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Supporting Information for

The Determination of the Rotational State and Interior Structure of Venus with VERITAS

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Text S1: Effect of the number of landmarks.

To explore the effectiveness of the inclusion of radar tie points, we performed a sensitivity analysis of the results to the number of included landmarks.
In Figure S1 we report the formal uncertainty improvement factor as a function of the number of observed landmark. Not surprisingly, the improvement factor P depends on the number of landmarks n as:

$$P(n) \sim n^{\frac{1}{2}}$$

A consequence of the assumption that the measurements are statistically independent. The results that we report can be easily scaled to an arbitrary higher number of landmarks.
The increase in the accuracy of the rotational parameters, MOIF and k_2 shows that, while the bulk of the information matrix comes from radio tracking data, tie points, being a largely independent data set, increase the overall information content by a quite significant amount.

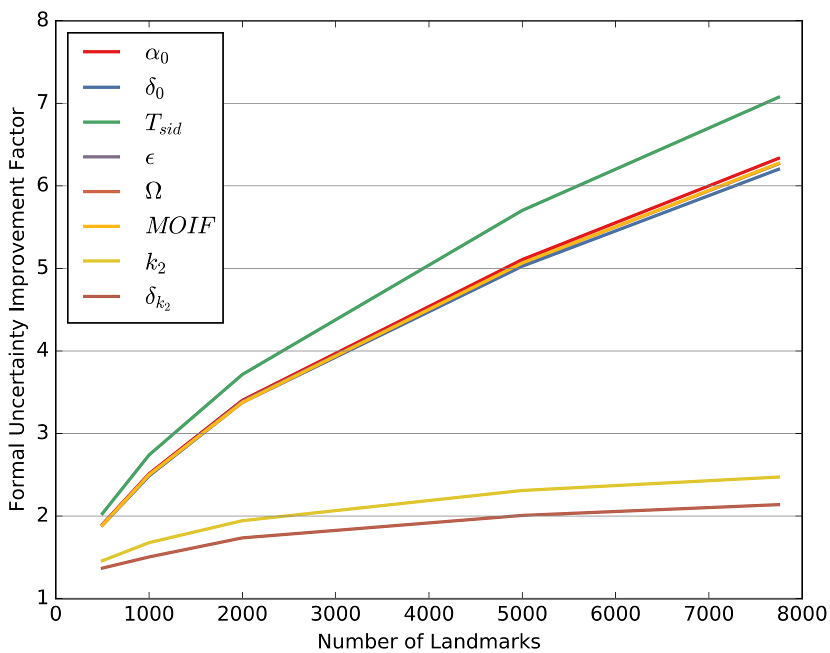

Figure S1 Tie points improvement factor as a function of the number of observed landmarks for the physical quantities object of our analysis

Text S2: Thermal tides modeling.

In the dynamical model used in our simulations we included the effect of atmospheric thermal tides. As shown by Bills et al., (2020) the mass transport induced by solar heating of the atmosphere is not a negligible factor for high precision radioscience experiments at Venus. For a realistic assessment of the attainable accuracies of VERITAS, we modeled the time-variable gravity field induced by solar heating-driven pressure variations of the atmosphere.

The spherical harmonics expansion of the total (static plus atmosphere) gravity field can be written as a function of time t as:

$$C_{lm}(t) = C_{lm}^{S} + \Delta C_{lm}(t) \qquad \text{(S2.1)}$$

C_{lm} is the total C coefficient of degree l and order m of the gravity field, C_{lm}^{S} is the static coefficient and $\Delta C_{lm}(t)$ is the correction due to the time variable mass transport (the same formulation applies for S_{lm} coefficients, here omitted for brevity).

To determine the time variable atmospheric contribution, we employed the model developed by Garate-Lopez and Lebonnois (2018) for retrieving surface pressure variations induced by solar heating and then converted these perturbations in the associated gravity field coefficient with a technique originally developed for Earth (Petrov, 2004) and applied also on Mars (Genova et al., 2016)). This procedure produces the time series of spherical harmonics expansions of the atmospheric gravity field. The gravity field perturbation induced by solar heating is a periodic signal of fundamental frequency f_1, equal to the main forcing effect (i.e. Venus solar day $\sim 116\ days$). We isolated the contribution of the fundamental frequency and its 3 first harmonics via a Fourier transform. Thus, we can expand equation (S2.1) as:

$$C_{lm}(t) = C_{lm}^{S} + \sum_{n} \Delta C_{lm}^{n}(t)$$

$$\Delta C_{lm}^{n} = A_{C_{lm}}^{n} \cos(2\pi f_n t) + B_{C_{lm}}^{n} \sin(2\pi f_n t)$$

Where $f_n = n f_0$ with $n = 1,2,3,4$ and A, B are coefficients derived from the Fourier analysis specific for each coefficient, degree and order.

