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# Dynamic event-triggered-based anti-disturbance control for uncertain LPV systems

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**Summary**

This investigation proposes a dynamic event-triggered-based anti-disturbance control technique for the uncertain linear parameter varying (LPV) systems subject to multiple disturbances. The disturbances are comprised of two parts including the unavailable modeled disturbances and the available unmodeled disturbances. First, an observer is constructed to capture the unavailable modeled disturbances. Then, a dynamic event-triggered-based feedback controller is proposed. Further, under the developed event-triggered controller, sufficient conditions are presented for the uncertain LPV systems to achieve the multiple disturbances suppression and communication transmission resources saving. In the end, the reasonability of the raised dynamic event-triggered based anti-disturbance control scheme is verified by an example of a turbofan.

**KEYWORDS:**
Uncertain LPV systems, multiple disturbances, dynamic event-triggered mechanism, anti-disturbance.

## 1 | INTRODUCTION

LPV systems are a special class of linear systems whose state-space matrix is a function of time-varying parameters. When these time-varying parameters change along a given parameter trajectory, the LPV system degenerates into a general linear time-varying system; and when these parameters are fixed, the system degenerates into a linear invariant system. Due to the high complexity in practical systems, linear time-invariant systems and their techniques have been unable to solve the problems encountered very well [1]-[2]. In order to better solve the problems in practical systems, LPV systems and technologies have been widely used in the fields ranging from ship autopilot driving to aerospace field [3]-[6]. The characteristic of the LPV system is that it is a separate linearized model for each parameter, which ensures that it can approximate the actual system model within a small range of parameter variation [7].

In engineering practice, the control systems are inevitably affected by a variety of disturbances [8]. Among which, the unavailable modeled disturbances are very complex but usually occur in the practical control systems [9]. Such as unknown constant load [10], harmonics with unknown phase and magnitude [11], periodic disturbances in vibrating structures with eccentricity [12]. Such disturbances often impose negative influences on the control systems and degrade the anticipated system performance [13]. Limited by the production level and cost, it is very difficult to change the equipment structure of the system to reduce the impact of disturbance on the system. For handling such unavailable modeled disturbances, the disturbance-observer-based control approach was introduced [14]. The main idea is to construct an observer to capture the disturbances, and then develop a controller with the observed information to counteract the influence of the disturbances on the control systems. On the other hand, for the measurable unmodeled disturbances, many control strategies have been reported . A widely recognized and frequently adopted control method is known as robust control. As a special robust control method, $H_\infty$ control plays a beneficial role in attenuating the effects of measurable unmodeled disturbances. The $H_\infty$ control issue for many control systems have been investigated, such as the switched systems [15-16] and the frctional systems [17-19]. There have been many studies on the anti-disturbance control issue for LPV system over the past decades. In [20,21,22], the $H_\infty$ anti-disturbance control issue were studied for LPV system. In [23], a bumpless transfer $H_\infty$ anti-disturbance control issue was proposed for switched LPV system. References [15-23] used robust control methods to study the available unmodeled disturbances, without considering the existence of multiple disturbances. At present, there are few researches on the multiple disturbances control scheme of LPV system, which motivates us to study this topic.

LPV systems need to use networked control equipment for actual control realization, however, the network resources are always limited, thus it is necessary to study the event-triggered control problem of LPV systems. The event-triggered control method has been widely exploited in decreasing the communication sources due to its additional flexibility in control design. Event-triggered logics are the key components of event-triggered control [24]. Recently, event-triggered control has been employed to various control systems, such as multi-agent systems [25], the impulse systems [26], the nonlinear systems [27] and so on. The author of [28] studied the event-triggered dynamic output feedback controller for discrete-time LPV systems. In [28], the event-triggered mechanism is static, which saves part of the communication resources. Usually, the proposed dynamic event-triggered is more popular among researchers because it has a longer trigger interval than static event-triggered. Thus we added non-negative dynamic variables to the event-triggered condition. Therefore, it is preferable to design the dynamic event-triggered-based control schemes for practical control systems. There have been some research results on the event-triggered-based anti-disturbance control issue in LPV systems. For example, the event-triggered-based anti-disturbance control problem of network LPV systems was studied [29]. In [30], the event-triggered-based anti-disturbance problem of discrete LPV systems was studied. In [31], the dynamic periodic event-triggered-based anti-disturbance control issue of quasi-LPV systems was analyzed. The event-triggered finite-time $H_\infty$ tracking control was researched for switched LPV systems [32]. None of these papers have studied the event-triggered control problem of LPV systems with multiple disturbances.

In order to address the multiple disturbances suppression and communication resources saving in uncertain LPV systems, in this article, we study the dynamic event-triggered-based anti-disturbance control issue for the uncertain LPV systems with multiple disturbances. The pivotal contributions of this paper can be encapsulated as follows.

i) Different from the existing investigations on the LPV systems with the single available unmodeled disturbances [20,21,34], the multiple disturbances (i.e., the unavailable modeled disturbances and the available unmodeled disturbances) alleviation issue is studied in the present research. In fact, the co-existence of both types of disturbances is more general for practice.

ii) Instead of using the system state to design the event-triggered condition in [22,31], we use control input as event-triggered criterion. This saves the signal transmission resources from the controller to the system instead of the signal transmission resources from the system to the controller like that in [35,36]. Unlike the event-triggered control investigations on the uncertain LPV systems in [22,30,33,37], the developed event-triggered condition is dynamic, which usually allows bigger triggering intervals than the static ones in [22,30,33,37].

iii) The event-triggered rule, controller and disturbance observer are co-designed to force the multiple disturbance suppression and communication transmission resource saving of the uncertain LPV systems. The corresponding sufficient conditions are developed, which ensure that the event-triggered-based anti-disturbance control problem of the uncertain LPV system is solvable.

Structure. In Section 2, we introduce the system description and the control objective. A new dynamic event-triggered-based anti-disturbance control method is proposed in Section 3. Via Section 4, the simulation verification is given. And the conclusions are developed in Section 5.

The symbols of this article are standard and summarized in Table 1.

