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Nonlinear Continuous-time System Identification by Linearization around a Time-varying setpoint†

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Summary

This paper handles the identification of nonlinear systems through linear time-varying (LTV) approximation. The mathematical form of the nonlinear system is unknown and regenerated through an experiment followed by LTV and linear parameter-varying (LPV) estimation and integration. By employing a well-designed experiment the linearized model of the nonlinear system around a time-varying trajectory is obtained. The result is an LTV approximation of the nonlinear system around that trajectory. Having estimated the LTV model, an LPV model is identified. It is shown that the parameter-varying (PV) coefficients of this LPV model are partial derivatives of the nonlinear system evaluated at the trajectory. In this paper, we will show that there exists a relation between the LPV coefficients. This structural relation in the LPV model ensures the integrability of PV coefficients for nonlinear reconstruction. Indeed, the vector of the LPV coefficients is the gradient of the nonlinear system evaluated at the trajectory. Then, the nonlinear system is reconstructed through symbolic integration of the coefficients. The proposed method is a data-driven scheme that can reconstruct an estimate of the nonlinear system and its mathematical form using input-output measurements. Finally, the use of the proposed method is illustrated via a simulation example.

KEYWORDS:
Data driven identification, nonlinear system identification, LTV approximation, frequency domain

1 | INTRODUCTION

Obtaining a sufficiently accurate model is the first step in the design, analysis, and control of dynamical systems. For many applications modeling based on the first principles is difficult or leads to very complex models which are computationally expensive with limited applicability. For such applications, data-driven methods could provide more compact and interpretable

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models. In this paper, we investigate the data-driven nonlinear system identification through LTV approximations around a time-varying trajectory.

Depending on the characteristics of the system and the underlying problem, various methods for identifying nonlinear systems have been developed. The nonlinear autoregressive moving average with exogenous input (NARMAX) method [1],[2], block-oriented methods [3],[4], nonlinear subspace identification [5] and neural networks [6],[7] are just a few to name. While methods based on a NARMAX structure have been applied successfully for the identification of nonlinear systems, the main limitation in their use is prior knowledge about the form and structure of nonlinear equations governing the system. Neural networks require a large data set for training due to their large parameter space, and selecting the structure of the neural networks is also challenging. Volterra series are very general tools to approximate nonlinear systems but they are cumbersome to handle and suffer from the curse of dimensionality. The Restoring Force Surface (RFS) method [8] provides satisfactory results for the identification of structured second-order systems with multiple degrees of freedom while extending this method to higher-order systems is challenging. In conclusion, the best choice for the identification method depends on the application at hand and associated assumptions. However, large parameter space, high-order systems, and prior knowledge of system structure are important issues in identifying nonlinear systems. The aim of this paper is to present a unified and systematic method to identify a class of continuous time nonlinear systems through linearization. The linearization approach in the identification of nonlinear systems leads to breaking the problem of identifying the nonlinear system into smaller sub-problems like LTV and LTI models, for which there exist systematic and optimal tools for estimation (see e.g. [9,10,11,12]).

In the Jacobian linearization framework, the linearized model around a single or family of equilibrium points corresponds to the Jacobian of the nonlinear system evaluated at the corresponding point (see for example [13], [14]). These local approximations are the basis for the local LPV modeling [15], [16], which is basically the interpolation of a collection of local LTI approximations of the nonlinear system. Local LPV modeling is one existing approach for modeling the nonlinear system through linearization. While the validity of the model is confined to the neighborhood of the isolated points. Instead of considering the equilibrium working points, linearization can be done on a time-varying trajectory. In many applications, this trajectory can be chosen to cover the envelope of the operational region of the system or the transition between equilibrium working points. In this case, the resulting system is a global LPV ([17], [9]), where the scheduling variable is the trajectory, and the validity of the model is extended to all the points along the trajectory and a neighborhood around it. However, since the equations of the nonlinear system are not available, analytical linearization of the system is not possible. Therefore, we use a data-driven method to identify the linearized system. In this paper, the experiment is designed in such a way that the nonlinear system can be approximated by an LTV system around a large trajectory. In [18,19] a small deviation of the nonlinear system from a periodic orbit is used to approximate the system by a linear time-periodic (LTP) system around this limit cycle. Then, by identifying this LTP system, one can reconstruct a nonlinear model through symbolic integration [20]. While the form of the nonlinear equations of the system is unknown, with this method, the nonlinear system is reconstructed through the identification of the linear system[20]. In fact, the perturbation data is used to identify the linear approximation of the system around the trajectory. In the present work, the more general case of an *arbitrarily* time-varying trajectory is considered[21],[22]. The nonlinear system is forced on this large slow (stable) trajectory, then we deviate the trajectory slightly via small input perturbations. The perturbation data around the trajectory is used to identify the local linear approximation of the system. Indeed, this is the nonlinear system linearized around the trajectory. In this paper, we show that this linear approximation is a trajectory-scheduled LPV system. Since the scheduling variable is a time-varying signal, the LPV has an LTV representation, so the optimal and systematic LTV estimators can be used (e.g.[23],[24]). Then the associated LPV model will be obtained from the estimated LTV model. We will show that the vector of the LPV coefficients is the gradient of the nonlinear system evaluated at the trajectory. Then, we will introduce a model structure for the corresponding LPV. This model ensures that the vector of the LPV coefficients is always the gradient of an unknown nonlinear system[25]. Hence, the proposed LPV parametrization ensures the integration of the LPV coefficients is path independence. Indeed, these coefficients are partial derivatives of the nonlinear system evaluated at the trajectory. Therefore, by having these coefficients one can regenerate the nonlinear system equations by symbolic integration. The main contribution of this paper is the proposed nonlinear identification approach. A new LPV parametrization and formal integration for nonlinear reconstruction are presented. In Section 2, the formulation of the problem is discussed. As a first step in Section 3.1, an experiment is designed so that the nonlinear system can be approximated around a large slow trajectory by an LTV system. Then the LTV system is identified from input-output measurements using the proposed method in[23]. In[23] a frequency domain kernel-based estimator is proposed for the identification of continuous-time LTV systems. This method is briefly summarized in Section 3.2. In Section 3.3 using the estimated LTV model, the corresponding LPV model is built. It is guaranteed that the coefficients of the LPV model is the gradient of an unknown nonlinear differential equation. Finally, the nonlinear differential equation is reconstructed through integration of the LPV model in Section 3.4. The proposed LPV parametrization ensures that the integration is unique in terms of path independence. Section 4 illustrates the method on a simulation example. The conclusions from this study are summarized in Section 5.