In our simulations we assessed the necessity of including these effects in the dynamical model of VERITAS as its extremely precise tracking system is sensitive to the main components of the thermal tides perturbation. In particular, we have assessed that if thermal tides are not accounted for, significant biases might arise in the gravity field and rotational state solution, in particular affecting the Love number k_2. The most recent works about Venus gravity field accounted for the atmospheric contribution by forward modelling its effect (Goossens et al., 2017, Goossens et al., 2018). We have chosen to adopt a conservative approach and account for the intrinsic uncertainty of the atmospheric model. We have chosen to model the thermal tide field up to the degree and order that guarantees that the higher degrees produce no residual signal in the Doppler residuals (i.e. degree and order 18 for f_1, 13 for f_2, 7 for f_3 and 10 for f_4) and considered the uncertainty associated to the correction coefficients $A_{C_{lm}}^{f}, A_{S_{lm}}^{f}, B_{C_{lm}}^{f}, B_{S_{lm}}^{f}$ for the frequencies f_1 through f_4.

In our simulation we evaluated the effect on the solution of the assumed apriori knowledge of the atmospheric model, without delving into a detailed analysis of atmospheric dynamics. In particular, we explored three cases by setting different apriori uncertainties on the thermal tides parameters. We considered an accurate model (model uncertainty equal to 10%), a medium-accuracy model (50% accuracy) and a coarse-accuracy model (100% accuracy). In table S1 we report the results relative to each of the three assumptions. It is important to note how the results, when combining tie points radar observation, become significantly less sensitive to the accuracy of the model, for all the parameters except the tidal response which, not surprisingly, is significantly sensitive to the atmospheric tides.

Table S1 Results comparison (In terms of formal uncertainties, 3σ) for different levels of apriori knowledge of the atmospheric thermal tides model parameters

Parameter	10%		50%		100%	
	Earth Doppler Only	Tie Points	Earth Doppler Only	Tie Points	Earth Doppler Only	Tie Points
α_0 [arcsec]	2.3	0.36	2.9	0.39	3.1	0.39
δ_0[arcsec]	1.3	0.21	1.6	0.21	1.8	0.21
T_{sid} [day]	1.1×10^{-5}	1.5×10^{-6}	1.3×10^{-6}	1.6×10^{-5}	1.4×10^{-5}	1.6×10^{-5}
ϵ [arcsec]	1.0	0.16	1.3	0.17	1.4	0.17
Ω [deg/century]	3.2×10^{-2}	5.1×10^{-3}	3.9×10^{-2}	5.4×10^{-3}	4.3×10^{-2}	5.4×10^{-3}
$MOIF$	8.6×10^{-3}	1.4×10^{-3}	1.1×10^{-2}	1.4×10^{-3}	1.2×10^{-2}	1.4×10^{-3}
k_2	9.6×10^{-4}	3.9×10^{-4}	1.4×10^{-3}	7.8×10^{-4}	1.8×10^{-3}	1.3×10^{-3}
δ_{k_2} [deg]	8.6×10^{-2}	4.0×10^{-2}	1.6×10^{-1}	1.3×10^{-1}	2.5×10^{-1}	2.2×10^{-1}

Text S3. Uncertainty on Venus' MOIF from pole precession measurements

In this section we will obtain the equations to express the motion of the pole as a function of the equatorial coordinates and their time derivatives. Finally, we will show the relation arising between these latter quantities after neglecting the nutations. This relation will be used in the simulations as a constraint to improve the measurement of the precession rate and the MOIF.

The Venus ecliptic (V_E) and the (usual) Earth ecliptic (E_E) reference frames are represented by the unit vectors {$\mathbf{u}_{v,x}$,$\mathbf{u}_{v,y}$,$\mathbf{u}_{v,z}$} and {$\mathbf{u}_{E,x}$,$\mathbf{u}_{E,y}$,$\mathbf{u}_{E,z}$} respectively. The equatorial frame is represented by {$\mathbf{u}_{eq,x}$,$\mathbf{u}_{eq,y}$,$\mathbf{u}_{eq,z}$}.

