

# A multi-fidelity framework for ocean temperature reconstruction based on model-inferred dynamics and real time satellite and buoy measurements

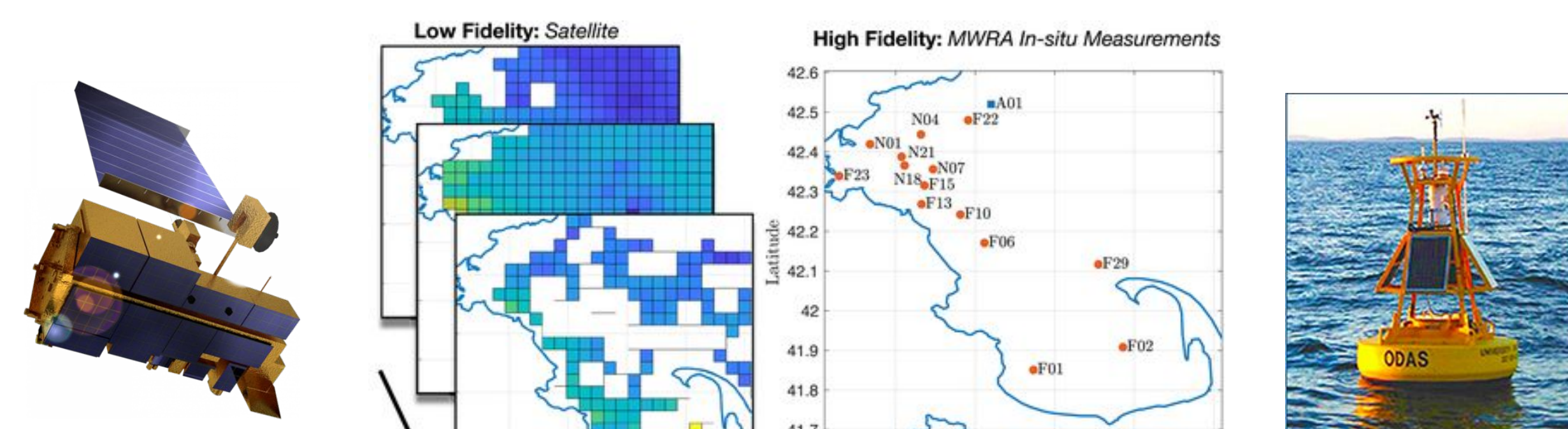


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## Objective

Estimate a full 3D temperature field of the ocean using a combination of numerical simulations and physical sensors.

## Background

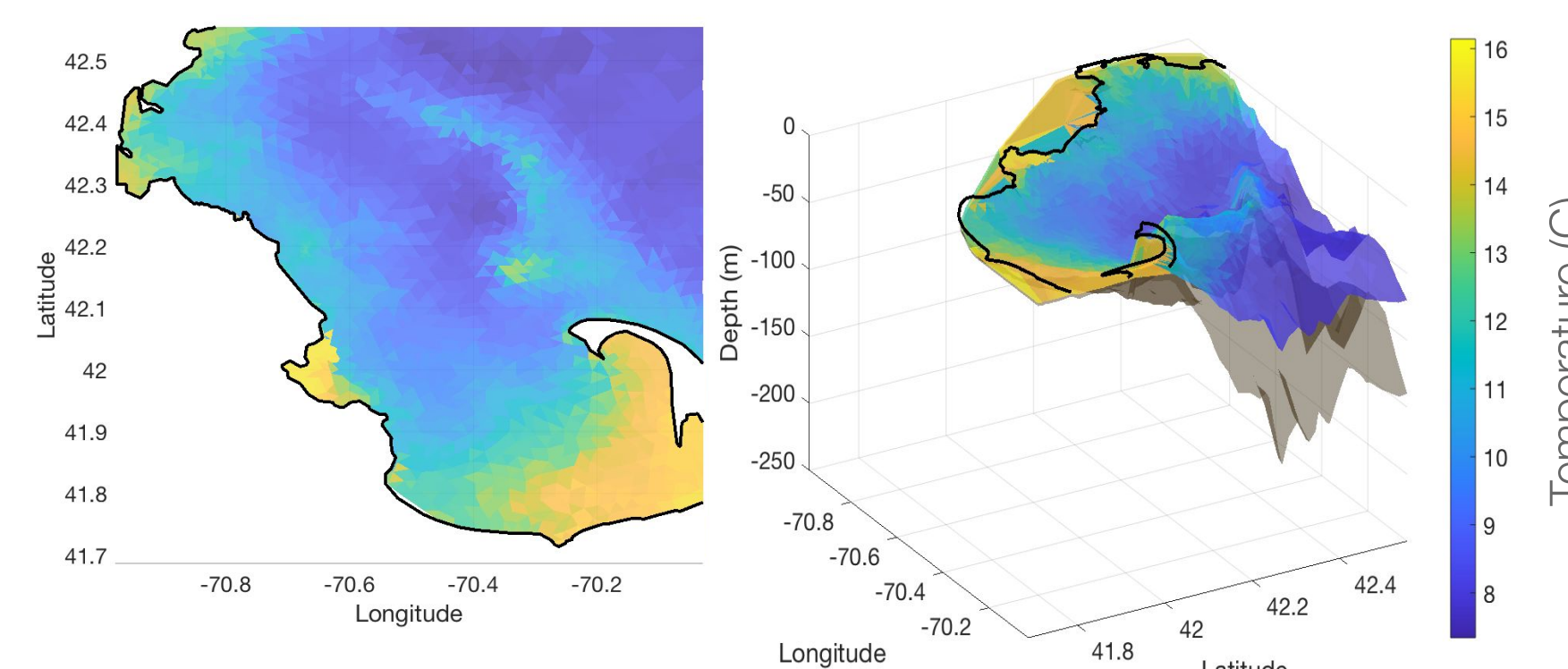


NASA Terra MODIS

MWRA NERACOOS Buoy

Babaei et al. 2020

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \frac{\partial}{\partial z} \left( K_h \frac{\partial T}{\partial z} \right) + F_T$$