## 2 | PROBLEM STATEMENT

### 2.1 | System statement

We take the following system

$$\begin{aligned} \dot{x}(t) &= A(\varpi(t))x(t) + B(\varpi(t))[u(t) + d_1(t)] + B(\varpi(t))d_2(t), \\ y(t) &= C(\varpi(t))x(t) + D(\varpi(t))d_2(t) \end{aligned} \tag{1}$$

into consideration, where $x(t) \in R^n$, $u(t) \in R^w$ and $y(t) \in R^q$ are the system state, control input and control output, respectively, $d_1(t) \in R^w$, $d_2(t) \in R^w$ are the unavailable modeled disturbance and available unmodeled disturbance,

Table1
Nomenclature

| Notation | Meaning |
|---|---|
| $N$ ($N^+$) | The set of all non-negative (positive) integers |
| $A > 0$ ( $A < 0$) | A is a symmetric positive (negative) definite matrix |
| $I$ | Identity matrix |
| $\lambda_{min}(A)$ | The smallest eigenvalue of $A$ |
| $x^T$ | Transpose of $x$ |
| $\|\|x\|\|$ | Euclidean norm of $x$ |
| $diag\{\}$ | Diagonal matrix |

respectively. $A(\varpi(t)), B(\varpi(t)), C(\varpi(t))$ and $D(\varpi(t))$ are the matrix of appropriate dimensions. The parameter $\varpi(t) = [\varpi_1(t), \varpi_2(t), ..., \varpi_s(t)]^T$, $\varpi_i(t)$ is completely measurable on the positive real axis, $i \in \{1, 2, \cdots, s\}$. And $d_1(t)$ is obtained from the following external model

$$\begin{aligned}\dot{\zeta}(t) &= G(\varpi(t))\zeta(t) + H(\varpi(t))d_3(t),\\ d_1(t) &= E(\varpi(t))\zeta(t),\end{aligned} \tag{2}$$

where the disturbance signal $d_3(t)$ is denoted as $d_3(t) \in R^w$ belonging to $L_2[0, \infty)$, $\zeta(t)$ is external system state, $G(\varpi(t))$, $H(\varpi(t))$ and $E(\varpi(t))$ are the matrices of appropriate dimensions.

*Remark 1.* Many systems can be modeled as LPV systems, such as the inverted pendulum control system [40], the aircraft control system [41], the missile control system [42], the aircraft control system [43], and the aircraft engine control system [44], etc. The disturbance $d_1(t)$ considered in system (1) is an unmeasurable disturbance, which exists widely in practice. This unmeasurable disturbance can be represented using model (2), where $d_3(t)$ is the additional perturbation generated by system uncertainties and perturbations.

*Remark 2.* $d_1(t)$ is an additional disturbance that results from uncertainties and perturbations in the exogenous system. Many kinds of disturbances in practical processes can be described by this model, such as unknown constant load [10], harmonics with unknown phase and magnitude [11], periodic disturbances in vibrating structures with eccentricity [12].

## 2.2 | Observer and event-triggered rule design

To capture the unavailable modeled disturbances $d_1(t)$, we design the following observer

$$\begin{aligned}\dot{\theta}(t) &= [G(\varpi) + \Lambda(\varpi)B(\varpi)E(\varpi)][\theta(t) - \Lambda(\varpi)x(t)] + \Lambda(\varpi)[A(\varpi)x(t) + B(\varpi)u(t))] + \frac{\partial\Lambda(\varpi)}{\partial\varpi}\dot{\varpi}x(t),\\ \hat{\zeta}(t) &= \theta(t) - \Lambda(\varpi)x(t),\\ \hat{d}_1(t) &= E(\varpi)\hat{\zeta}(t),\end{aligned} \tag{3}$$

where $\theta(t)$ is the observer state, $\Lambda(\varpi)$ is the observer gain to be yield, $\hat{d}_1(t)$ is the estimation of $d_1(t)$.

The observation error is defined as follows

$$e(t) = \zeta(t) - \hat{\zeta}(t). \tag{4}$$

Substituting (2) and (3) into (4) produces the following error system

$$\begin{aligned}\dot{e}(t) &= \dot{\zeta}(t) - \dot{\hat{\zeta}}(t)\\ &= G(\varpi)e(t) + \Lambda(\varpi)B(\varpi)d_2(t) + H(\varpi)d_3(t), -\Lambda(\varpi)B(\varpi)E(\varpi)\hat{\zeta}(t) + \Lambda(\varpi)B(\varpi)d_1(t)\\ &\quad + \frac{\partial\Lambda(\varpi)}{\partial\varpi}\dot{\varpi}x(t) - \frac{\partial\Lambda(\varpi)}{\partial\varpi}\dot{\varpi}x(t)\\ &= (G(\varpi) + \Lambda(\varpi)B(\varpi)E(\varpi))e(t) + \Lambda(\varpi)B(\varpi)d_2(t) + H(\varpi)d_3(t).\end{aligned} \tag{5}$$

For the system (1), usually, the following feedback controller can be designed

$$u(t) = -\hat{d}_1(t) + K(\varpi)x(t), \tag{6}$$

where $K(\varpi)$ is the controller gain to be designed.

For reducing the communication resources from the controller to the actuator, we design the following dynamic event-triggered rule

$$\mathrm{t}_{k+1} = \min\{t \geq t_k | \mu e_1^T(t)e_1(t) \geq c_1\beta^T(t)\beta(t) + \kappa(t) + m\}, k \in N, \tag{7}$$

where $\beta(t) = \left[\, x^T(t) \;\; e^T(t) \,\right]^T$, $e_1(t) = u(t_k) - u(t)$, $c_1 \geq 0$, $m > 0$, $\mu \geq 1$ and

$$\dot{\kappa}(t) = -b\kappa(t) + c_2\beta^T(t)\beta(t) + m - {e_1}^T(t)e_1(t),$$

where

$$\kappa(0) > 0, b \geq 1, c_2 \geq c_1.$$

*Remark 3.* The proposed dynamic event-triggered mechanism can save more communication resources than static event-triggered in [37], thus we added non-negative dynamic variables $\kappa(t)$ to the event-triggered condition (7). We will prove its non-negativity below.
When $t \in [0, +\infty)$,
from