2 | PROBLEM FORMULATION

The class of the nonlinear systems that we are interested in is defined as below:

Definition 1 (System under test)**.** We consider a single-input single-output (SISO), continuous in time, nonlinear system which is defined by the following equation:

$$f(y(t), \dots, y^{(n_y)}(t), u(t), \dots, u^{(n_u)}(t)) = 0 \tag{1}$$

Where $u(t)$ and $y(t)$ denote the time domain input and output of the system respectively and n_u and n_y are known positive integers. Also, $\bullet^{(n)}$ denotes the n^{th} order derivative of $\bullet$. By convention, $\bullet^{(0)} = \bullet$. Also, $f : \mathbb{D} \longrightarrow \mathbb{R}$ is a static nonlinear function where $\mathbb{D} \subset \mathbb{R}^{n_u+n_y+2}$.

The aim is to model the system (1) along an arbitrary, smooth system trajectory in a finite time interval. The modeling procedure involves the linearization of the system along a system trajectory which is called large trajectory, as defined next.

Definition 2 (Large trajectory)**.** Any output of the system (1) with its derivatives up to the order n_y along with the corresponding input $u(t)$ with its derivatives up to the order n_u within the time interval $[t_i, t_f]$ form a vector function which is called large trajectory $R(t)$, as below:

$$R(t) = [y(t), \dots, y^{(n_y)}(t), u(t), \dots, u^{(n_u)}(t)]^T, t \in [t_i, t_f] \tag{2}$$

The specific large trajectory that the system is linearized around is denoted by $R_L(t)$.

We would like to estimate (1) along $R_L(t)$. For this purpose, another large trajectory other than $R_L(t)$ is required which is called $R(t)$. To this end, the following assumptions are made on the system and the domain of large trajectories.

Assumption 1. The domain $\mathbb{D}$ where the system trajectory $R_L(t)$ and $R(t)$ lie in, is a simply connected domain. This means that this domain is an open set containing $R_L(t)$, $R(t)$ and neighbouring trajectories of the nonlinear system without a hole within it. More precisely:

$$\begin{aligned} R_L(t) \in \mathbb{D} \subset \mathbb{R}^{n_u+n_y+2} \\ R(t) \in \mathbb{D} \subset \mathbb{R}^{n_u+n_y+2} \end{aligned} \tag{3}$$

Assumption 2. f has continuous second order partial derivatives in $\mathbb{D}$.

Considering Assumption 2, it is possible to obtain Taylor's representation of $f(R(t))$ along $R_L(t)$:

$$\begin{aligned} f(R(t)) = f(R_L(t)) + \sum_{n=0}^{n_y} \frac{\partial f}{\partial y^{(n)}}\bigg|_{R_L(t)} (y^{(n)}(t) - y_L^{(n)}(t)) + \sum_{m=0}^{n_u} \frac{\partial f}{\partial u^{(m)})}\bigg|_{R_L(t)} (u^{(m)}(t) - u_L^{(m)}(t)) + \\ \frac{1}{2}(R(t) - R_L(t))^T H\bigg|_{(1-k)R_L(t)+kR(t)} (R(t) - R_L(t)) \end{aligned} \tag{4}$$

Where H stands for the Hessian matrix and $\bullet_L$ are components of the $R_L(t)$. $k \in [0, 1]$ is a real number.

> $R_L(t)$ can be the result of an excitation experiment with an arbitrary input $u_L(t)$ or simply measurements of a system in operation. Here, we don't assume anything about how $R_L(t)$ is acquired.

Taylor's theorem is valid for all $t \in [t_i, t_f]$ because both $R_L(t)$ and $R(t)$ are in $\mathbb{D}$. Equation (1) enforces $f(R_L(t)) = f(R(t)) = 0$. So, we can deduce the below equation:

$$\sum_{n=0}^{n_y} \frac{\partial f}{\partial y^{(n)}}\bigg|_{R_L(t)} (y^{(n)}(t) - y_L^{(n)}(t)) + \sum_{m=0}^{n_u} \frac{\partial f}{\partial u^{(m)})}\bigg|_{R_L(t)} (u^{(m)}(t) - u_L^{(m)}(t)) + \frac{1}{2}(R(t) - R_L(t))^T H\bigg|_{(1-k)R_L(t)+kR(t)} (R(t) - R_L(t)) = 0 \tag{5}$$

The goal is to obtain a linear model around $R_L(t)$. But, second order terms are detrimental here. Therefore, second order term in (5) must be as small as possible such that it is possible to assume that this term is negligible.

Assumption 3. It is assumed that second order terms are negligible compared to the linear part

$$\left\| \frac{1}{2}(R(t) - R_L(t))^T H\bigg|_{(1-k)R_L(t)+kR(t)} (R(t) - R_L(t)) \right\| \ll \left\| \sum_{n=0}^{n_y} \frac{\partial f}{\partial y^{(n)}}\bigg|_{R_L(t)} (y^{(n)}(t) - y_L^{(n)}(t)) + \sum_{m=0}^{n_u} \frac{\partial f}{\partial u^{(m)})}\bigg|_{R_L(t)} (u^{(m)}(t) - u_L^{(m)}(t)) \right\|$$

Definition 3 (slightly perturbed large trajectory)**.** A large trajectory $R(t)$ that fullfils assumption (3) is a slightly perturbed large trajectory.