FVCOM Reanalysis Data

Chen et al. 2003

## Techniques

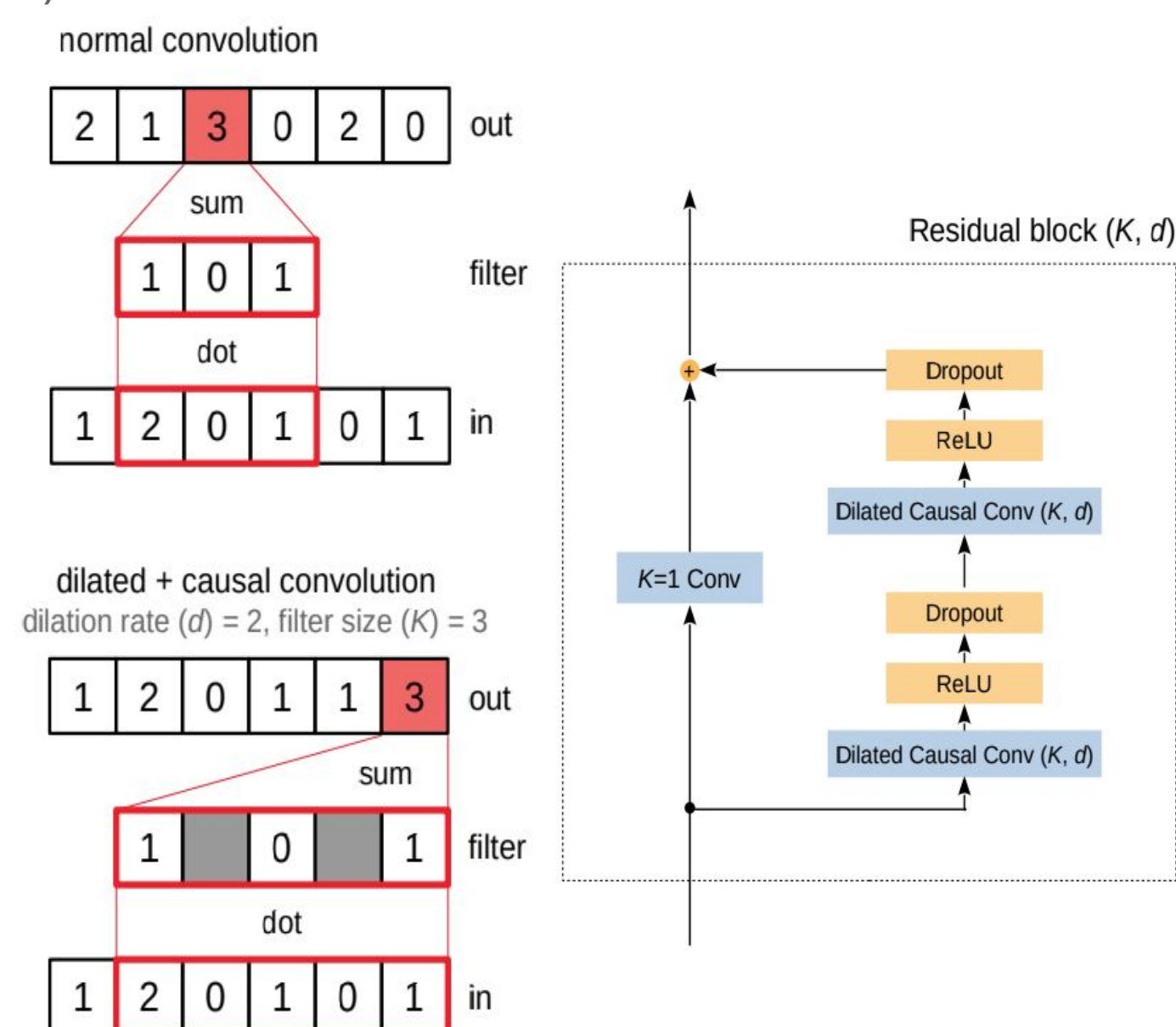
Principal Component Analysis (PCA)

$$\mathbf{T}_{proj}(\cdot, t) = \sum_{i=1}^2 q_i(t) \phi_i + \bar{\mathbf{T}}(t)$$

Gaussian Process Regression (GPR)

$$\bar{\mathbf{f}}(\mathbf{x}_*) = \mathbf{K}(\mathbf{X}_*, \mathbf{X}) [\mathbf{K}(\mathbf{X}, \mathbf{X}) + \sigma_n \mathbf{I}]^{-1} \mathbf{y}$$

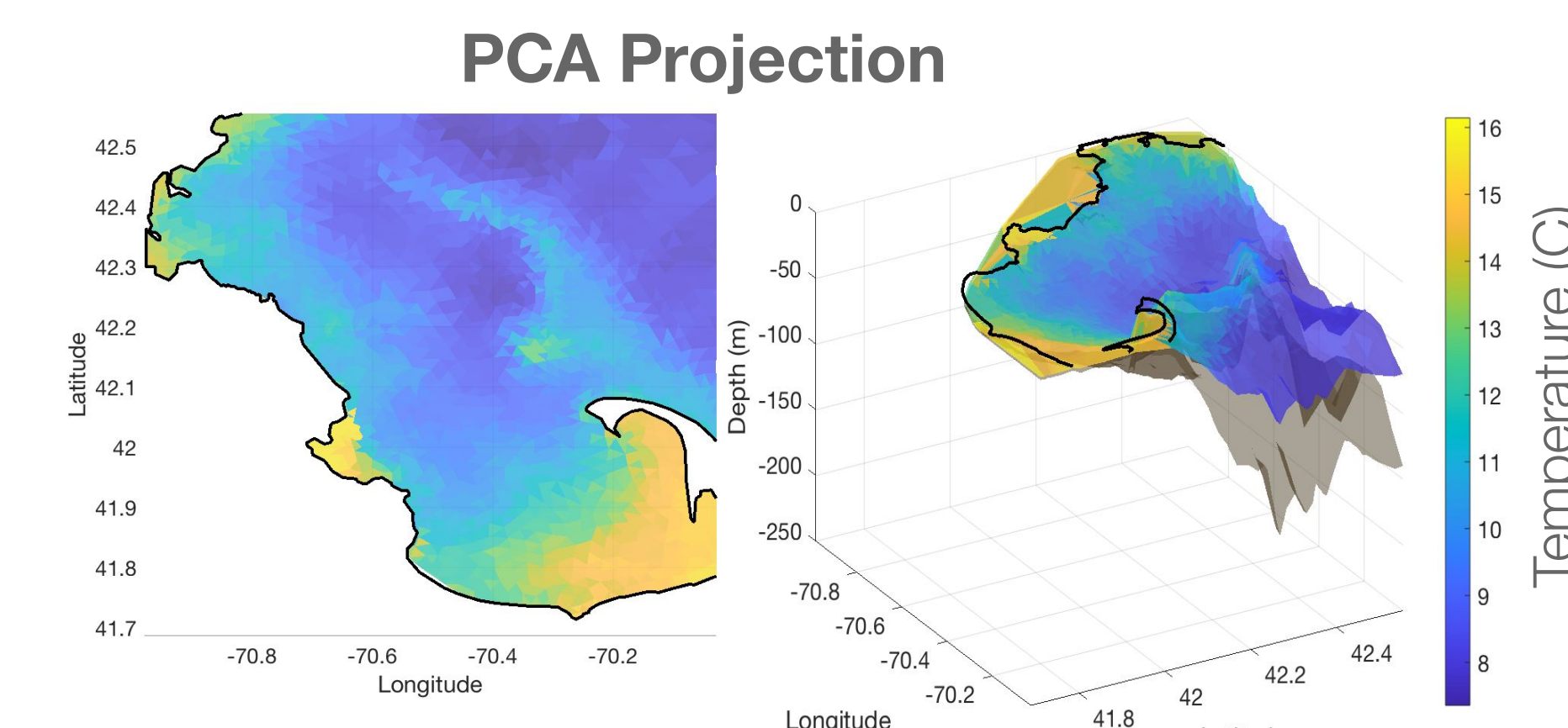
Temporal Convolutional Network (TCN)