We will use the following coordinates:

- $\alpha(t)$, $\delta(t)$ are right ascension and declination (equatorial J2000 coordinates);
- $\lambda(t)$, $\beta(t)$ are ecliptic coordinates referred to the E_E reference frame at J2000.0;
- $\lambda_V(t)$, $\beta_V(t)$ are ecliptic coordinates referred to the V_E reference frame at J2000.0

We define:

- the direction (as a unit vector) P_V of the Venus' pole;
- the direction (as a unit vector) P_{0V} of the normal to the Venus orbital plane (hereafter the "orbital pole").

All coordinates above will be referred to the pole position (in particular, $\beta_V(t)$ is the nutation in obliquity and $\lambda_V(t)$ is the sum of the precession and the nutation in longitude.
The direction P_V in the three reference frames is:

$$P_{V,eq} = cos[\delta(t)]cos[\alpha(t)]\boldsymbol{u}_{eq,x} + cos[\delta(t)]sin[\alpha(t)]\boldsymbol{u}_{eq,y} + sin[\delta(t)]\boldsymbol{u}_{eq,z} \tag{S3.1}$$
$$P_{V,E_E} = cos[\beta(t)]cos[\lambda(t)]\boldsymbol{u}_{E,x} + cos[\beta(t)]sin[\lambda(t)]\boldsymbol{u}_{E,y} + sin[\beta(t)]\boldsymbol{u}_{E,z} \tag{S3.2}$$
$$P_{V,V_E} = cos[\beta_V(t)]cos[\lambda_V(t)]\boldsymbol{u}_{V,x} + cos[\beta_V(t)]sin[\lambda_V(t)]\boldsymbol{u}_{V,y} + sin[\beta_V(t)]\boldsymbol{u}_{V,z} \tag{S3.3}$$

while the orbital pole direction in the E_E frame is

$$P_{0V,E_E} = sini_0 sin\Omega_0 \boldsymbol{u_{E,x}} - sini_0 cos\Omega_0 \boldsymbol{u_{E,y}} + cosi_0 \boldsymbol{u_{E,z}} \tag{S3.4}$$

where $i_0 = 3.39466189°$ (inclination) and $\Omega_0 = 76.67992019°$ (longitude of the ascending node), at J2000.0 (Simon et al., 1994).

The transformations of P_V from equatorial to E_E (and vice versa) coordinates are

$$P_{V,E_E} = R^{-1}P_{V,eq} \quad \text{and} \quad P_{V,eq} = RP_{V,E_E} \tag{S3.5}$$

where

$$R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\epsilon & -\sin\epsilon \\ 0 & \sin\epsilon & \cos\epsilon \end{pmatrix} \tag{S3.6}$$

and $\epsilon = 23.43662deg$ is the Earth's obliquity.

Eq. 3.5 corresponds to

$$\cos\beta\cos\lambda = \cos\propto\cos\delta \tag{S3.7}$$

$$\cos\beta\sin\lambda = \cos\epsilon\cos\delta\sin\propto + \sin\epsilon\sin\delta \tag{S3.8}$$

$$\sin\beta = \cos\epsilon\ \sin\delta - \sin\epsilon\cos\delta\sin\propto \tag{S3.9}$$

At J2000.0 ($t = 0$) the equatorial coordinates of the pole of Venus are $\alpha_0 = \alpha(0)\ 272.76°$ and $\delta_0 = \delta(0) = 67.16°$ (Archinal et al., 2009).

By solving Equations 3.7-3.9, we obtain the pole position at the same epoch in E_E coordinates:

$$\lambda_0 = 30.079869° \ ; \ \beta_0\ 88.762332° \tag{S3.10}$$

The $\boldsymbol{u}_{V,x}, \boldsymbol{u}_{V,y}, \boldsymbol{u}_{V,z}$ directions of the V_E reference frame are:

- The z-axis points towards the pole $\boldsymbol{u}_{V,z} = P_V$;

- The x-axis is the direction of the vernal equinox of Venus: the vernal equinox of Venus coincides with the coordinates of the ascending node of the orbit of Venus at J2000.0 with respect to the equator of Venus at the same date, so $\boldsymbol{u}_{V,x} = P_{0V} \times P_V$

From Equations 3.2 and 3.4 we obtain

$$\boldsymbol{u}_{V,x} = u_{V,x_1}\boldsymbol{u}_{E,x} + u_{V,x_2}\boldsymbol{u}_{E,y} + u_{V,x_3}\boldsymbol{u}_{E,z} \tag{S3.11}$$