Definition 4 (Small trajectory)**.** A small trajectory is a difference between a slightly perturbed large trajectory $R(t)$ and $R_L(t)$. The small trajectory will be denoted by $\tilde{R}(t)$. More precisely:

$$\begin{aligned} &\tilde{R}(t) = R(t) - R_L(t) \\ &\tilde{R}(t) = [\tilde{y}(t), \dots, \tilde{y}^{(n_y)}(t), \tilde{u}(t), \dots, \tilde{u}^{(n_u)}(t)]^T, t \in [t_i, t_f] \end{aligned} \tag{6}$$

where $R(t)$ is a "slightly perturbed large trajectory" according to definition 3.

In theory, it is possible to make the second order terms negligible by making $\tilde{R}(t)$ as small as possible. But, in practice, this may lead to a poor SNR. So, there is a trade-off between SNR and nonlinear distortions. It is possible to tune this trade-off by prior knowledge about the system, by gradually increasing the small trajectory in an iterative procedure to check for nonlinear distortions (possibly via a non-parametric method like[26]) or simply by estimating the linear model and validating it. Anyway, if it is not possible to maintain this trade-off, it is not possible to use this approach towards nonlinear identification.

Considering assumption (3), we can proceed and deduce the below equation from (5):

$$\sum_{n=0}^{n_y} \frac{\partial f}{\partial y^{(n)}}\bigg|_{R_L(t)} \tilde{y}^{(n)}(t) + \sum_{m=0}^{n_u} \frac{\partial f}{\partial u^{(m)})}\bigg|_{R_L(t)} \tilde{u}^{(m)}(t) = 0 \tag{7}$$

The main contribution of this paper is the estimation of the system (1) along $R_L(t)$. This goal is achieved in a sequential procedure with (7) as its first step. As (7) is linear time-varying, it is also possible to express it explicitly as a linear time-varying model as below :

Definition 5. The linear time-varying system which is constructed in a linearization procedure described as above is called "true LTV system" from now on and is defined precisely as :

$$\begin{aligned} &\sum_{n=0}^{n_y} a_n(t)\tilde{y}^{(n)}(t) + \sum_{m=0}^{n_u} b_m(t)\tilde{u}^{(m)}(t) = 0 \\ &a_n(t) = \left.\frac{\partial f}{\partial y^{(n)}}\right|_{R_L(t)}, b_m(t) = \left.\frac{\partial f}{\partial u^{(m)}}\right|_{R_L(t)} \end{aligned} \tag{8}$$

The model (8) will be estimated as an intermediate step towards model (7), which will in turn be used to reconstruct the full nonlinear model (1).

3 | PROPOSED APPROACH

In this section, the LTV and LPV models (8) and (7) are estimated in Sections 3.2 and 3.3 respectively, from which the nonlinear model will be reconstructed in Section 3.4. First, the design of the experiment is considered.

3.1 | Experiment Design

While the nonlinear function (1) is unknown, we use experimental data to obtain its linearized model. Consider a large trajectory $R_L(t)$ of the system in Definition 2. Then, $R_L(t)$ is perturbed by adding some small input signals $\tilde{u}(t)$, which leads to a slightly perturbed large trajectory $R(t)$ under Definition 3. This input perturbation is typically a small-fast signal which leads to small fast variations around $R_L(t)$. To identify the LTV system (8) we need to obtain the large trajectory $R_L(t)$ and small trajectory $\tilde{R}(t)$ separately. Therefore, we perform two experiments:

1. With only a large trajectory $u_L(t)$ as excitation, resulting in $R_L(t)$.
2. By adding some input perturbations, resulting in the slightly perturbed large trajectory $R(t)$.

Then the small trajectory $\tilde{R}(t)$ can be computed in (6).

Assumption 4. The influence of input and output derivatives on LPV coefficients is negligible and the coefficients can be approximated as follows

$$\begin{aligned} a_n(t) &\approx \frac{\partial f}{\partial y^{(n)}}\bigg|_{[y_L(t),u_L(t)]} \\ b_m(t) &\approx \frac{\partial f}{\partial u^{(m)}}\bigg|_{[y_L(t),u_L(t)]} \end{aligned} \tag{9}$$

This assumption makes the scheduling vector of the LPV a measurable quantity. And (9) can be satisfied through experiment, by making the large trajectory slow enough. However, if the trajectory of the system is directly measurable then this assumption can be relaxed.

3.2 | LTV Estimation

The obtained perturbation data around $R_L(t)$ satisfy the model structure (8) which is the LTV approximation of the system around $R_L(t)$. The goal of this section is to estimate the time-varying coefficients $a_n(t)$ and $b_m(t)$ in (8). This is mainly achieved by parametrization of the LTV coefficients and minimizing an empirical cost function over the measurement time. Many estimators have been proposed to identify LTV systems [27,28,29], however, any LTV estimator for the model structure (8) is applicable.

Definition 6 (Estimated LTV coefficients)**.** We define the estimation of time-varying coefficients of the LTV system (8) as $\hat{a}_n(t)$, $\hat{b}_m(t)$, respectively. Which are the result of any appropriate LTV estimation with the model structure (8).

For example, a consistent estimate of the coefficients is presented in [24]. And in [23], a regularized weighted least squares method is proposed to estimate the coefficients in a smooth reproducing kernel Hilbert space.

Since the considered nonlinear system is continuous and in the input-output form, we employ a frequency domain kernel-based estimator [23] to identify these LTV systems. This estimator adopts a mixed time and frequency domain formulation which enables the identification of continuous-time LTV systems. The time-varying coefficients $a_n(t)$, $b_m(t)$ are estimated as minimizers of a regularized weighted least squares cost function using input/output measurements in the frequency domain in order to impose the smoothness of the time-varying coefficients. While the LTV approximation is based on the small perturbation of the large trajectory, measurement noise could be an issue in the LTV identification. The estimator [23] is used because it can provide reliable results in the presence of input-output measurement noise. On the other hand, the model complexity selection of the time-varying parameters can be formulated as an optimization problem with continuous variables.