Wan et al. 2020

## Framework

**Step 1:** Apply PCA to Each (x,y) Point of the FVCOM Data



$$\mathbf{T} = \begin{bmatrix} T(z_1, t_1) & \dots & T(z_1, t_m) \\ T(z_2, t_1) & \dots & T(z_2, t_m) \\ \vdots & \vdots & \vdots \\ T(z_n, t_1) & \dots & T(z_n, t_m) \end{bmatrix}$$

only keep the first two modes

$$\mathbf{T}_{proj}(\cdot, t) = \sum_{i=1}^2 q_i(t) \phi_i + \bar{\mathbf{T}}(t)$$

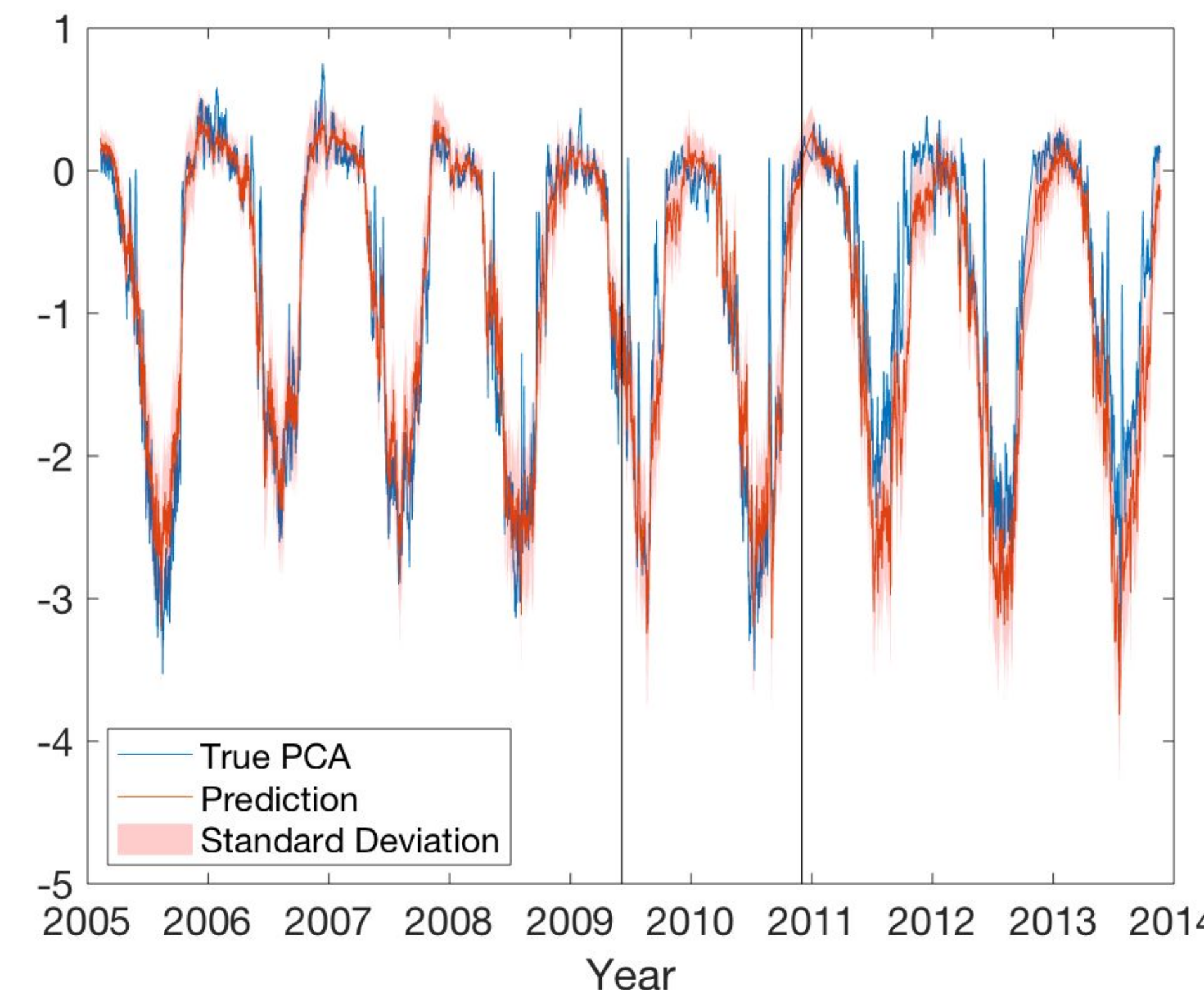
PCA coefficients

PCA modes

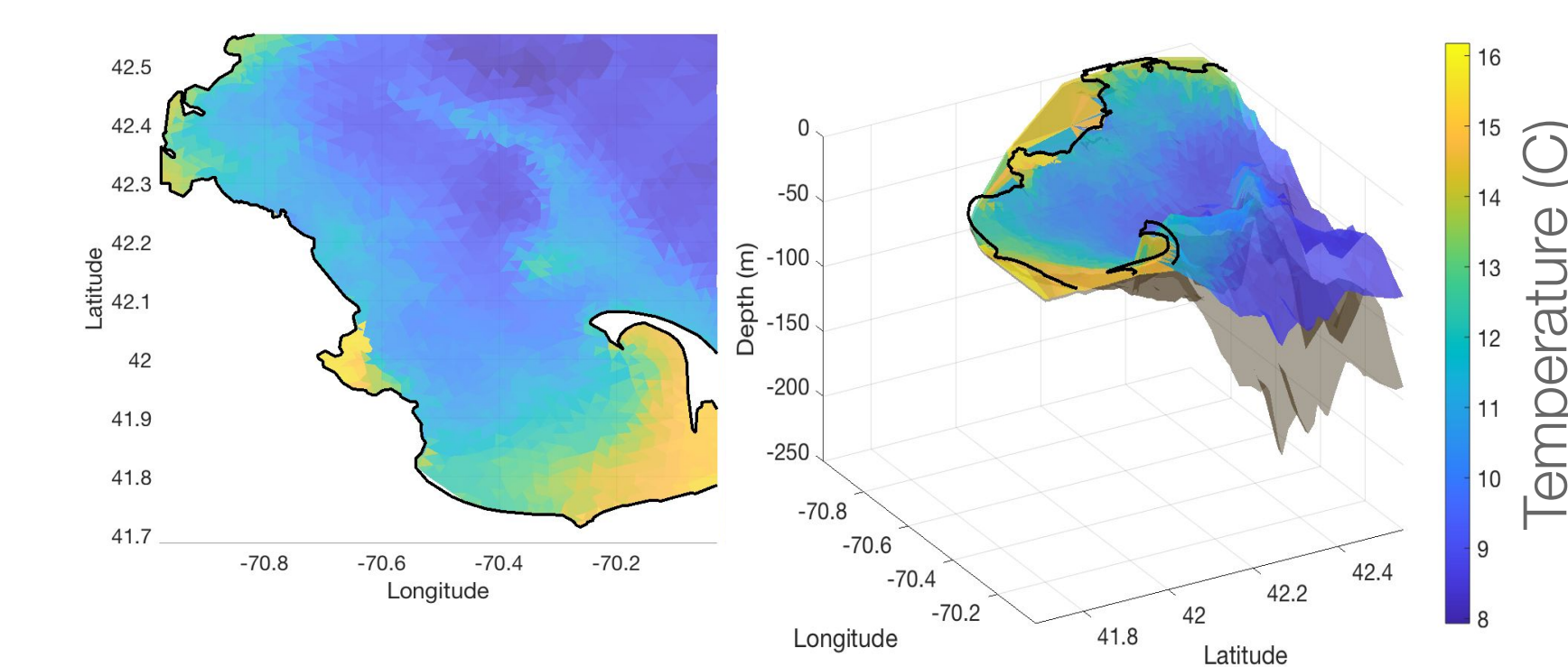
Holmes et al. 1996

**Step 2:** Build a TCN to Predict the PCA Coefficients as a