$$\mathrm{t}_{k+1} = \min\{t \geq t_k | \mu e_1^T(t)e_1(t) \geq c_1\beta^T(t)\beta(t) + \kappa(t) + m\},$$

one can get

$$c_1\beta^T(t)\beta(t) + \kappa(t) + m - \mu e_1^T(t)e_1(t) \geq 0,$$

thus

$$c_1\beta^T(t)\beta(t) + m - \mu e_1^T(t)e_1(t) \geq -\kappa(t).$$

Then, from the dynamics of

$$\dot{\kappa}(t) = -b\kappa(t) + c_2\beta^T(t)\beta(t) + m - {e_1}^T(t)e_1(t),$$

we have

$$\dot{\kappa}(t) + (b+1)\kappa(t) \geq 0, \kappa(0) > 0 \text{ and } b \geq 1,$$

thus

$$\kappa(t) \geq e^{(-b-1)t}\kappa(0) + \frac{1}{(b+1)}(1 - e^{(-b-1)t})$$

which guarantees that $\kappa(t)$ is non-negative.

When the event is triggered, the true control signal is

$$u(t) = u(t_k) = -\hat{d}_1(t_k) + K(\varpi)x(t_k). \tag{8}$$

Substituting (8) into (1) , the following closed-loop system can be get

$$\begin{aligned} \dot{x}(t) &= A(\varpi)x(t) + B(\varpi)[d_1(t) - \hat{d}_1(t_k) + K(\varpi)x(t_k) + d_2(t)], \\ &= [A(\varpi) + B(\varpi)K(\varpi)]x(t) + B(\varpi)[e_1(t) + d_1(t) - \hat{d}_1(t) + d_2(t)], \\ &= [A(\varpi) + B(\varpi)K(\varpi)]x(t) + B(\varpi)[e_1(t) + E(\varpi)e(t) + d_2(t)]. \end{aligned} \tag{9}$$

Let $d(t) = [d_2^T(t) \;\; d_3^T(t)]^T$, $\beta(t) = [x^T(t) \;\; e^T(t)]^T$, combining (5) and (9) one can deduce the augmented dynamics

$$\begin{aligned} \dot{\beta}(t) &= M(\varpi)\beta(t) + N(\varpi)d(t) + P(\varpi)e_1(t), \\ y(t) &= \bar{C}(\varpi)\beta(t) + \bar{D}(\varpi)d(t), \end{aligned} \tag{10}$$

where

$$\begin{aligned} M(\varpi) &= \begin{bmatrix} A(\varpi) + B(\varpi)K(\varpi) & B(\varpi)E(\varpi) \\ 0 & G(\varpi) + \Lambda(\varpi)B(\varpi)E(\varpi) \end{bmatrix}, \\ N(\varpi) &= \begin{bmatrix} B(\varpi) & 0 \\ \Lambda(\varpi)B(\varpi) & H(\varpi) \end{bmatrix}, P(\varpi) = \begin{bmatrix} B(\varpi) \\ 0 \end{bmatrix}, \\ \bar{C}(\varpi) &= \begin{bmatrix} C(\varpi) & 0 \end{bmatrix}, \bar{D}(\varpi) = \begin{bmatrix} D(\varpi) & 0 \end{bmatrix}. \end{aligned} \tag{11}$$

## 2.3 | Control objectives

The purpose of this article is to address the issue of dynamic event-triggered-based anti-disturbance control of the system (9).

For the system (9), if there exist the disturbance estimator (3), the controller (8) and such the following properties are ensured:
i) If $d(t) \equiv 0$, the system (10) is practically stable;
ii) If $d(t) \neq 0$, the $L_2$-gain index

$$\int_0^{\infty} y^T(s)y(s)ds \leq \theta^2 \int_0^{\infty} d^T(s)d(s)ds + \aleph \tag{12}$$

holds, where $\theta > 0$ is a constant specially labelled as the $L_2$-gain index, $\aleph$ is a positive constant.

Then the dynamic event-triggered-based anti-disturbance control problem of system (9) is said to be solvable.

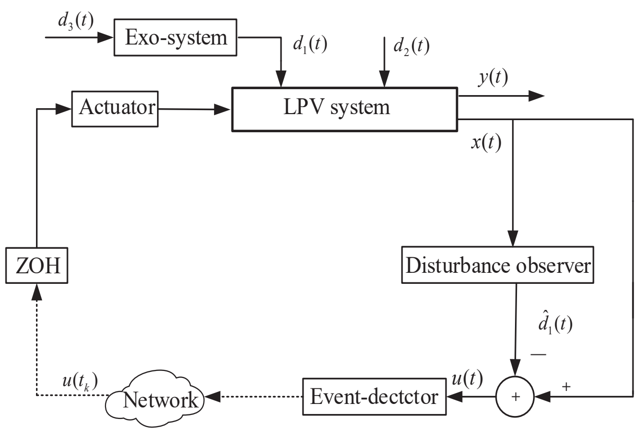


**FIGURE 1** The configuration of the dynamic event-triggered-based anti-disturbance control scheme.

The following lemma will be used in this article.

**Lemma 1.** [36] For any matrix $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$, if $\mathbf{C^T C} \leq \mathbf{I}$ holds, then for any positive constant $\mathbf{h}$, the inequality:
$\mathbf{A + BCD + BCD^T \leq A + hBB^T + h^{-1}DD^T}$
holds.

# 3 | MAIN RESULT

In this section, we display how the event-triggered schemes (7) can exclude Zeno behavior, and how the dynamic event-triggered-based anti-disturbance issue is solved by the disturbance estimator (3) and the controller (8). Fig. 1 shows the structure of the dynamic event-triggered-based anti-disturbance control strategy.

First of all, the avoidance of Zeno behavior for the event-triggered strategy (7) is exhibited.

