_{m-1}^+\right] \end{cases} \end{aligned} \tag{7}$$

Estimation method: From Eq. 7, the phase function of FI-DAC can be expressed as:

$$\begin{aligned} &\theta(\Omega) \\ &= \begin{cases} \varphi_1(\Omega) & \Omega \in \left[0, \Omega_2^-\right] \\ \varphi_m(\Omega) & \Omega \in \left[\Omega_{m-1}^+, \Omega_{m+1}^-\right] \\ \varphi_M(\Omega) & \Omega \in \left[\Omega_{M-1}^+, \Omega_M^+\right] \\ \arctan\frac{C_{m-1}(\Omega)\sin\varphi_{m-1}(\Omega)+C_m(\Omega)\sin\varphi_m(\Omega)}{C_{m-1}(\Omega)\sin\varphi_{m-1}(\Omega)+C_m(\Omega)\sin\varphi_m(\Omega)} & \Omega \in \left[\Omega_m^-, \Omega_{m-1}^+\right] \end{cases} \end{aligned} \tag{8}$$

The FI-DAC system consists of multiple sub-band channels with different frequency responses. In the overlapping band, the frequency of the FI-DAC system is jointly determined by two adjacent sub-bands. In the non-overlapping band, the frequency response of the FI-DAC system is equal to the frequency response of the sub-band in this frequency band. Furthermore, the analog filter usually has small nonlinear phase error in the passband , which will not cause serious nonlinear phase distortion to the FI-DAC system in the non-overlapping band$\Omega \in \left[\Omega^{+}_{m-1}, \Omega^{-}_{m+1}\right]$. Therefore, in the passband of each sub-band channel, the phase function is approximately linear, and can be linearly fitted by LS to estimate the time delay t_m and initial phase δ_m of each sub-band. As shown in Figure 2, the dark blue solid line is the phase function of the m-th sub-band, the dark purple solid line is the phase function of the (m+1)-th sub-band, and the dark brown solid line is phase function of the FI-DAC in the overlapping band. The phase function of the sub-band channels in the non-overlapping band (gray area) can be approximated by a straight line, which can be expressed as:

$$\hat{\theta}_m(\Omega) = \hat{t}_m \Omega + \hat{\delta}_m \tag{9}$$

In a system with linear phase, the time delay is equal the group delay of the system, then the time delay of the m-th sub-band channel is equal to the slope of the approximation line:

$$t_m \approx \frac{d\hat{\theta}_m(\Omega)}{d\Omega} = \hat{t}_m \tag{10}$$

The initial phase is the intercept of the approximation line and the phase axis

$$\delta_m \approx \hat{\theta}_m(0) = \hat{\delta}_m \tag{11}$$

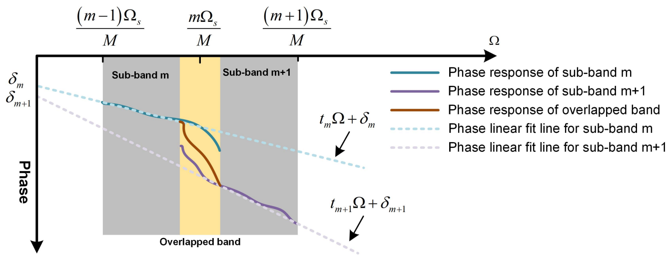

Fig 2 *Diagram of proposed estimation method.*

From Eq. 2, the ideal frequency response of FI-DAC system should have the same time delay at all frequencies and the initial phase of each sub-band should be 0. Therefore, the time delay error Δt_m and initial phase error $\Delta\delta_m$ for each sub-band are:

$$\begin{cases} \Delta t_m = \hat{t}_m - \hat{t}_1 \\ \Delta\delta_m = \hat{\delta}_m \end{cases} \tag{12}$$

The estimation of system's nonlinear phase error first needs to compensate the time delay error and initial phase error between sub-bands. After compensating, the phase function of FI-DAC system should be:

$$\theta_{comp}(\Omega) = \Omega\hat{t}_1 + \hat{\delta}'(\Omega) \tag{13}$$

Where, $\hat{\delta}'(\Omega)$ is the nonlinear phase error of system. $\Omega\hat{t}_1$is known. And $\theta_{comp}(\Omega)$ is the phase function of system after compensation, that can be measured.