Function of Surface Temperature

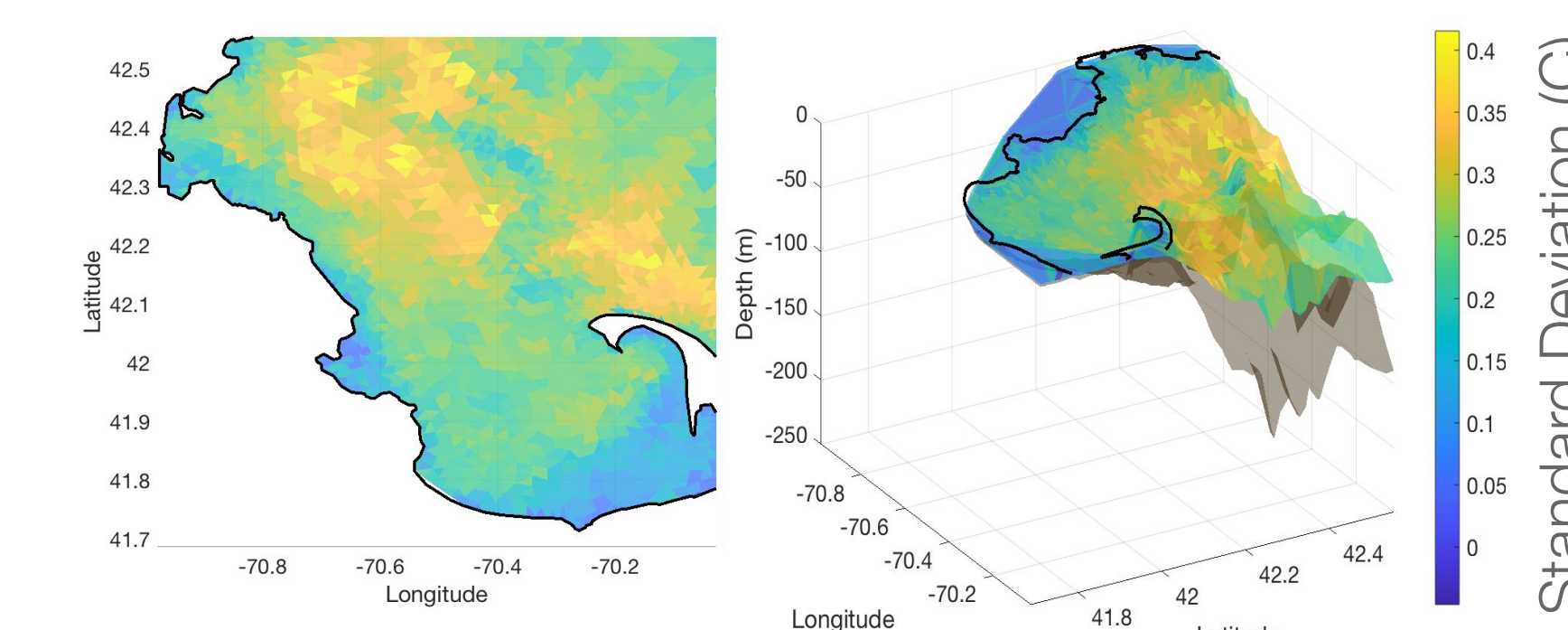
First PCA Coefficient Prediction



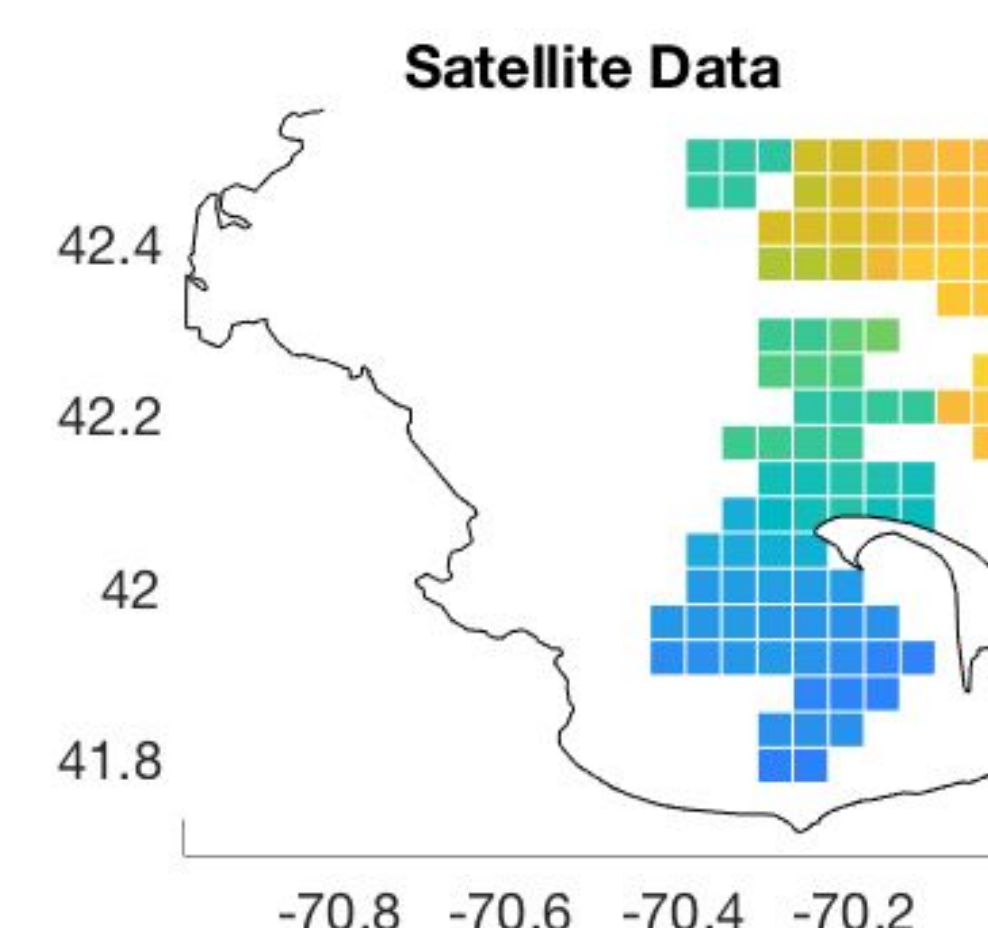
TCN Mean Prediction



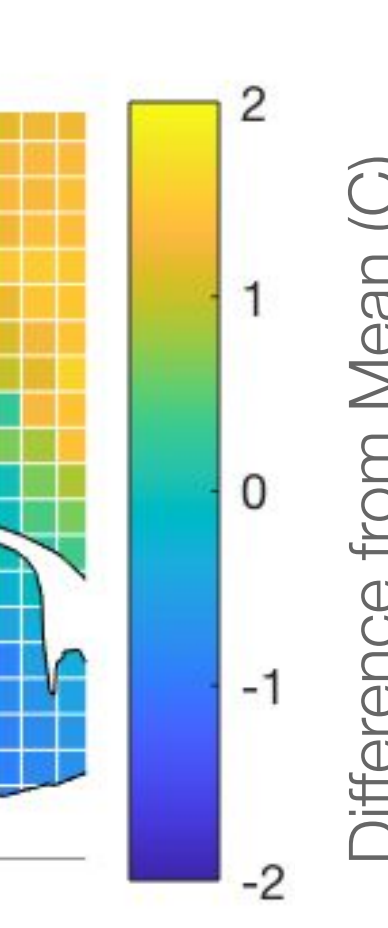
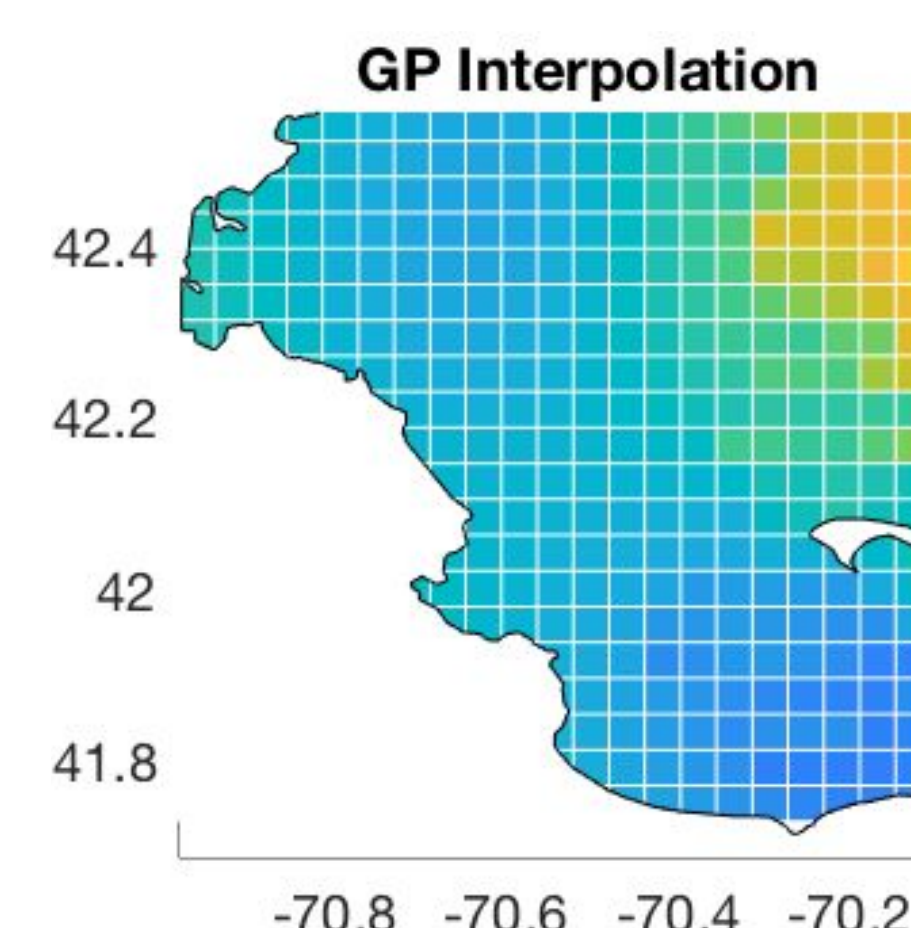
TCN Standard Deviation Prediction