**Theorem 1.** Consider the event-triggered rule defined in (7). The Zeno phenomenon can be prevented with the triggering interval given by

$$t_{k+1} - t_k \geq \frac{m}{m_0 + m + m_1}, \tag{13}$$

where

$$m_0 = \left\| \frac{d}{dt}\hat{d}_1(t) \right\|,$$
$$m_1 = \| K(\varpi)\dot{x}(t) \|.$$

**Proof.** In the triggering interval $[t_k, t_{k+1})$, we can get

$$\begin{aligned} \frac{d\left\|e_1(t)\right\|}{dt} &\leq \left\|\frac{de_1(t)}{dt}\right\| = \left\|\frac{d}{dt}(-Kx(t) + \hat{d}_1(t))\right\|, \\ &\leq \|K\dot{x}(t)\| + \left\|\frac{d}{dt}\hat{d}_1(t)\right\| + m, \\ &= m_0 + m + m_1, \end{aligned} \tag{14}$$

where $m > 0$ is a constant.

Solving (14) for $t$ under the initial condition, we can get

$$\left\|e_1(t)\right\| \leq (t_{k+1} - t_k)(m_0 + m + m_1).$$

When $t \in [t_k, t_{k+1})$, $\left\|e_1(t)\right\| \geq m$, the event is triggered. Accordingly

$$\begin{aligned} m \leq \left\|e_1(t)\right\| &\leq (t_{k+1} - t_k)(m_0 + m + m_1), \\ (t_{k+1} - t_k) &\geq \tfrac{m}{m_0+m+m_1} > 0. \end{aligned}$$

Then, a criterion is established to ensure the solvability of the dynamic event-triggered-based anti-disturbance issue for the system (1).

**Theorem 2.** Consider the system (10). If there exist symmetric matrix $Q(\varpi) > 0$, scalars $c_2 > 0, b > 0$, $\bar{m} > 0$, $2\bar{m} - b < 0$ and $m > 0$ satisfying the following constraint

$$\begin{bmatrix} \Gamma_{11} & \Gamma_{12} & 0 & \Gamma_{14} \\ * & -I & 0 & 0 \\ * & * & \Gamma_{33} & 0 \\ * & * & * & \Gamma_{44} \end{bmatrix} < 0, \tag{15}$$

where

$$\begin{aligned} \Gamma_{11} &= M^T(\varpi)Q(\varpi) + Q(\varpi)M(\varpi) + \frac{\partial Q}{\partial \varpi}\dot{\varpi} + 2\bar{m} + c_2 + Q(\varpi)Q(\varpi) + \bar{C}^T(\varpi)\bar{C}(\varpi), \\ \Gamma_{12} &= Q(\varpi)P(\varpi), \\ \Gamma_{14} &= \bar{C}^T(\varpi)\bar{D}(\varpi), \\ \Gamma_{33} &= (2\bar{m} - b)I, \\ \Gamma_{44} &= N^T(\varpi)N(\varpi) + \bar{D}^T(\varpi)\bar{D}(\varpi) - \alpha^2 I, \end{aligned}$$

the controller (6) can be applied to realize the $L_2$-gain property (12) of the system (1).

**Proof.** Choose the following Lyapunov-like function

$$V(t) = \beta^T(t)Q(\varpi)\beta(t) + \kappa(t), \tag{16}$$

one can get

$$\begin{aligned} \dot{V}(t) &= \dot{\beta}^T(t)Q(\varpi)\beta(t) + \beta^T(t)Q(\varpi)\dot{\beta}(t) + \beta^T(t)\frac{\partial Q}{\partial \varpi}\dot{\varpi}\beta(t) + \dot{\kappa}(t) \\ &= \dot{\beta}^T(t)Q(\varpi)\beta(t) + \beta^T(t)Q(\varpi)\dot{\beta}(t) + \beta^T(t)\frac{\partial Q}{\partial \varpi}\dot{\varpi}\beta(t) - b\kappa(t) + c_2\beta^T(t)\beta(t) + m - e_1^T(t)e_1(t) \\ &= \beta^T(t)[M^T(\varpi)Q(\varpi) + Q(\varpi)M(\varpi)]\beta(t) + 2d^T(t)N^T(\varpi)Q(\varpi)\beta(t) + 2e_1^T(t)P^T(\varpi)Q(\varpi)\beta(t) \\ &+ \beta^T(t)\frac{\partial Q}{\partial \varpi}\dot{\varpi}\beta(t) - b\kappa(t) + c_2\beta^T(t)\beta(t) + m - e_1^T(t)e_1(t) \\ &\leq \beta^T(t)[M^T(\varpi)Q(\varpi) + Q(\varpi)M(\varpi)]\beta(t) + 2e_1^T(t)P^T(\varpi)Q(\varpi)\beta(t) + m + \beta^T(t)\frac{\partial Q}{\partial \varpi}\dot{\varpi}\beta(t) - b\kappa(t) \\ &+ c_2\beta^T(t)\beta(t) - e_1^T(t)e_1(t) + d^T(t)N^T(\varpi)N(\varpi)d(t) + \beta^T(t)Q(\varpi)Q(\varpi)\beta(t) \\ &= \begin{bmatrix} \beta(t) \\ e_1(t) \\ \sqrt{\kappa(t)} \end{bmatrix}^T \begin{bmatrix} \hat{\Gamma}_{11} & \Gamma_{12} & 0 \\ * & -I & 0 \\ * & * & -b \end{bmatrix} \begin{bmatrix} \beta(t) \\ e_1(t) \\ \sqrt{\kappa(t)} \end{bmatrix} + d^T(t)N^T(\varpi)N(\varpi)d(t) + m, \end{aligned}$$

where

$$\hat{\Gamma}_{11} = M^T(\varpi)Q(\varpi) + Q(\varpi)M(\varpi) + \frac{\partial Q}{\partial \varpi}\dot{\varpi} + c_2 + Q(\varpi)Q(\varpi).$$