Where

$$u_{V,x_1} = \frac{\cos\beta_0\ \cos i_0 \sin\lambda_0 + \sin\beta_0 \sin i_0 \cos\Omega_0}{\sqrt{1-[\sin\beta_0 \cos i_0 - \cos\beta_0 \sin i_0 \cos(\lambda_0-\Omega_0)]^2}} \tag{S3.12}$$

$$u_{V,x_2} = \frac{\sin\beta_0 \sin i_0 \sin\Omega_0 - \cos\beta_0\ \cos i_0 \cos\lambda_0}{\sqrt{1-[\sin\beta_0 \cos i_0 - \cos\beta_0 \sin i_0 \cos(\lambda_0-\Omega_0)]^2}} \tag{S3.13}$$

$$u_{vx3} = -\frac{\cos\beta_0 \sin i_0 \cos(\lambda_0-\Omega_0)}{\sqrt{1-[\sin\beta_0 \cos i_0 - \cos\beta_0 \sin i_0 \cos(\lambda_0-\Omega_0)]^2}} \tag{S3.14}$$

and

$$\boldsymbol{u}_{V,z} = u_{V,z_1}\boldsymbol{u}_{E,x} + u_{V,z_2}\boldsymbol{u}_{E,y} + u_{V,z_3}\boldsymbol{u}_{E,z} \tag{S3.15}$$

with

$$u_{v,z1} = \sin i_0 \sin\Omega_0\ ;\ u_{V,z_2} = -\sin i_0 \cos\Omega_0\ ;\ u_{V,z_3} = \cos i_0 \tag{S3.16}$$

and finally, $\boldsymbol{u}_{V,y} = \boldsymbol{u}_{V,z} \times \boldsymbol{u}_{V,x}$.

We define M (at J2000.0) as

$$M = \begin{pmatrix} u_{V,x_1} & u_{V,y_1} & u_{Vz_1} \\ u_{V,x_2} & u_{V,y_2} & u_{V,z_2} \\ u_{V,x_3} & u_{V,y_3} & u_{V,z_3} \end{pmatrix} \approx \begin{pmatrix} 0.531509 & -0.84509 & 0.0576204 \\ 0.846837 & 0.531678 & -0.0136422 \\ -0.0.191066 & 0.0560461 & 0.998245 \end{pmatrix} \tag{S3.17}$$

The $\alpha(t)$ and $\delta(t)$ coordinates as functions of $\beta_V(t)$, λ_V are given by the following relations

$$P_{V,eq}(t) = RMP_{V,V_E}(t) \qquad P_{V,V_E}(t) = M^{-1}R^{-1}P_{V,eq}(t) \tag{S3.18}$$

The pole motion around the orbit pole is (t=0 corresponds to J2000.0)

$$\lambda_V(t) = \lambda_V(0) + \Omega t + \delta\lambda_V(t) \tag{S3.19}$$
$$\beta_V(t) = \beta_V(0) + \delta\beta_V(t) \tag{S3.20}$$

where $\delta\beta_V$, $\delta\lambda_V$ are the nutations in obliquity and in longitude, respectively and Ω is the precession rate of the Venus pole. The precession rate is the sum of the solar precession (~44.74''/yr) and the planetary precession (-10''/yr, Simon et al 1994). By solving for β_V in Eq. (S3.18b) we get