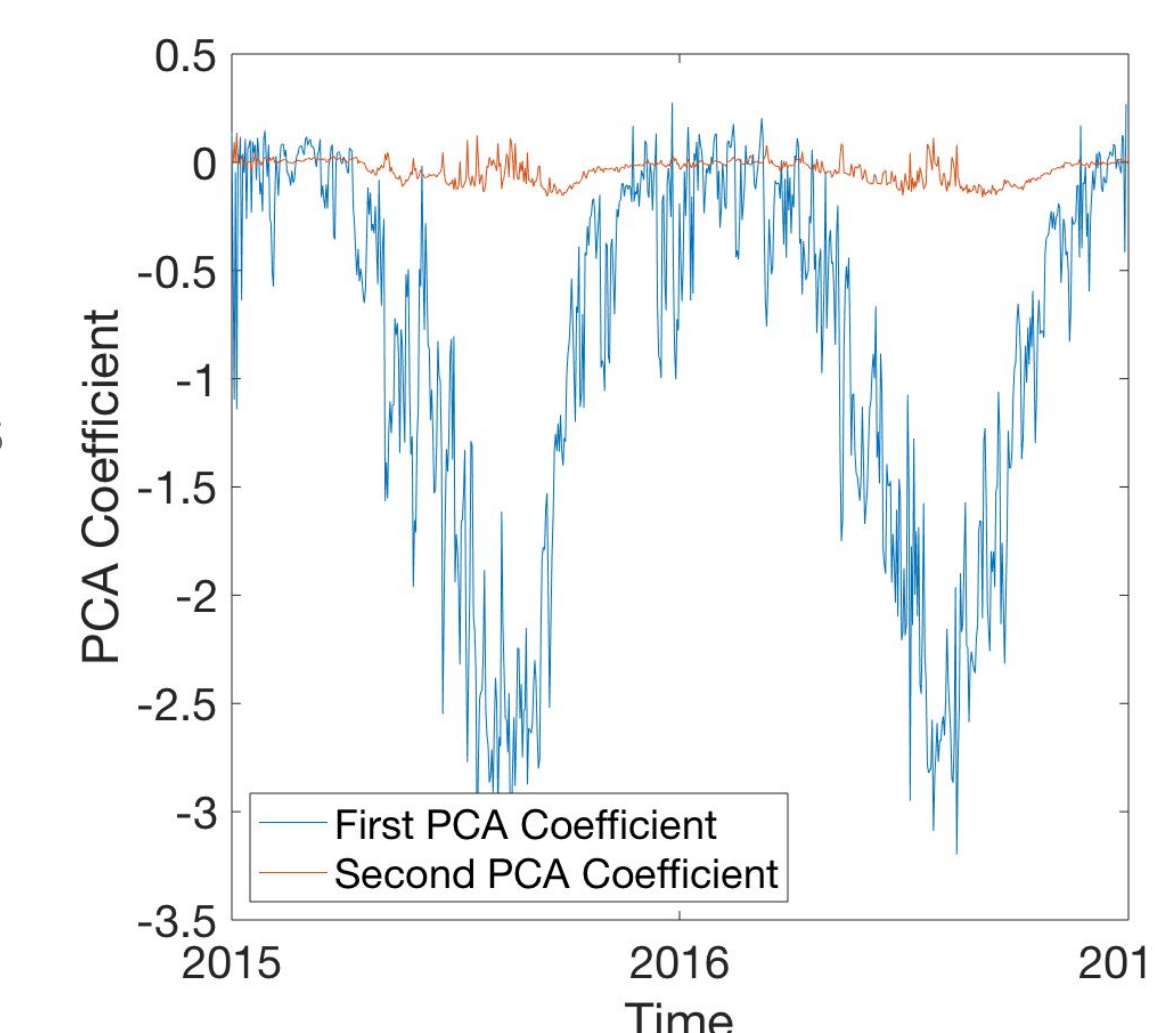
**Step 3:** Predict the PCA Coefficients from the Satellite Data to Reconstruct the 3D Field



interpolate with GPR



predict coefficients



**Step 4:** Build a Multi-Fidelity GPR Model Using the TCN Predictions and Buoy Data

$$\bar{\mathbf{f}}_1(\mathbf{x}_*) = \mathbf{K}(\mathbf{X}_*, \mathbf{X}_1) [\mathbf{K}(\mathbf{X}_1, \mathbf{X}_1) + \sigma_{n1} \mathbf{I}]^{-1} \mathbf{y}_1$$

$$\text{cov}(\bar{\mathbf{f}}_1) = \mathbf{K}(\mathbf{X}_*, \mathbf{X}_*) - \mathbf{K}(\mathbf{X}_*, \mathbf{X}_1) [\mathbf{K}(\mathbf{X}_1, \mathbf{X}_1) + \sigma_{n1} \mathbf{I}]^{-1} \mathbf{K}(\mathbf{X}_1, \mathbf{X}_*)$$

$$\bar{\mathbf{f}}_2(\mathbf{x}_*) = \rho \bar{\mathbf{f}}_1(\mathbf{x}_*) + \mu_d + \mathbf{K}(\mathbf{X}_*, \mathbf{X}_2) [\mathbf{K}(\mathbf{X}_2, \mathbf{X}_2) + \sigma_{n2} \mathbf{I}]^{-1} (\mathbf{y} - \rho \bar{\mathbf{f}}_1(\mathbf{x}_2) - \mu_d)$$

$$\text{cov}(\bar{\mathbf{f}}_2) = \rho^2 \text{cov}(\bar{\mathbf{f}}_1) + \mathbf{K}(\mathbf{X}_*, \mathbf{X}_*) - \mathbf{K}(\mathbf{X}_*, \mathbf{X}_2) [\mathbf{K}(\mathbf{X}_2, \mathbf{X}_2) + \sigma_{n2} \mathbf{I}]^{-1} \mathbf{K}(\mathbf{X}_2, \mathbf{X}_*)$$