Thus, the following inequality can be held

$$
\begin{aligned}
&\dot{V}(t) + 2\bar{m}V(t) + y^T(t)y(t) - \alpha^2 d^T(t)d(t) \\
&\leq \begin{bmatrix} \beta(t) \\ e_1(t) \\ \sqrt{\kappa(t)} \end{bmatrix}^T \begin{bmatrix} \hat{\Gamma}_{11} + 2\bar{m} & \Gamma_{12} & 0 \\ * & -I & 0 \\ * & * & \Gamma_{33} \end{bmatrix} \begin{bmatrix} \beta(t) \\ e_1(t) \\ \sqrt{\kappa(t)} \end{bmatrix} \\
&+ d^T(t)N^T(\varpi)N(\varpi)d(t) + \beta^T(t)\bar{C}^T(\varpi)\bar{C}(\varpi)\beta(t) \\
&+ 2\beta^T(t)\bar{C}^T(\varpi)\bar{D}(\varpi)d(t) + d^T(t)\bar{D}^T(\varpi)\bar{D}(\varpi)d(t) \\
&- \alpha^2 d^T(t)d(t) + m \\
&= \begin{bmatrix} \beta(t) \\ e_1(t) \\ \sqrt{\kappa(t)} \\ d(t) \end{bmatrix}^T \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & 0 & \Gamma_{14} \\ * & -I & 0 & 0 \\ * & * & \Gamma_{33} & 0 \\ * & * & * & \Gamma_{44} \end{bmatrix} \begin{bmatrix} \beta(t) \\ e_1(t) \\ \sqrt{\kappa(t)} \\ d(t) \end{bmatrix} + m.
\end{aligned}
$$

From (15), we can get

$$
\dot{V}(t) + 2\bar{m}V(t) + y^T(t)y(t) \leq \alpha^2 d^T(t)d(t) + m,
$$

when $d(t) = 0$, one have

$$
\dot{V}(t) + 2\bar{m}V(t) \leq m.
$$

It is not difficult to draw that

$$
V(t) \leq e^{-2\bar{m}t}V(0) + \frac{m}{2\bar{m}}(1 - e^{-2\bar{m}t})
$$

which means

$$
\beta^T(t)Q(\varpi)\beta(t) \leq e^{-2\bar{m}t}V(0) + \frac{m}{2\bar{m}}(1 - e^{-2\bar{m}t}).
$$

Thus, both the system state $x(t)$ and the estimation error $e(t)$ converge exponentially to the region

$$
s(v) = \{v \in R : \|\beta(t)\| \leq \sqrt{\frac{m}{2\bar{m}\lambda_{\min}(Q(\varpi))}}\}.
$$

This implies the practical stability of the system (1) with $d(t) = 0$ and the convergence of the observer (3) can be obtained.

When $d(t) \neq 0$, for $\forall d(t) \in L_2[0, \infty]$, integral of inequality $\dot{V}(t) + 2\bar{m}V(t) + y^T(t)y(t) - \alpha^2 d^T(t)d(t) \leq m$ at zero initial conditions respecting to the variable $p$ produces, one can get

$$
\int_0^\infty [y^T(p)y(p) - \alpha^2 d^T(p)d(p)]dp \leq \frac{m}{2\bar{m}}, \tag{17}
$$

thus, we can get the $L_2$-gain property (12) while $t \to \infty$ and where $\frac{m}{2\bar{m}} \leq \aleph$.

*Remark 4.* Theorem 2 gives a condition by which the dynamic event-triggered-based anti-disturbance control problem of the system (1) with (2) is solved. Furthermore, the linear matrix inequality (LMI) expressed in the (15) contains nonlinear terms, which makes it difficult to solve.

Now let us explain in detail how to solve LMI (15).

**Theorem 3.** If there exist $\Pi_1(\varpi) > 0, \Pi_2(\varpi) > 0$, matrices $K(\varpi), \Lambda(\varpi)$, scalars $\alpha > 0$, $b > 0$, $\bar{m} > 0$, $2\bar{m} - b < 0$ and $c_2 > 0$ satisfying the following constraint

$$\begin{bmatrix} \phi_{11} & \phi_{12} & B(\varpi) & 0 & \phi_{15} & 0 & \Pi_1(\varpi) & 0 & \phi_{19} & 0 & 0 \\ * & \phi_{22} & 0 & 0 & 0 & 0 & 0 & I & 0 & \Upsilon(\varpi) & 0 \\ * & * & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \phi_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \phi_{55} & \phi_{56} & 0 & 0 & 0 & 0 & \phi_5^{11} \\ * & * & * & * & * & \phi_{66} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \phi_{77} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \phi_{77} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0, \tag{18}$$

where

$$\begin{aligned} \phi_{11} &= \Pi_1(\varpi)A^T(\varpi) + \Pi_1(\varpi)K^T(\varpi)B^T(\varpi) + A(\varpi)\Pi_1(\varpi) + B(\varpi)K(\varpi)\Pi_1(\varpi) - \dot{\Pi}_1(\varpi) + I, \\ \phi_{12} &= B(\varpi)E(\varpi), \\ \phi_{15} &= \Pi_1(\varpi)C^T(\varpi)D(\varpi), \\ \phi_{19} &= \Pi_1(\varpi)C^T(\varpi), \\ \phi_{22} &= \Upsilon(\varpi)G(\varpi) + \Upsilon(\varpi)\Lambda(\varpi)B(\varpi)E(\varpi) + \dot{\Pi}_2(\varpi)E^T(\varpi)B^T(\varpi)\Lambda^T(\varpi)\Upsilon(\varpi) + G^T(\varpi)\Upsilon(\varpi) + I, \\ \phi_{44} &= (2\bar{m} - b)I, \\ \phi_{55} &= B^T(\varpi)B(\varpi) + D^T(\varpi)D(\varpi) - \alpha^2, \\ \phi_5^{11} &= B^T(\varpi)\Lambda^T(\varpi), \\ \phi_{56} &= B^T(\varpi)\Lambda^T(\varpi)H(\varpi), \\ \phi_{66} &= H^T(\varpi)H(\varpi) - \alpha^2, \\ \phi_{77} &= -(2\bar{m} + c_2)^{-1}I, \end{aligned}$$

then, the controller (6) is a solution to the dynamic event-triggered-based anti-disturbance control issue of the system (1).

**Proof.** From Theorem 2, it is clear that if (15) is ensured, then the issue of dynamic event-triggered-based anti-disturbance for the system (1) is addressed.