$$\beta_V(0) = 87.3638°;\ \lambda_V(0) = 90° \tag{S3.21}$$

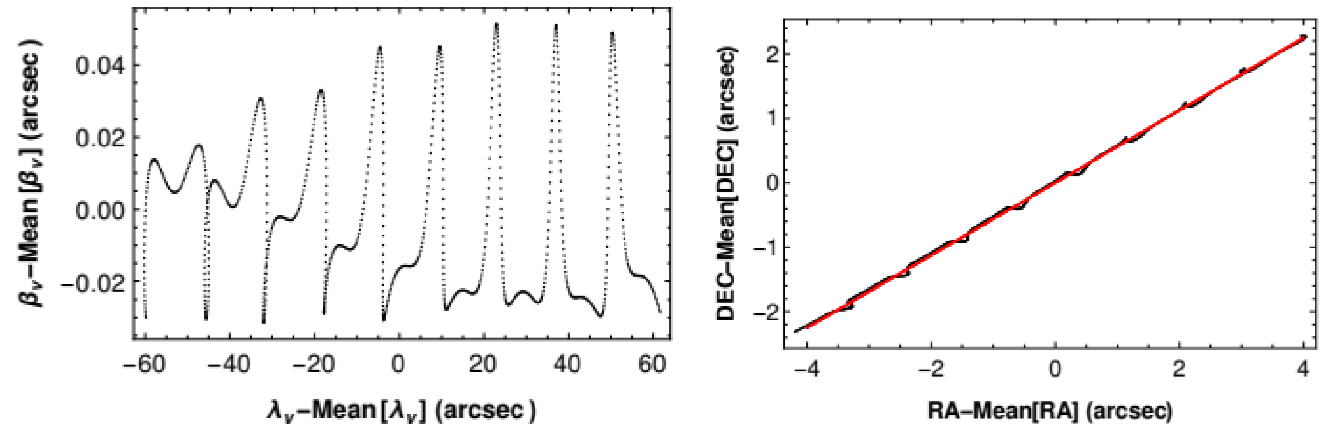

Figure S2. *Left: longitude vs. obliquity nutations and precession about the orbit pole (time span: 4 Venus cycles, 972d). The displacement in longitude is about 45"/yr. However, the overall displacement in the sky is very small. Right: the same path but in equatorial coordinates over the same interval of time (red: precession only, black: precession and nutations).*

By deriving Eq. (18b) with respect to time and neglecting the nutations, we get

$$\frac{\dot{\alpha}(t)}{\dot{\delta}(t)} = \frac{\sin\alpha(t)\tan\delta(t)\,(\cos\epsilon\ \sin i_0\cos\Omega_0 + \sin\epsilon\ \cos i_0)}{\cos\alpha\,(t)(\cos\epsilon\sin i_0\cos\Omega_0 + \sin\epsilon\ \cos i_0) + \sin i_0\sin\Omega_0\sin\alpha(t)} +$$

$$+\frac{\cos\epsilon\cos i_0 - \sin i_0(\sin\Omega_0\cos\alpha(t)\tan\delta(t) + \sin\epsilon\cos\Omega_0)}{\cos\alpha(t)(\cos\epsilon\sin i_0\cos\Omega_0 + \sin\epsilon\ \cos i_0) + \sin i_0\sin\Omega_0\sin\alpha(t)} \tag{S3.22}$$

This quantity depends on the geometry of the orbit (i_0, Ω_0) and on the pole position but it is independent of Ω .

The link between $\frac{d\alpha}{dt}$ and $\frac{d\delta}{dt}$ is a consequence of the circular path of the pole. For t = 0 (i.e. at J2000.0) we obtain

$$\dot{\alpha}(0) = 1.77694\dot{\delta}(0) \tag{S3.23}$$

From Eq (18a) we get

$$\cos\delta\cos\alpha = u_{V,x_1}\cos\beta_V\cos\lambda_V + u_{V,y_2}\cos\beta_V\ \sin\lambda_V + u_{V,z_1}\sin\beta_V \tag{S3.24}$$

$$\cos\delta\ \sin\alpha = \cos\epsilon\left(u_{V,x_2}\cos\beta_V\cos\lambda_V + u_{V,y_2}\cos\beta_V\ \sin\lambda_V + u_{V,z_2}\sin\beta_V\right) - \sin\epsilon\left(u_{V,x_3}\cos\beta_V\cos\lambda_V + u_{V,y_3}\cos\beta_V\sin\lambda_V + u_{V,z_3}\sin\beta_V\right) \tag{S3.25}$$

$\sin\delta = \cos\beta_V \cos\lambda_V \left(u_{V,x_2}\sin\epsilon + u_{V,x_3}\cos\epsilon\right) + \cos\beta_V \sin\lambda_V \left(u_{V,y_2}\sin\epsilon + u_{Vy_3}\cos\epsilon\right) + \sin\beta_V \left(u_{V,z_2}\sin\epsilon + u_{V,z_3}\cos\epsilon\right)$
(S3.26)

The functions $\alpha(t)$, $\delta(t)$, obtained by solving Eqs. S3.24-S3.26, can be expanded to first order in Taylor series around the pole position. We get (units are radians)

$$\alpha(t) \approx -1.52263 + 2.11423\delta\beta_V(t) - 0.0672264(\delta\lambda_V(t) + \Omega t) \quad \text{(S3.27)}$$
$$\delta(t) \approx 1.17216 - 0.568672\delta\beta_V(t) - 0.0378326(\delta\lambda_V(t) + \Omega t) \quad \text{(S3.28)}$$