Linear fit: This section describes how to measure the phase response of the system and make a linear approximation to the phase response of each sub-band. The square wave signal has equally interval frequency components in the frequency domain make it suitable for the measurement of the phase response of the system. The frequency domain expression of the square wave signal with frequency Ω_{test} and amplitude A is

$$S(j\Omega) = \frac{4A}{j}\sum_{k=1}^{\infty}\frac{1}{(2k-1)}\left[\delta\left(\Omega-(2k-1)\Omega_{\text{test}}\right) - \delta\left(\Omega+(2k-1)\Omega_{\text{test}}\right)\right] \tag{14}$$

The frequency domain expression of the output signal after the square wave signal passes through the system is

$$\begin{aligned} &Y(j\Omega) = S(j\Omega)\cdot H(j\Omega) \\ &= \frac{4A}{j}\sum_{k=1}^{\infty}\frac{1}{(2k-1)}\left[H\left(\Omega-(2k-1)\Omega_{\text{test}}\right) - H\left(\Omega+(2k-1)\Omega_{\text{test}}\right)\right] \\ &= \sum_{k=1}^{\infty}\frac{4A}{(2k-1)}\left[H\left(\Omega-(2k-1)\Omega_{\text{test}}\right)e^{j-\frac{\pi}{2}} + H\left(\Omega+(2k-1)\Omega_{\text{test}}\right)e^{j\frac{\pi}{2}}\right] \end{aligned} \tag{15}$$

Therefore, in positive axis, the difference between the phase of output signal and the system is only a fixed value $-\frac{\pi}{2}$. The phase of the output signal at the frequency point $\mathbf{\Omega} = [\Omega_{\text{test}}, 3\Omega_{\text{test}}, 5\Omega_{\text{test}}, \cdots, (2K-1)\Omega_{\text{test}}]$ can be get via discrete Fourier transform (DFT), denoted as $\boldsymbol{\theta}_{\text{test}}(\mathbf{\Omega})$ .The K is the largest integer that satisfies $(2K-1)\Omega_{\text{test}} < \Omega_s$.Within the passband of the sub-band channel, the linear fit of the phase function can be transformed into a LS optimization as follows:

$$\begin{aligned} &\min \sum_{\Omega\in[\Omega^{+}_{m-1},\Omega^{-}_{m+1}]\cap\Omega\in\mathbf{\Omega}} \left\|\hat{t}_m\Omega + \hat{\delta}_m - \boldsymbol{\theta}_{\text{test}}(\Omega) + \frac{\pi}{2}\right\|^2 \\ &\text{s.t.} \ -\pi < \hat{\delta}_m < \pi \end{aligned} \tag{16}$$

Experimental results: To verify the performance of the proposed phase estimation method in real circuit. We built a dual-channel FI-DAC system. The setup as shown in Figure 3, is utilized and the experimental test bench as shown in Figure 4 below. For the D/A conversion, Tektronix AWG5014B with 14 bits vertical resolution was used at a sampling rate of 900 MSPS provided by Analog Signal Generator (Keysight N5173B). The Low-pass Filters are Mini-Circuits SLP-450 with cut-off frequency of 440 MHz. The mixer Mini-Circuits ZFM-150-S is driven by the same signal from N5173B. The band-pass filter is combined by a low-pass filter SLP-850 and a high-pass filter SHP-500. Finally, the signals of the first and the second signal path are combined by means of the passive power combiner Mini-Ciruit ZX10R-14and acquired with the Tektronix Mixed Signal oscilloscope (MSO6B series). The digital signal part is realized by using MATLAB on personal computer (PC).

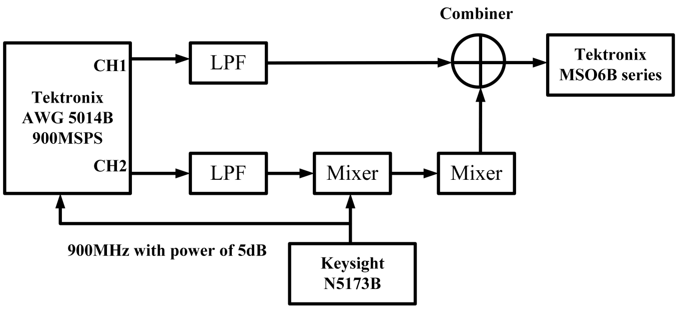

Fig 3 *Experimental setup.*

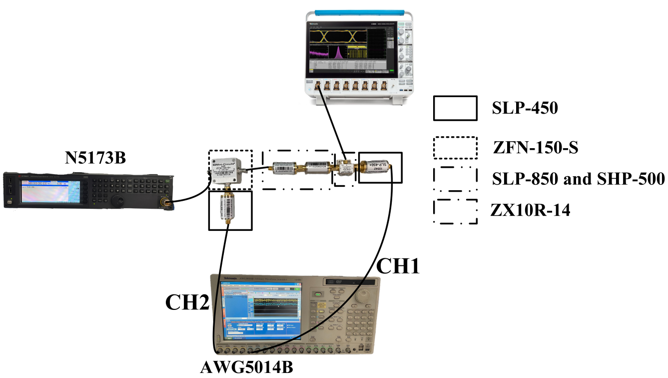

Fig 4 *The experimental test bench.*

The sampling rate of a single channel is 900 MSPS, so that the sampling rate of the built dual-channel FI-DAC system is 1.8 GSPS,