Letting $\Upsilon(\varpi) = \Pi_2^{-1}(\varpi)$, $Q(\varpi) = \begin{bmatrix} \Pi_1(\varpi) & 0 \\ 0 & \Pi_2(\varpi) \end{bmatrix}$, then substituting (11) into (15) , one can get

$$\begin{bmatrix} \phi_{11} & \Phi_{12} & B(\varpi) & 0 & \phi_{15} & 0 & \Pi_1(\varpi) & 0 & \phi_{19} \\ * & \Phi_{22} & 0 & 0 & 0 & 0 & 0 & \Pi_2(\varpi) & 0 \\ * & * & -I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \phi_{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Phi_{55} & \phi_{56} & 0 & 0 & 0 \\ * & * & * & * & * & \phi_{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & \phi_{77} & 0 & 0 \\ * & * & * & * & * & * & * & \phi_{77} & 0 \\ * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0, \tag{19}$$

where

$$\begin{aligned} \Phi_{12} &= B(\varpi)E(\varpi)\Pi_2, \\ \Phi_{22} &= G(\varpi)\Pi_2(\varpi) + \Lambda(\varpi)B(\varpi)E(\varpi)\Pi_2(\varpi) + \Pi_2(\varpi)E^T(\varpi)B^T(\varpi)\Lambda^T(\varpi) + \Pi_2(\varpi)G^T(\varpi) - \dot{\Pi}_2(\varpi) + I, \\ \Phi_{55} &= B^T(\varpi)B(\varpi) + B^T(\varpi)\Lambda^T(\varpi)\Lambda(\varpi)B(\varpi) + D^T(\varpi)D(\varpi) - \alpha^2, \end{aligned}$$

and then applying the Schur complement lemma, we can get

$$\begin{bmatrix} \phi_{11} & \Phi_{12} & B(\varpi) & 0 & \phi_{15} & 0 & \Pi_1(\varpi) & 0 & \phi_{19} & 0 \\ * & \Phi_{22} & 0 & 0 & 0 & 0 & 0 & \Pi_2(\varpi) & 0 & 0 \\ * & * & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \phi_{44} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \phi_{55} & \phi_{56} & 0 & 0 & 0 & \phi_5^{11} \\ * & * & * & * & * & \phi_{66} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \phi_{77} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \phi_{77} & 0 & 0 \\ * & * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0. \tag{20}$$

Multiplying the inequality (19) to the left and right by $diag\{I, \Upsilon(\varpi), I\}$ and applying the Schur complement lemma again we can obtain the inequality (18).

*Remark 5.* In fact, a smaller $L_2$ gain lever indicates that the considered system has a better disturbance attenuation performance. In other words, a smaller $L_2$ gain lever is more desirable. To obtain a minimal $L_2$ gain lever, the following optimization problem can be utilized in the design process:

$$\min_{\substack{s.t.(18) \\ Q(\varpi)>0}} \alpha .$$

*Remark 6.* By solving Theorem 3, we can get the controller and observer gains and design the controller and observer to meet the control requirements.

## 4 | SIMULATION EXAMPLE

For purpose of the effectiveness illustration, a turbofan example is given to carry the simulation study.

Here, the turbofan mode of[39] represented by

$$\begin{aligned} \begin{bmatrix} \Delta\dot{\alpha}(t) \\ \Delta\dot{\beta}(t) \end{bmatrix} &= A(\varpi(t)) \begin{bmatrix} \Delta\alpha(t) \\ \Delta\beta(t) \end{bmatrix} + B(\varpi(t)) \left[u(t) + d_1(t)\right] + B(\varpi(t))d_2(t), \\ y(t) &= C(\varpi(t)) \begin{bmatrix} \Delta\alpha(t) \\ \Delta\beta(t) \end{bmatrix} + D(\varpi(t))d_2(t) \end{aligned} \tag{21}$$

is considered, in which $\Delta\alpha(t)$ indicates the turbofan state representing the fan speed increment and $\Delta\beta(t)$ presents core speed increment, respectively, $u(t)$ is the input signal representing the fuel flow increment, $y(t)$ is the measurable output signal , $d_1(t)$ and $d_2(t)$ are the noise and disturbance representing the turbofan deterioration parameters.

The parameters in the mode (21) are provided as follows:

$$\begin{aligned} \mathrm{A}(\varpi) &= \begin{bmatrix} -3.6284 & -0.5373 \\ 0.9017 & -4.6475 \end{bmatrix} + \varpi \begin{bmatrix} -1.8470 & -0.7489 \\ -0.0996 & -1.4302 \end{bmatrix}, \\ B(\varpi) &= \begin{bmatrix} 0.01 \\ 0.03 \end{bmatrix} + \varpi \begin{bmatrix} 0.01 \\ 0.05 \end{bmatrix}, \\ C(\varpi) &= \begin{bmatrix} 1 & 0 \end{bmatrix} + \varpi \begin{bmatrix} 0.11 & 0 \end{bmatrix}, \\ D(\varpi) &= 0.12 + 0.11\varpi, \\ G(\varpi) &= \begin{bmatrix} -1.01 & 3 \\ -3.10 & 0 \end{bmatrix} + \varpi \begin{bmatrix} -0.01 & 0.02 \\ -0.10 & 0 \end{bmatrix}, \end{aligned}$$

$$H(\varpi) = \begin{bmatrix} 0.11 \\ -0.02 \end{bmatrix} + \varpi \begin{bmatrix} 0.02 \\ -0.01 \end{bmatrix},$$
$$E(\varpi) = \begin{bmatrix} -1 \\ -15 \end{bmatrix} + \varpi \begin{bmatrix} -1.1 \\ -0.5 \end{bmatrix},$$
$$d_2(t) = 5e^{-t}\sin(2t), \quad d_3(t) = 5e^{-t}\cos(2t),$$
$$a = 10, c_1 = 1.1, c_2 = 5.2, b = 1.6, \mu = 2.1, \bar{m} = 1.03.$$