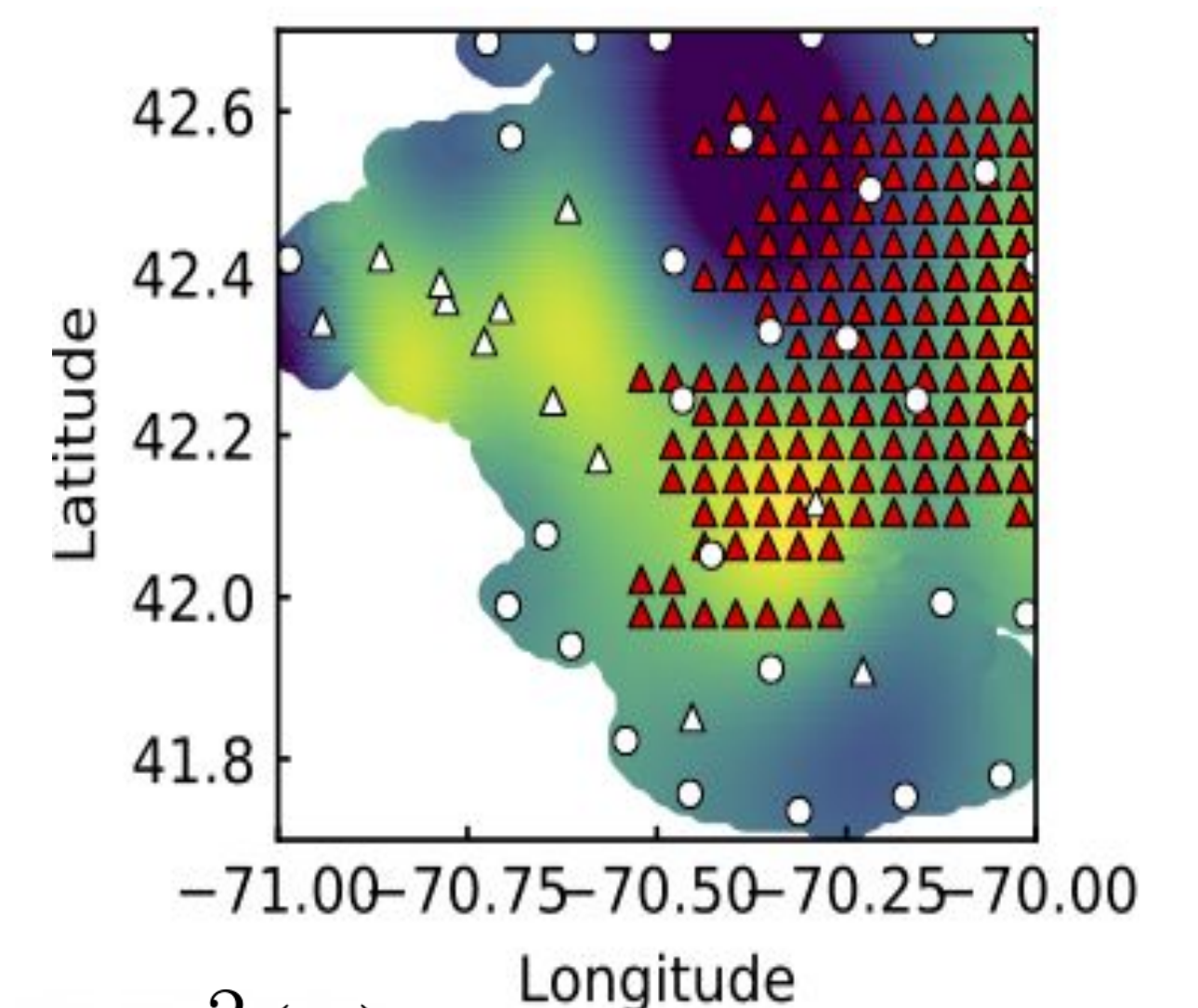
low fidelity

high fidelity

Rasmussen and Williams 2004

## Future Work

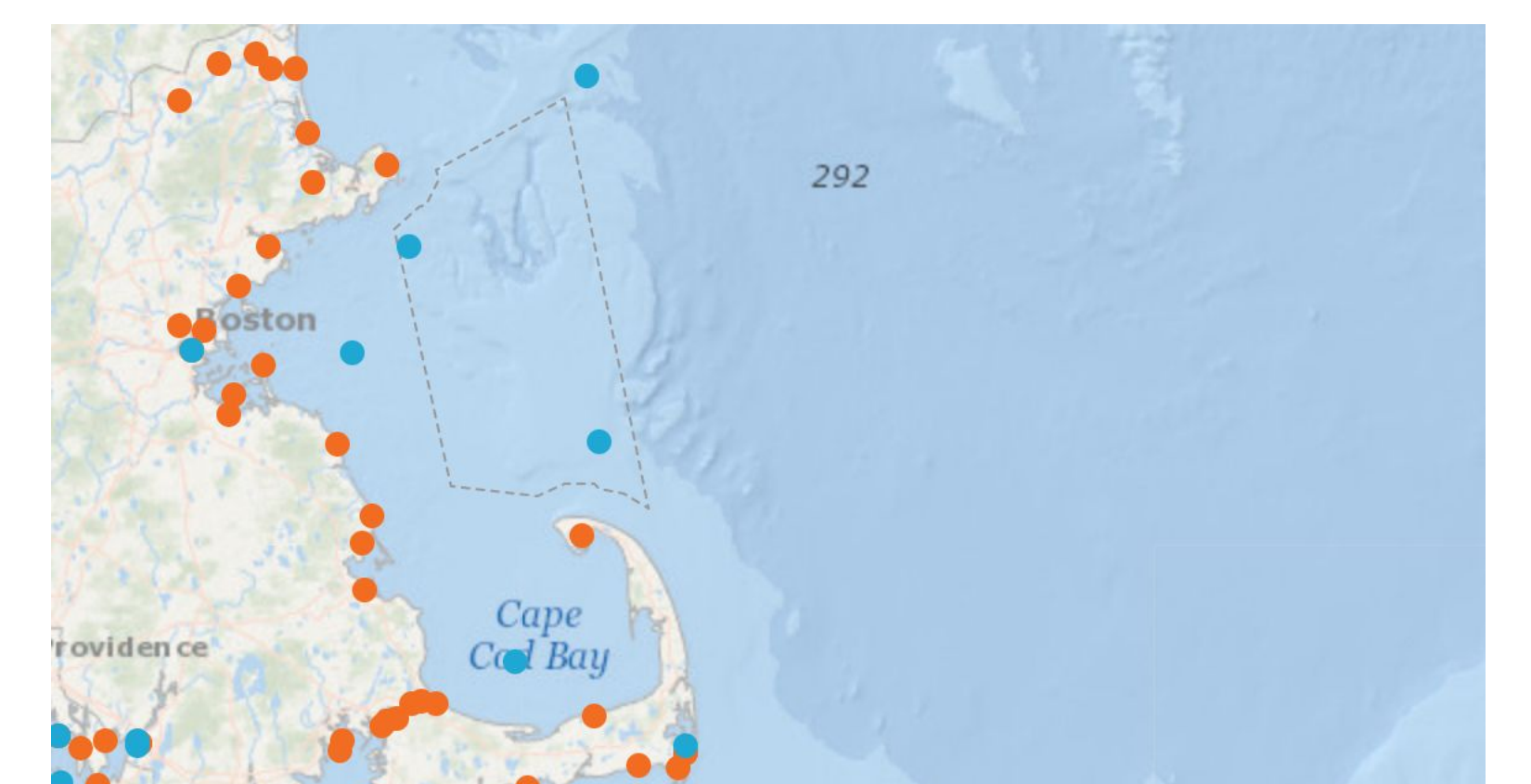
**Optimal Sampling and Path Planning**



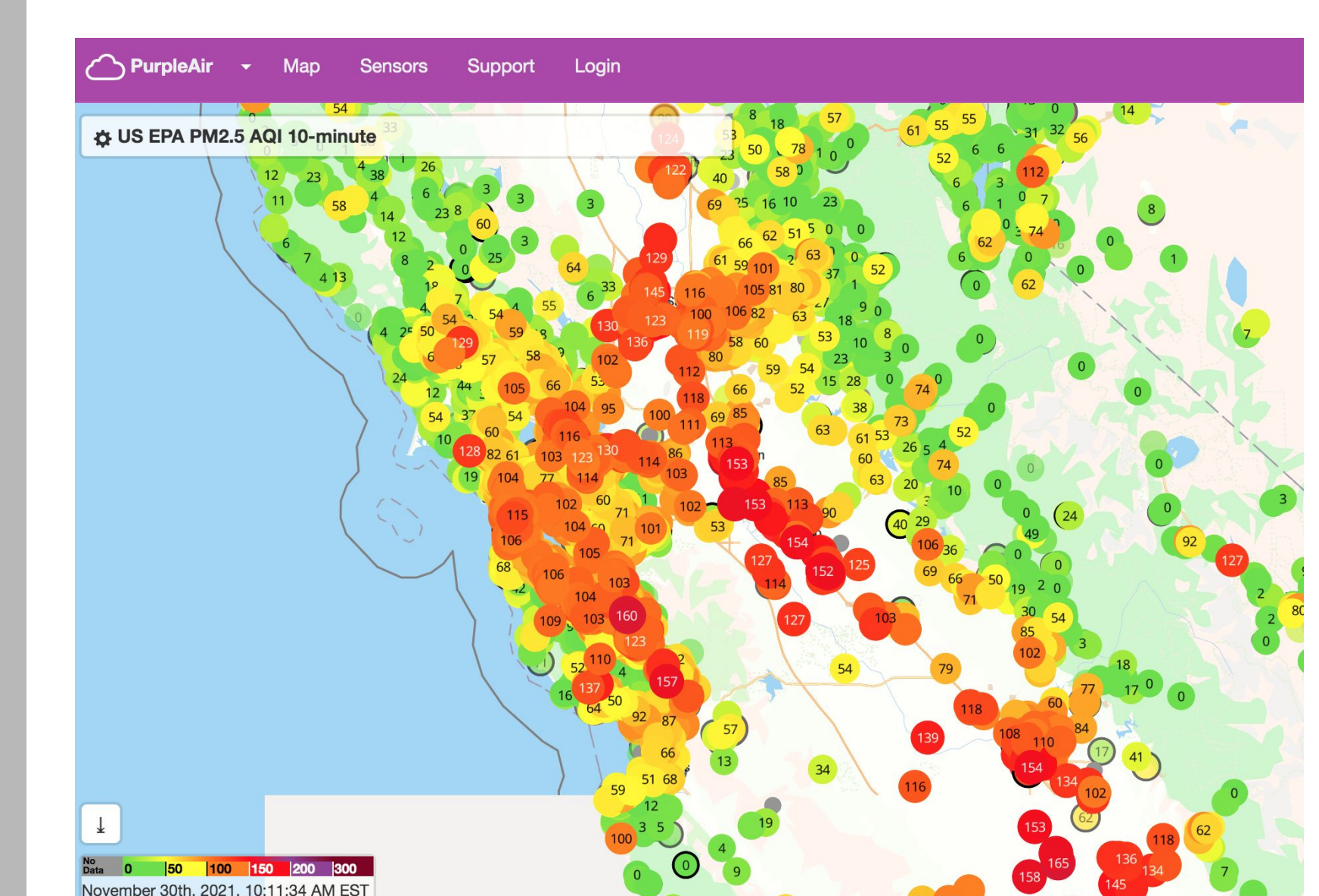
$$a_{US}(x) = \sigma^2(x)$$

Blanchard and Sapsis 2020

**Incorporate Results Into SEAGLASS**



## Other Applications



## Acknowledgements

This material is based upon work supported by the National Science Foundation Graduate Research Fellowship under Grant No. 1745302, MIT Sea Grant, and the Harrington Fellowship. Thank you to Dr. Carolina Bastidas and Michael Defilippo for the data.