3.3 | LPV Model

The LTV model (8) has an LPV representation. In the most general form, the time-varying coefficients $a_n(t)$, $b_m(t)$ are functions of the large trajectory:

Definition 7 (Parameter-varying coefficients)**.** We define the parameter-varying coefficients of the system (8) as follows

$$
\begin{aligned}
a_n(t) &= a_n(R_L(t)) \\
b_m(t) &= b_m(R_L(t))
\end{aligned}
\tag{10}
$$

Then the LTV (8) can be seen as an LPV system (7) in which the scheduling vector is the large trajectory. The coefficients (10) are the partial derivatives of the function $f(.)$ evaluated at the large trajectory (see (7)). Having these derivatives, the function $f(.)$ can be reconstructed through symbolic integration. But modeling these coefficients as a combination of general basis functions does not necessarily lead to an LPV with integrable coefficients [25]. In the following, we impose a set of constraints that ensure that the vector of parameter-varying coefficients of the LPV model is always a gradient vector.

Definition 8. For the sake of clarity, we define the following two vectors:

$$
\mathrm{C}(R_L(t)) = [a_0(R_L(t)), \dots, a_{n_y}(R_L(t)), b_0(R_L(t)), \dots, b_{n_u}(R_L(t))] = [c_1(R_L(t)), c_2(R_L(t)), \dots, c_{n_u+n_y+2}(R_L(t))] \tag{11}
$$

$$
R_L(t) = [r_1(t), \dots, r_{n_u+n_y+2}(t)] = [y_L(t), \dots, y_L^{(n_y)}(t), u_L(t), \dots, u_L^{(n_u)}(t)] \tag{12}
$$

Definition 9 (Curl-free vector field)**.** Consider a vector field $C \in C^1$ defined on $\mathbb{D}$, then the Curl operator is defined as:

$$
\mathrm{Curl}(C) = \nabla \times C \tag{13}
$$

where ∇ is the vector differential (gradient) operator and $\times$ is the cross product. Then, C is curl-free when its curl is equal to zero (see [30, Section 16]).

Since the integration of the gradient vector field is independent of the integration path [30, Section 16], to reconstruct the NL system from the LPV coefficients, the parameterization of the LPV coefficients must be in such a way that it is always a gradient.

Theorem 1. Consider the nonlinear system (1) and the LPV system (7). The vector of LPV coefficients (7) is the gradient of an unknown function, if the large trajectory lies in a simply connected domain $\mathbb{D}$ and the vector of PV coefficients are curl-free. Then the parametrization of the LPV must satisfy the following constraint:

$$
\nabla \times \mathrm{C}(R_L(t)) = 0 \tag{14}
$$

which can be expanded as:

$$
\frac{\partial c_i(R_L(t))}{\partial r_j} = \frac{\partial c_j(R_L(t))}{\partial r_i} \tag{15}
$$

with $i, j = 1, 2, \ldots, (n_u + n_y + 2), i \neq j$, and under the Assumption 4 simplified to

$$\frac{\partial a_0(y_L(t), u_L(t))}{\partial u_L} = \frac{\partial b_0(y_L(t), u_L(t))}{\partial y_L}. \tag{16}$$

Proof. The parameter-varying coefficients $a_n(R_L(t))$ and $b_m(R_L(t))$ are partial derivatives of an unknown function $f(.)$. We put all of these partial derivatives into a vector in (11). Then, we guarantee that this vector is always a gradient. The necessary and sufficient condition for a vector field to be a gradient is that its curl is zero in a simply connected domain (see Section 16[30]). This is obtained by satisfying the condition (15) in $\mathbb{D}$. □

Theorem 1 shows that the LPV coefficients are related, and relation (15) must be considered in the LPV estimation. Otherwise, there is no guarantee that the coefficients are integrable in terms that the integration is unique and path independent.

We estimate the LPV model by having the estimated LTV coefficients in Definition 6. We start by parameterizing the LPV coefficients by a linear combination of known/chosen basis functions so that the scheduling signal is the large trajectory $R_L(t)$. It is possible to approximate the LPV coefficients (10) as follows:

$$\begin{aligned} \breve{a}_n(R_L(t)) &= \sum_{i=0}^{N_p} \alpha_{n,i} \phi_i(R_L(t)) \\ \breve{b}_m(R_L(t)) &= \sum_{i=0}^{N_p} \beta_{m,i} \phi_i(R_L(t)) \end{aligned} \tag{17}$$

where $\alpha_{n,i}, \beta_{m,i}$ are unknown constants to be estimated, $\phi_i(R_L(t))$ are basis functions, and N_p is the expansion order.

To estimate the LPV model, as a first step, we select the type of basis functions and the expansion order N_p in (17). To guarantee the zero curl of the LPV model coefficients, condition (15) is applied to the parametrization (17). Note that applying (15) to the LPV parametrization can change the expansion order N_p for some coefficients (as shown in[25]).

Then, the parameters $\alpha_{n,i}, \beta_{m,i}$ in (17) are obtained using linear regression by considering the estimated LTV coefficients in Definition 6 as the output of the regression problem.

Definition 10 (Estimated LPV coefficients)**.** We define the estimates of the parameter-varying coefficients of the LPV representation of (7) as $\breve{a}_n(R_L(t)), \breve{b}_m(R_L(t))$, respectively. These are the solutions of the following problem:

$$\underset{\alpha_{n,i}, \beta_{m,i}}{\arg\min}\, \mathcal{E}\left(\begin{pmatrix} \breve{a}_n(R_L(t)) \\ \breve{b}_m(R_L(t)) \end{pmatrix} - \begin{pmatrix} \hat{a}_n(t) \\ \hat{b}_m(t) \end{pmatrix}\right) \tag{18}$$

Where $\mathcal{E}(.)$ is an empirical cost function aiming to obtain coefficients through data fit (e.g. least squares).