By solving the relation of Theorem 3, we derive $\theta = 2.5$,

$$Q(\varpi) = \begin{bmatrix} 1.2141 & 0.8957 & 0 & 0 \\ 0.8957 & 1.2409 & 0 & 0 \\ 0 & 0 & 1.3915 & -0.1453 \\ 0 & 0 & -0.1453 & 2.5927 \end{bmatrix} + \varpi \begin{bmatrix} 1.1211 & 0.8277 & 0 & 0 \\ 0.8277 & 1.8044 & 0 & 0 \\ 0 & 0 & 2.1798 & -0.2099 \\ 0 & 0 & -0.2099 & 3.5132 \end{bmatrix},$$
$$\Lambda(\varpi) = \begin{bmatrix} 9 & 0 \\ 0 & 27 \end{bmatrix} + \varpi \begin{bmatrix} 18 & 0 \\ 0 & 36 \end{bmatrix},$$
$$K(\varpi) = \begin{bmatrix} 3.5 & 0 \\ 0 & 1.75 \end{bmatrix} + \varpi \begin{bmatrix} 7 & 0 \\ 0 & 2.333 \end{bmatrix}.$$

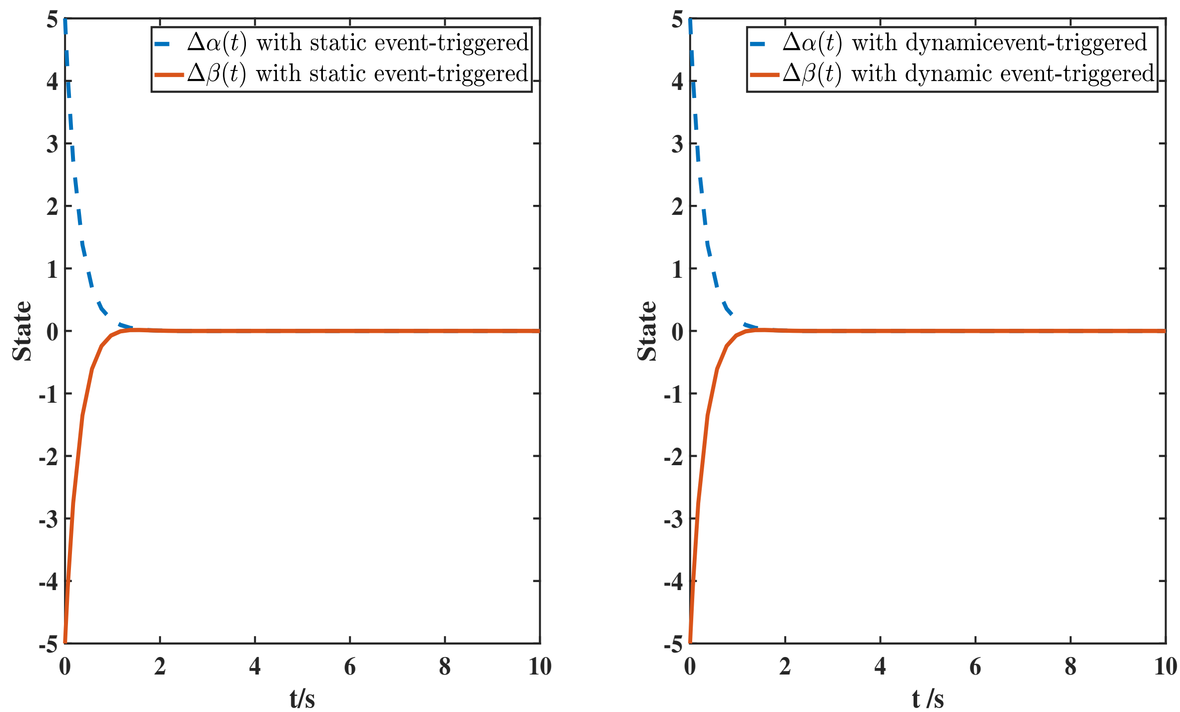


**FIGURE 2** The fan speed increment $\Delta\alpha(t)$ and core speed increment $\Delta\beta(t)$ .

The simulation results are shown by Figs. 2-5. Fig. 2 exhibits the fan speed increment $\Delta\alpha(t)$ and core speed increment $\Delta\beta(t)$. Fig. 3 shows the disturbance estimation errors $e_1(t)$) and $e_2(t)$. The trajectory of control signal $u(t)$ and $u(t_k)$ are shown by Fig. 4. Fig. 5 depicts the inter-intervals of the ET rule (7) expressed by $\{t_{k+1} - t_k\}$. Easily, it can be observed from Figs. 2 and 3 that the system state and the observation error tend to zero. From Fig. 4 we can see that the control signal $u(t)$ also tend to zero in the case of without , static, dynamic event-triggered and the communication resources are reduced. And Fig. 5 shows that the event-triggered interval is greater than zero. Especially, we can see that the proposed dynamic event-triggered method