Finally, we obtain the components of the initial velocity of the pole. The evolution of the Venus' orbital elements due to planetary effects are already included in our setup, so here we will consider the solar precession rate only.
Moreover, we neglect the nutations since, to our purposes, they are fast and zero-mean oscillations. Therefore, neglecting the nutations, the relations between $\dot{\alpha}$, $\dot{\delta}$ and Ω:

$$\dot{\alpha}(0) \approx -0.0672264\Omega = -4.62097 \times 10^{-13}[rad/s] \quad \text{(S3.29)}$$
$$\dot{\delta}(0) \approx -0.0378326\Omega = -2.60051 \times 10^{-13}[rad/s] \quad \text{(S3.30)}$$

Finally, the ratio $\dot{\alpha}\,(0)/\dot{\delta}\,(0)$ (Eq. 22) can be used as an apriori constraint between the two quantities.

Text S4. Tie points simulation

The match covariance matrix is given by

$$M_{cov} = k_c H^{-1}\left(2\sigma_n^2 H + \frac{1}{2}A_w\sigma_n^4 I\right)H^{-1} \quad \text{(S4.1)}$$

Where k_c is an empirical constant inferred from Magellan match statistics, I is the identity matrix, H is the hessian of the match correlation function $c(x,y)$. For a given image offset (x,y) the hessian is given by

$$H = \begin{bmatrix} \frac{\partial^2 c}{\partial x^2} & \frac{\partial^2 c}{\partial x \partial y} \\ \frac{\partial^2 c}{\partial x \partial y} & \frac{\partial^2 c}{\partial y^2} \end{bmatrix} \quad \text{(S4.2)}$$

A_w is the area in pixel of the marching window, σ_n is a measure of the backscatterer difference between the two images in the matching window

$$\sigma_n = \frac{1}{2A_w}\sum_{x\in W}\left[I_1(\vec{x}) - \bar{I}_1 - I_2(\vec{x}-\vec{o}) + \bar{I}_2(\vec{o})\right]^2 \quad \text{(S4.3)}$$

where $I_1(\vec{x})$ and $I_2(\vec{x})$ are the pixel intensities for the two images at position $\vec{x}$, $\vec{o}$ is the offset vector between the images, and $\bar{\bar{I}}_1$ and $\bar{\bar{I}}_2(\vec{o})$ are mean intensities in the match window. We approximate the correlation function for a good match by the product of $sinc$ functions given by:

$$c(x,y) = sinc\left[\frac{\pi x}{2\sigma_{m_x}}\right] sinc\left[\frac{\pi y}{2\sigma_{m_y}}\right] \tag{S4.4}$$

where σ_{m_x} and σ_{m_y} are the matching accuracy in pixel in the x and y directions. Differentiating Equation S4.4 twice and evaluating at the peak yields

$$H = \begin{bmatrix} -\frac{1}{12}\frac{\pi^2}{\sigma_{m_x}^2} & 0 \\ 0 & -\frac{1}{12}\frac{\pi^2}{\sigma_{m_y}^2} \end{bmatrix} \tag{S4.5}$$

where σ_{m_x} and σ_{m_y} *are given by*

$$\sigma_{m_q} = \frac{1}{2} - k_{m_q} \frac{\sigma_{10\times10}}{\bar{\sigma}_{10\times10}\frac{1}{\sqrt{N_L}}\left(1+\frac{1}{SNR}\right)} \tag{S4.6}$$

where $q = x, y$, $k_{m_q}$are empirical parameters based on Magellan match statistics, *SNR* is the signal to noise ratio, N_L are the number of looks (number of single look pixel intensities averaged together in a 30 m multi-looked pixel to reduce thermal and speckle noise, $\bar{\sigma}_{10\times10}$, $\sigma_{\sigma_{10\times10}}$are the mean and standard deviation of the backscatter in a 10 by 10 pixel window centered at the match point obtained from Magellan imagery and where

$$SNR = \frac{\bar{\sigma}_{10\times10}}{NES0} \tag{S4.6}$$

where *NES0* is the radar noise equivalent sigma naught (backscatter value where the SNR equals to 1). To obtain an approximate value for σ_n^2 we use the

$$\sigma_n^2 = \left[\sqrt{2}\bar{\sigma}_{10\times10}\left(\frac{1}{\sqrt{N_L}}\frac{1}{SNR} + k_s\frac{\bar{\sigma}_v}{\sigma_{10\times10}}\right)\right] \tag{S4.7}$$

where k_s is an empirical value derived from Magellan match statistics and $\bar{\sigma}_v$ is the mean X-band backscatter value for Venus, roughly-10.5 dB.