Since the LPV coefficients are dependent (through (15)), the regression problem (18) is a multi-input multi-output problem that must be solved simultaneously for all coefficients. The LPV estimation procedure is summarized as follows:

1. Consider LPV coefficients (17) and fix the order of expansions for a chosen/known combination of basis functions.

2. Apply the curl-free constraint (15) to the LPV coefficients.
3. Having the estimated LTV coefficients in Definition 6, and LPV parametrization of step (2), the parameters of the LPV coefficients are obtained through (18).

The vector (11) is the gradient of an unknown function and once this vector is obtained , the nonlinear map can be reconstructed by integration. This is elaborated in the following part.

3.4 | Nonlinear Reconstruction

As a final step, the nonlinear model reconstruction from the LPV model is provided. The main idea here is to use the fact that the LPV model coefficients are the gradient of the scalar multivariable function f in (1), and that it is possible to recover the function f by the techniques for retrieving a potential function from a conservative field. To this purpose, equation (7) is re-written in the following form:

$$\nabla f(t) \cdot \tilde{R} = \nabla f(R_L(t)) \cdot \tilde{R} = 0 \tag{19}$$

Where ∇f and $\tilde{R}$ denote the gradient of f and the small trajectory respectively, and "$\cdot$" denotes the standard inner product. Also,

$$\begin{aligned} &\nabla f(t) = [a_0(t), \dots, a_{n_y}(t), b_0(t), \dots, b_{n_u}(t)] \\ &\nabla f(R_L(t)) = \mathrm{C}(R_L(t)) = [a_0(R_L(t)), \dots, a_{n_y}(R_L(t)), b_0(R_L(t)), \dots, b_{n_u}(R_L(t))]. \end{aligned} \tag{20}$$

Where $\nabla f(t)$ is the true vector of LTV system coefficients and $\nabla f(R_L(t))$ is the true vector of LPV system coefficients. Similarly, $f(t)$ describes f as a function of time and $f(R_L(t))$ describes f as a function of the large signal elements. Provided that $\nabla f(t)$ is known exactly everywhere in $\mathbb{D}$, the "fundamental theorem of line integrals" or "gradient theorem"[31] allows to obtain f as:

$$\begin{aligned} &\int_{R_L(t)} \nabla f.dr = f(R_L(t)) - f(R_L(t_i)) \\ &t \in [t_i, t_f] \end{aligned} \tag{21}$$

Where t is any general arbitrary time between t_i and t_f.

$R_L(t)$ is a function of time and the standard way to calculate this integral is to calculate $\nabla f(t)$ on $R_L(t)$ and the derivative of $R_L(t)$. ($dr = dR_L(t) = \dot{R}_L(t)dt$). Now, everything is a function of time and it can be calculated as a single integral of time t. The result will be a function of time ($f(t)$) for an arbitrary t. For our problem, this procedure leads to an absolute 0 as we know that $f(R_L(t)) = f(t) = 0$. Even if it was possible to obtain f as a function of time, it could not be useful as we want $f(R_L(t))$ not $f(t)$. So, we have to find a way to calculate the integration (21) which leads to $f(R_L(t))$.

Fortunately, there is a standard way to retrieve a potential field from a known static conservative field via integration along a sequence of orthogonal paths which relies on the path independency of the integration of a gradient field. Inspired by this algorithm, we come up with a technique to calculate the $f(R_L(t))$ properly. In the remainder of this section, we elaborate this technique in more details. First, some auxiliary functions will be introduced.

Definition 11 (Points)**.** Define $n_y + n_u + 3$ functions called "points" $P_l(R_L(t)) \in \mathbb{D}, l = 0, 1, \dots, n_y + n_u + 2$, as below:

$$P_{lj}(R_L(t)) = \begin{cases} r_j(t) & j \le l \\ r_j(t_i) & j > l, \end{cases}$$

With $P_{lj}(R_L(t))$ being the j-th component of $P_l(R_L(t))$ and $r_j(t)$ are as (12).

Using definition (11), a line segment is defined as follows.

Definition 12 (Line segment)**.** define $L_l, l = 1, 2, \dots, n_y + n_u + 2$ as a line segment that connects P_{l-1} to P_l and lies in $\mathbb{D}$. More specifically:

$$L_l(\xi, R_L(t)) = \xi P_l + (1-\xi)P_{l-1}\,, \quad 0 \le \xi \le 1$$

Where P_l are points, from Definition 11.

The elementwise equivalence of Definition 12 is given by

$$\begin{aligned} L_l =&(r_1(t), \dots, r_{l-1}(t), \xi r_l(t) + (1-\xi)r_l(t_i), r_{l+1}(t_i), \\ &\dots, r_{n_u+n_y+2}(t_i)), \quad \text{for } 0 \le \xi \le 1 \end{aligned}$$

Obviously, L_l evolves only along the l^{th} component of $\nabla f(R_L(t))$ (i.e. $\frac{\partial f}{\partial r_l(t)}$) when ξ varies.) So, integration along L_l (when ξ varies from 0 to 1) turns into a single integral where just $\frac{\partial f}{\partial r_l(t)}$ (or $c_l(R_L(t))$) involves and just ξ (or $r_l(t)$) takes part in integration. (See Fig. 1.) More specifically :

$$\int\limits_{L_l} C(R_L(t)).dr = \int\limits_{\xi P_l + (1-\xi)P_{l-1}} C(R_L(t)).dr = \int\limits_0^1 c_l(L_l) r_l(t) d\xi$$

where

$$dr = dL_l = (0, \dots, 0, (r_l(t) - r_l(t_i))d\xi, 0, \dots, 0)$$

and

$$C(R_L(t)).dr = \frac{\partial f(R_L(t))}{\partial r_l(t)}\bigg|_{L_l} (r_l(t) - r_l(t_i))d\xi$$