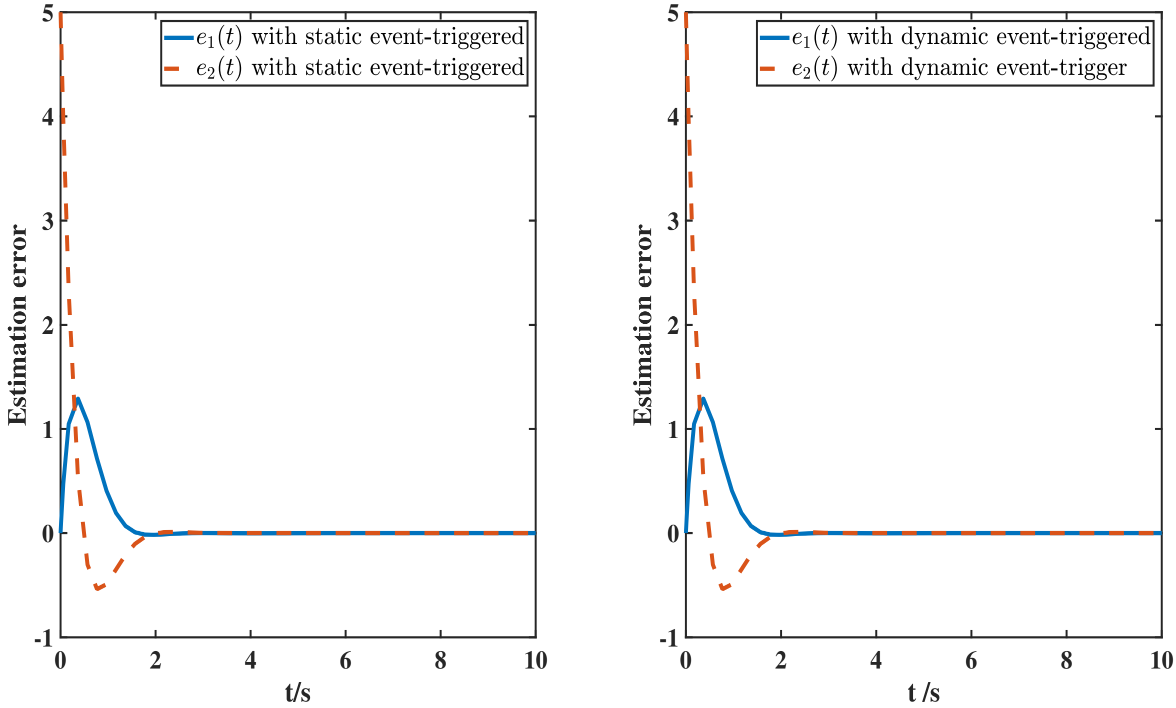


**FIGURE 3** The disturbance estimation error $e_1(t)$ and $e_2(t)$

.

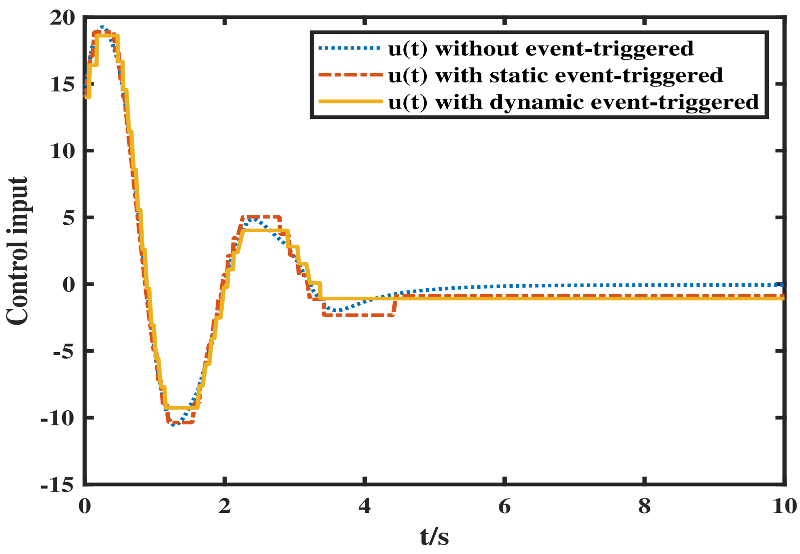


**FIGURE 4** The control signal $u(t)$.

has a longer event-triggered interval than the static event-triggered strategy[45], and thus, saves more communication resources. Therefore, we can declare that the designed dynamic event-triggered-based anti-disturbance control scheme is effective.

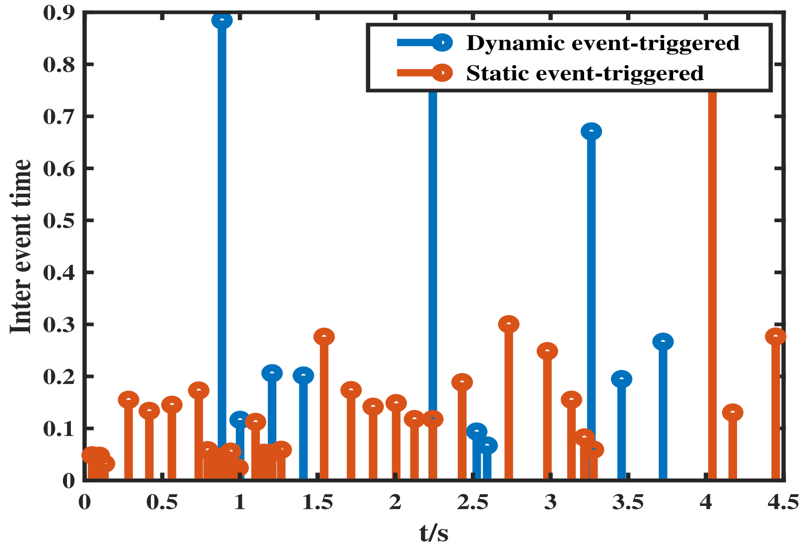


**FIGURE 5** The event-triggered interval $\{t_{k+1} - t_k\}$.

## 5 | CONCLUSIONS

In this paper, we have studied the dynamic event-triggered-based anti-disturbance control technique for the uncertain linear parameter varying systems subject to multiple disturbances. First, the unavailable disturbances have been estimated by a disturbance estimator. Second, the dynamic event-triggered criterion has been set based on the system input signal. The proposed dynamic event-triggered mechanism has a longer event-triggered interval than most existing static event-triggered strategy. Third, the dynamic event-triggered-based anti-disturbance control strategy has been established under which the theoretical condition has been developed to assure the solvability of the dynamic event-triggered based anti-disturbance control issue for the uncertain LPV system. Under the presented dynamic event-triggered-based anti-disturbance control method, the unmeasurable modeling disturbance is compensated, the measurable unmodeled disturbance is suppressed. At last, the turbofan case study has been provided to exhibit how the developed dynamic event-triggered-based anti-disturbance control strategy effectively works. The main difficulties is how to achieve the multi-resource disturbance alleviation, while saving the communication transmission resources for the uncertain LPV systems. In addition, in this paper, the considered LPV system is continuous-time, and the delay influence may be cased by the event-triggered mechanism is also not considered. In the future, we will make the further study in the dynamic event-triggered-based anti-disturbance control for the discrete time LPV systems with time delay.

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