Numerical Simulation Research Lab

University of Saskatchewan

The SUMMA–SUNDIALS Hydrological Model

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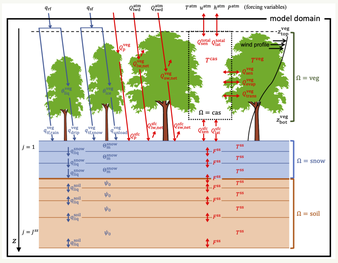
Motivation

- Upcoming Earth System models promise unmatched predictive power.
- A key component to the accuracy, efficiency, and robustness of Earth System simulations is the time integration of differential equations describing the physical processes.
- Many existing Earth System models use low-order, constant-stepsize time-integration methods with no error control.

What is SUMMA?

The Structure for Unifying Multiple Modelling Alternatives (SUMMA) model is a software framework that enables the exploration and analysis of different hydrological models and their constituent components in a systematic way[1].



What is SUNDIALS?

The SUite of Nonlinear and DIfferential/ALgebraic equation Solvers (SUNDIALS) is a set of six software librairies that provide robust numerical solutions and sensitivity analysis for nonlinear algebraic equations and ordinary and differential-algebraic equations[2].

Experiments

- 517,315 basins in North America (median 33 km^2; mean 40 km^2)
- Time span: 1986-01-01 00:00 to 1986-12-31 23:00; hourly forcing.
- SUMMA running backward Euler (BE) with constant step $\Delta t/64$ (BE64) is taken as the ground truth.

Results

- BE with constant step size Δt requires less computation time than SUNDIALS.
- SUNDIALS is generally more accurate than BE.

Conclusions

- One-off time integration with low-order methods and constant step-sizes is fraught with risk of inaccuracy and inefficiency.
- State-of-the-art time-integration software with error control generally produces accurate solutions in an efficient manner and can be applied on large domains where manual tuning is infeasible.

Future Work

- Global simulations.
- Adaptive error control in space.
- Fault tolerance and automatic restart.
- Dynamic load balancing.

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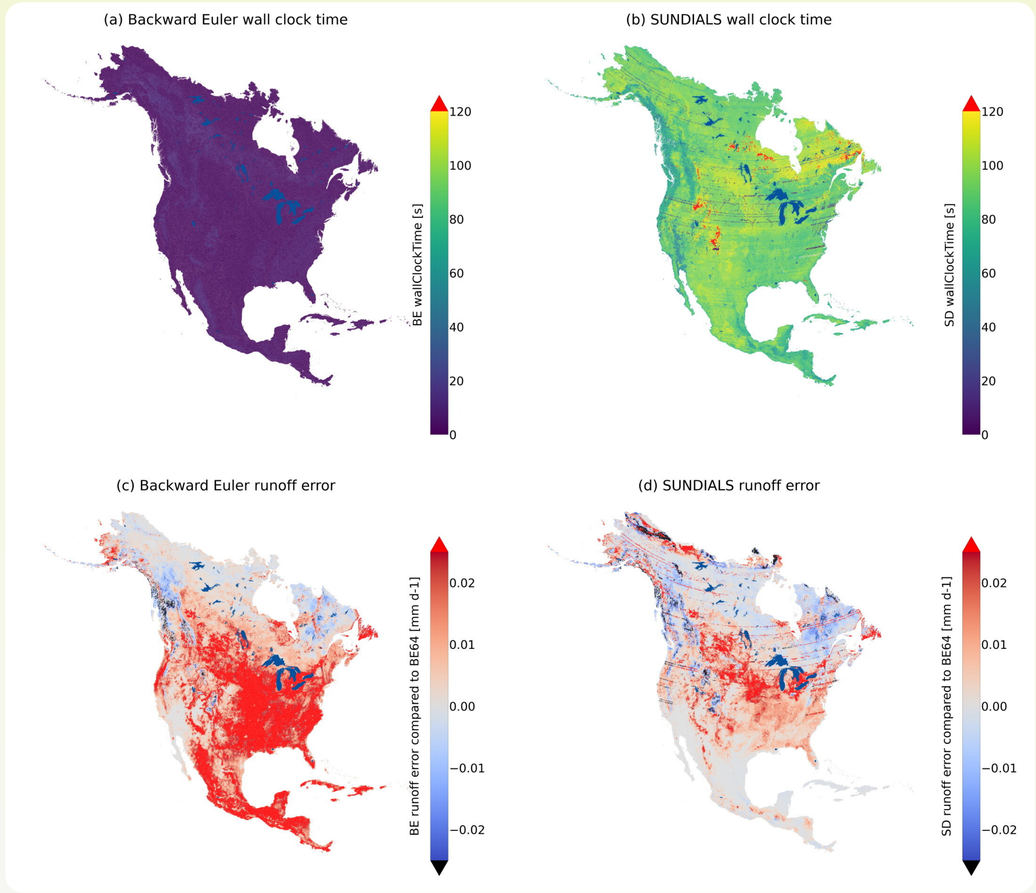


Details: SUMMA

SUMMA solves coupled conservation equations for thermodynamics and hydrology for the sub-domains of the canopy air space (cas), vegetation (veg), snow, and soil. For a given model sub-domain Ω, the state equations for thermodynamics and hydrology can be written as

$$\frac{\partial H^\Omega}{\partial t} = -\frac{\partial F^\Omega}{\partial z} + \mathcal{F}^\Omega_{\text{sink}}, \qquad \Omega = \text{cas, veg, snow, soil,} \tag{1a}$$

$$\frac{\partial \Theta^\Omega_{\text{m}}}{\partial t} = -\frac{\partial q^\Omega_{\text{ice}}}{\partial z} - \frac{\partial q^\Omega_{\text{liq}}}{\partial z} + \mathcal{M}^\Omega_{\text{sink}}, \qquad \Omega = \text{veg, snow, soil.} \tag{1b}$$

Details: SUNDIALS

Equations (1) are discretized vertically in space à la method of lines.
Time integration of the resulting system is performed using the IDA package from SUNDIALS that solves initial-value problems for implicit ordinary and differential-algebraic equations of the form $\mathbf{f}(t, \mathbf{y}, \dot{\mathbf{y}}) = \mathbf{0}$ via variable-stepsize, variable-order backward differentiation formulas.

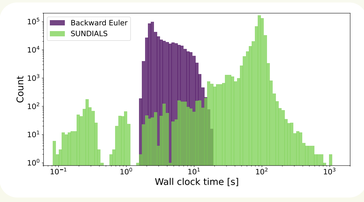
Details: Experiments

Model configuration was generated through the Community Workflows to Advance Reproducibility in Hydrologic Modeling (CWARHM) project:
`https://github.com/CH-Earth/CWARHM`.

Details: Results

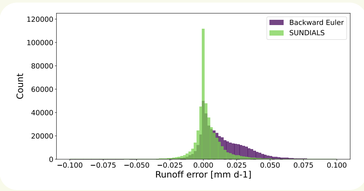
Distribution of runtimes

- Note logarithmic scales.
- Many SUNDIALS runs are much slower than BE.



Distribution of errors

- Many BE runs have larger errors than SUNDIALS.
- BE runs are biased to overestimate runoff.



References

1. Clark, M. P., et al. (2015), A unified approach for process-based hydrologic modeling: 1. Modeling concept, Water Resour. Res., **51**, 2498– 2514, Research, doi:10.1002/2015WR017198.
2. Hindmarsh, A. C., et al. (2005), SUNDIALS: Suite of nonlinear and differential/algebraic equation solvers. ACM Trans. Math. Softw. **31**, 3, 363–396. doi:10.1145/1089014.1089020