By changing the variable $r_{ll} = \xi r_l(t) + (1-\xi)r_l(t_i)$, which leads to $dr_{ll} = (r_l(t) - r_l(t_i))d\xi$, we have:

$$\int\limits_0^1 \frac{\partial f(R_L(t))}{\partial r_l(t)}\bigg|_{L_l} r_l(t) d\xi = \int\limits_{r_l(t_i)}^{r_l(t)} \frac{\partial f(R_L(t))}{\partial r_l(t)}\bigg|_{L_l} dr_{ll}$$

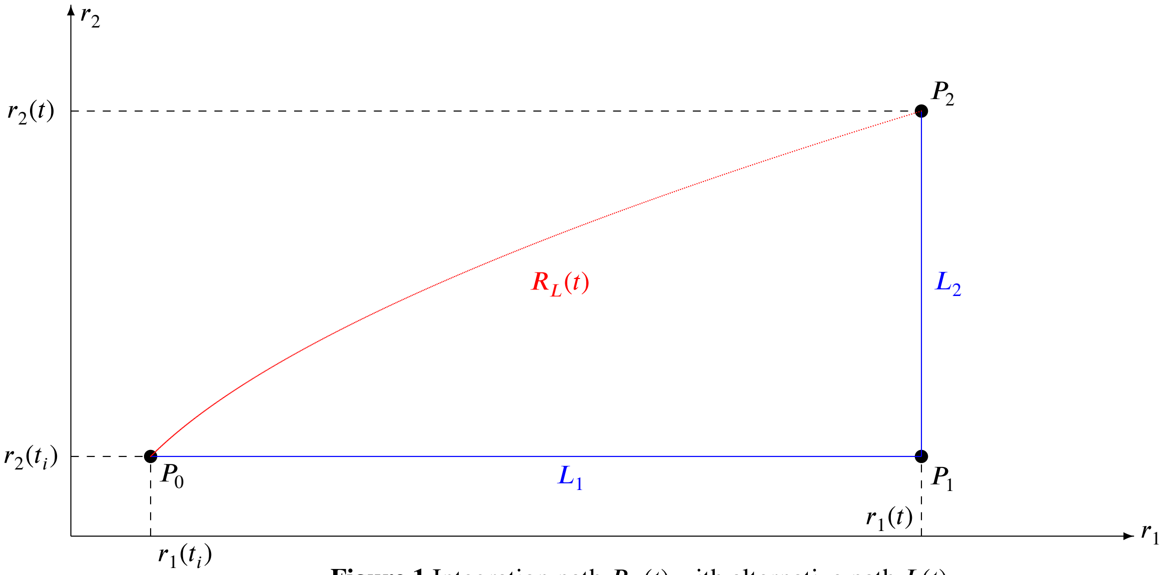

Figure 1 Integration path $R_L(t)$ with alternative path $L(t)$

where L_l is defined as before, except that $\xi r_l(t) + (1-\xi) r_l(t_i)$ is replaced with $r_{ll}(t)$. This integral can be calculated by fixing all the components, except the l^{th} one along which the integration is performed and the lower and upper bounds of integration are P_{l-1} and P_l respectively.

Next, we have to show how to calculate the integral in (21) as $f(R_L(t))$. The next theorem answers this question.

Theorem 2. : With the line segments L_l from definition 12, define the path L as (see Fig. 1)

$$L(\xi, R_L(t)) = \bigcup_{l=1}^{n_u+n_y+2} L_l(\xi, R_L(t)). \tag{22}$$

Where L is the concatenation of all L_l. In other words, L is constructed by joining all L_l.

Then,

$$\begin{aligned} &\int_{R_L} \nabla f.dr = \int_{L} \nabla f.dr = f(R_L(t)) - f(R_L(t_i)) \\ &t \in [t_i, t_f] \end{aligned} \tag{23}$$

Proof. Given that

$$\nabla f = \left[\frac{\partial f}{\partial y}, \dots, \frac{\partial f}{\partial y^{(n_y)}}, \frac{\partial f}{\partial u}, \dots, \frac{\partial f}{\partial u^{(n_u)}}\right]^T,$$

we have

$$\begin{aligned}\int_{L} \nabla f.dr &= \sum_{l=1}^{n_u+n_y+2} \int_{L_l} \nabla f.dr = \sum_{l=1}^{n_u+n_y+2} \int_{P_{l-1}}^{P_l} \nabla f.dr \\ &= \sum_{l=1}^{n_u+n_y+2} f(P_l) - \sum_{l=0}^{n_u+n_y+1} f(P_l) = f(P_{n_u+n_y+2}) - f(P_0) = f(R_L(t)) - f(R_L(t_i))\end{aligned} \tag{24}$$

Where the third line is derived from the gradient theorem.

$R_L(t_i)$ is a constant term and from (1) we know that $f(R_L(t_i)) = 0$. Eliminating this term in the final line concludes the proof. □

Theorem 2 provides us with a practical way of calculating $f(R_L(t))$. It is shown before how to calculate $\int_{P_{l-1}}^{P_l} \nabla f.dr$. Note that $P_l \in \mathbb{D}$ is an extremely restricting condition. It is relaxed by the following theorem.

Theorem 3. Assume $\exists t_m \in [t_i, t_f]\ s.t.\ L(\xi, R_L(t_\circ)) \subset \mathbb{D}, \forall t_\circ \in [t_i, t_m]$. Define $P_l(R_L(t_\circ))$ as in Definition (11). Then:

$$\begin{aligned}&\int_{R_L} \nabla f.dr = \int_{L(R_L(t))} \nabla f.dr = \int_{L(R_L(t_\circ))} \nabla f.dr = f(R_L(t)) \\ &t \in [t_i, t_f]\end{aligned} \tag{25}$$

Proof. The upper bound evaluation is done in a symbolic way and so the expression for the two evaluations must be the same. Thus, replacing terms related to $t_\circ$ with terms related to t doesn't change f. □

Theorem 3 tells us that L may or may not lie in $\mathbb{D}$ and in any case, the f can be calculated precisely along all points of the $R_L(t)$.