Table 1. The phase error between two sub-bands estimated by proposed method

Error setting	$\Delta t = 0$ $\Delta\delta = 0$	$\Delta t = 0$ $\Delta\delta = 1.0472$rad	$\Delta t = 3.5$samples $\Delta\delta = 0$	$\Delta t = 3.5$samples $\Delta\delta = 0.7854$rad	$\Delta t = 6$samples $\Delta\delta = 0$	$\Delta t = 6$samples $\Delta\delta = 1.5708$rad
Time delay estimation error	7.9554 samples	7.9648 samples	11.5179 samples	11.5017 samples	14.0243 samples	14.0119 samples
Initial phase estimation error	-0.0111 rad	1.0421 rad	-0.0117 rad	0.7819 rad	0.0015 rad	1.5664 rad

and the output bandwidth should be 900 MHz. We set the frequency of the square wave signal used for testing as 9 MHz, then the frequency components are at $\mathbf{\Omega} = [9, 27, 45, \ldots, 18(2k-1), \ldots, 873, 891]$ MHz. By consulting the datasheet of the used analog filters, components at $[459, 477, 495, 513, 531, 549]$ MHz are in the overlapping band. Therefore, each sub-band channel has 22 frequency components for estimation, $[9, 27, 45, \ldots, 441]$ MHz for sub-band 1, and $[567, 585, 603, \ldots, 897]$ MHz for sub-band 2. With the proposed estimation method, the delay time error between two channels is 7.9554 samples, and the initial phase error is -0.0111 rad . To verify the accuracy of the estimation method, we added known time delay ($\Delta t = t_2 - t_1$) and initial phase offset ($\Delta\delta = \delta_2 - \delta_1$) to sub-band 2. The phase error between two sub-bands estimated by proposed method is shown in Table 1.

After estimating the delay time error and initial phase error between sub-band channels, the error can be compensated by digital finite impulse response (FIR) filter. And then, the nonlinear phase error of FI-DAC system can be estimated. According to [5], the nonlinear phase error can be compensated by digital all-pass filter. We compensated the built FI-DAC system according to the phase error estimated by the proposed method, and used a square wave signal with a frequency of 45 MHz to verify the performance of system. As shown in Figure 5, in the FI-DAC system without phase compensation, the square wave signal is seriously distorted due to the phase error, and its rise time is 1.254 ns. As shown in Figure 6, in the FI-DAC system with phase compensation, the quality of square wave signal has been greatly improved, and its rise time is 600.2 ps. The relationship between rise time and bandwidth is as follow:

$$BW = \frac{N}{t_{rise}} \tag{17}$$

Where, BW is the bandwidth of system, t_{rise} is the rise time, and N is a fixed value. After the phase compensation, the rise time of output was reduced by half, which proves the effective bandwidth of the dual-channel FI-DAC system is increased by 2 times. Therefore, the experimental results show that the proposed phase estimated method can accurately estimate the phase error between the sub-band channels.

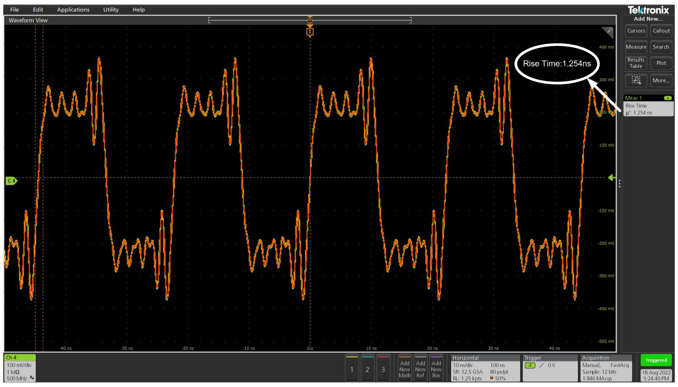

Fig 5 *The output square signal by FI-DAC system without compensation.*

Conclusion: This letter proposed a comprehensive estimation method based on linear fitting for the phase error estimating, which are caused by the delay time error, initial phase error, nonlinear error between the sub-band channels in FI-DAC system. And a FI-DAC test bench was built to verify the accuracy and effectiveness of the proposed estimation method.

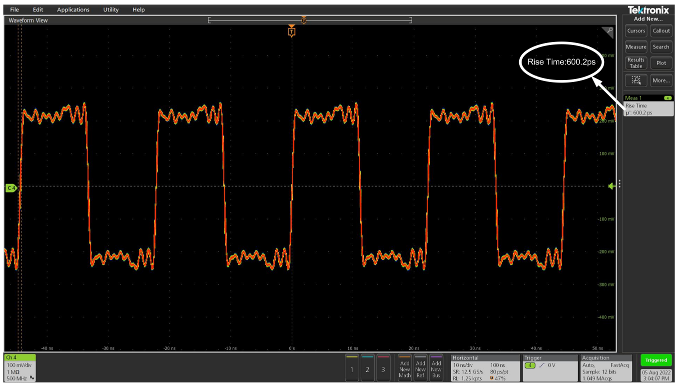

Fig 6 *The output square signal by FI-DAC system with compensation.*

After compensating the estimated phase error by digitally method, the output signal quality was greatly improved.

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