As $R_L(t_i)$ is generally unknown, it is easier to consider point (0,0,...,0) as the lower bound of integral in (23). Change of the lower bound will add a bias term to our calculation of $f(R_L(t))$. As $f(R_L(t)) = 0$ at $\forall t \in [t_i, tf]$, this bias term can be calculated by forcing $f(R_L(t)) = 0$ at an arbitrary $t \in [t_i, t_f]$.

Until now, it is assumed that ∇f is known everywhere in $\mathbb{D}$ which is not realistic. It is possible to relax this assumption such that ∇f needs only to be known on $R_L(t)$. This is elaborated in the following theorem:

Theorem 4. ∇f and its estimate $\breve{\nabla f}$ are defined in $\mathbb{D}$ which contains $R_L(t)$. It is assumed that $\nabla f = \breve{\nabla f}$ on $R_L(t)$ and $\breve{\nabla f}$ is arbitrary elsewhere. Then, provided that $\breve{\nabla f}$ itself is a gradient of a scalar function then :

$$f(R_L(t)) = \check{f}(R_L(t))$$

Proof.

$$f(R_L(t)) = \int\limits_{R_L(t)} \nabla f.dr = \int\limits_{R_L(t)} \breve{\nabla f}.dr = \int\limits_{L(\xi,R_L(t))} \breve{\nabla f}.dr = \hat{f}(R_L(t)) \tag{26}$$

The above equations are deduced based on the fact that the integrals are path independent. L is defined as in definition 12.

□

As a result, we need an accurate estimate of ∇f just on R_L to obtain an exact estimate of f.

But, as we know, it is impossible to have ∇f exact, even on a path $R_L(t)$ because of various uncertainties including measurement noise. In this regard, it is important to ensure that smaller variations on ∇f will lead to a smaller variation on f and, consequently, a consistent estimate of ∇f will result in a consistent estimate of f itself. In other words, it is desirable to show that the estimation error of f decreases as the error on the estimate of ∇f decreases and in the ideal case, it will converge to zero as the uncertainty on the LPV estimate converges to zero.

Theorem 5. Let ∇f be the true gradient of the system (1) and let $\breve{\nabla f}$ be its estimate on $R_L(t)$. Denote:

$$\begin{aligned} \triangle \frac{\partial f}{\partial y^{(j)}} &= \frac{\partial f}{\partial y^{(j)}} - \breve{\frac{\partial f}{\partial y^{(j)}}} \\ \triangle \frac{\partial f}{\partial u^{(j)}} &= \frac{\partial f}{\partial u^{(j)}} - \breve{\frac{\partial f}{\partial u^{(j)}}} \end{aligned} \tag{27}$$

Then,

$$\left| \int\limits_{L(R_L(t))} (\nabla f - \breve{\nabla f}).dr \right| \leq \sum_{j=0}^{n_y} \left\| \triangle \frac{\partial f}{\partial y^{(j)}} \right\|_\infty (y^{(j)}|_{t_f}) + \sum_{j=0}^{n_u} \left\| \triangle \frac{\partial f}{\partial u^{(j)}} \right\|_\infty (u^{(j)}|_{t_f}) \tag{28}$$

Proof.

$$\begin{aligned}
\Big| \int\limits_{L(R_L(t))} (\nabla f - \breve{\nabla} f).dr \Big| &= \Big| \sum_{j=0}^{n_y} \int\limits_{L_{j+1}} (\nabla f - \breve{\nabla} f).dr + \sum_{j=n_y+1}^{n_y+n_u+2} \int\limits_{L_{j+1}} (\nabla f - \breve{\nabla} f).dr \Big| \\
&= \Big| \sum_{j=0}^{n_y} \int\limits_{L_{j+1}} (\triangle \frac{\partial f}{\partial y^{(j)}}).dy^{(j)} + \sum_{j=n_y+1}^{n_y+n_u+2} \int\limits_{L_{j+1}} (\triangle \frac{\partial f}{\partial u^{(j)}}).du^{(j)} \Big| \\
&\leq \sum_{j=0}^{n_y} \Big| \int\limits_{L_{j+1}} (\triangle \frac{\partial f}{\partial y^{(j)}}).dy^{(j)} \Big| + \sum_{j=n_y+1}^{n_y+n_u+2} \Big| \int\limits_{L_{j+1}} (\triangle \frac{\partial f}{\partial u^{(j)}}).du^{(j)} \Big| \\
&\leq \sum_{j=0}^{n_y} \Big\| \triangle \frac{\partial f}{\partial y^{(j)}} \Big\|_{\infty} (y^{(j)}|_{t_f}) + \sum_{j=0}^{n_u} \Big\| \triangle \frac{\partial f}{\partial u^{(j)}} \Big\|_{\infty} (u^{(j)}|_{t_f})
\end{aligned} \tag{29}$$

Where the last inequality comes from the triangle inequality for line integrals. □

Inequality (29) might seem too conservative but it is sufficient for our purpose. It clearly shows that a more accurate $\breve{\nabla} f$ will result in a more accurate $\breve{f}$. Now, we have the adequate theoretical tools to estimate f from an estimate of ∇f.

Since the symbolic integral can recover the nonlinear function up to a constant term, this unknown constant should be obtained to prevent bias in the nonlinear response. We obtain this constant value so that the response of the reconstructed system matches the actual response of the system. This DC value only adjusts the response of the system to match the large trajectory and does not have any influence on the dynamics around the large trajectory $R_L(t)$. Indeed, it can be recovered by evaluating the reconstructed system at the large trajectory through simulation. But due to the modeling error, this will not be a constant term for all sample times, and an averaging method can recover the DC value.

3.5 | Procedure summary

In this short subsection, we summarize the steps of our proposed framework for nonlinear identification of continuous time systems. The steps can be summarized as below :

1. Excite the system twice with and without small trajectory as described in Section 3.1.
2. Estimate the LTV model in (8) by the proposed algorithm in Section 3.2.
3. Having estimated the LTV model in the previous step and the large trajectory, estimate the LPV model (17) subject to the curl-free condition.
4. Reconstruct the nonlinear part from the LPV model by the algorithm as elaborated in Section 3.4.

4 | SIMULATION RESULTS

In this section, the proposed identification algorithm is evaluated on a second-order simulation example. The nonlinear system is given as follows:

$$\begin{aligned}\ddot{y} &= 0.5u - 0.1y\dot{y}u - 0.981\sin(y) - 0.8\dot{y} \\ &+ 0.2(-2y^2 + 2)\cos(0.286u - \pi/4)\end{aligned} \tag{30}$$

System dynamics include nonlinear terms of the input and system states. A large slow reference signal is applied to the system, the large input and the corresponding large output are shown in Fig. 2 (top). The nonlinear system is stable around this trajectory. A small fast signal is used to perturb this stable trajectory slightly (See Fig. 2 (bottom)). For this example, we use a periodic signal as input perturbations: a zero-mean multisine with period time $T_p = 50\,\mathrm{s}$. This multisine excites 190 harmonics in the band $[0.02, 3.8]$Hz with RMS (root-mean-square) value of 0.15. The measured data in the time interval $[50, 450]\,\mathrm{s}$ are used to identify the nonlinear system. Both the input and output are measured with additive filtered noise, with the signal-to-noise ratio (SNR) 50 dB for both input and output. Fig. 3 shows the input-output and noise in the frequency domain. According to Section 3.1 two experiments are performed to obtain input/output small and large trajectory. One experiment with only a large input, and a second experiment with the sum of small and large inputs are applied to the system. As a result, input, and output small trajectory are retrieved.

In the identification setup, the LTV system is modeled as follows

$$\ddot{y} = \sum_{n=0}^{2} a_n(t)\tilde{y}^{(n)}(t) + \sum_{m=0}^{2} b_m(t)\tilde{u}^{(m)}(t) \tag{31}$$

and in the LTV estimator[23] the order of the transient term is chosen $N_{\mathrm{tr}} = 7$. This transient term is an additive term modeled by an expansion of Legendre polynomials of $j\omega$ (with $\omega = 2\pi f$) to capture the sum of transient response and high-order terms in the Taylor series expansion (4). In practice, selecting the orders N_a, N_b and N_{tr} is a model order selection problem. The estimated time-varying coefficients are shown in Fig. 4. The other three time-varying coefficients are estimated to be zero, so we neglect them for the LPV modeling. Fig. 5 shows the performance of the LTV estimator in the frequency band of interest. The parameter-varying coefficients are modeled by bivariate monomials. For $a_0(R_L(t))$, $b_0(R_L(t))$ two sets of 3rd order bivariate monomials of $R_L(t) = [y_L(t), u_L(t)]$ with the constraint (16) are used and for the $a_1(R_L(t))$ a second-order bivariate monomial is chosen. The LPV system is obtained by fitting these polynomials to the estimated time-varying coefficients $a_0(t)$, $a_1(t)$, and $b_0(t)$. In Fig. 6 the reconstructed nonlinear terms are obtained by symbolic integration of the parameter-varying coefficients.

Finally, the performance of the reconstructed nonlinear system is shown in Fig. 7, which presents the validation of the reconstructed nonlinear system for large trajectory and broadband small input. Therefore an initial estimate of the nonlinear terms of the system are reconstructed uniquely based on LTV estimations.

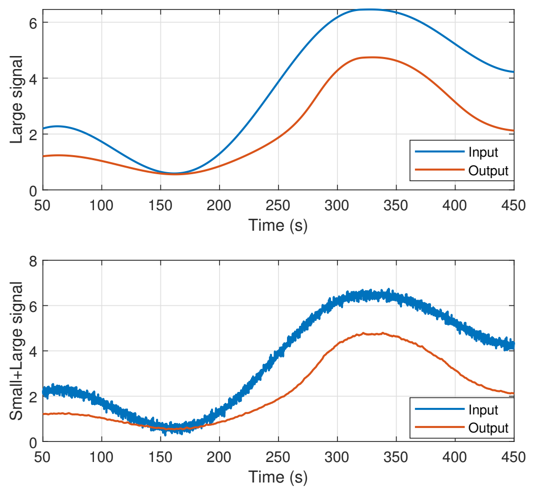

Figure 2 Two performed experiments. Top: The first experiment with the only large trajectory. Bottom: The second experiment with the same large input as the first experiment and some added input perturbations.

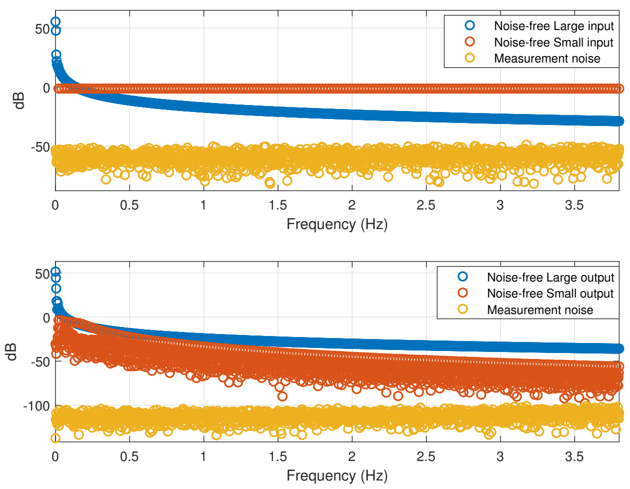

Figure 3 Large, small and noise contribution of input (Top) and output (Bottom).

5 | CONCLUSION

In this paper, we introduce a novel modeling procedure to model a broad class of continuous-time nonlinear SISO systems under mild assumptions on the system (specifically assumptions (1), (2) and (4)). In particular, there is greater flexibility in choosing the structure of the nonlinear model compared to other methods. The main part of the identification process is done by linear tools that are more established than their nonlinear counterparts. In addition, the LTV modeling procedure is not bound to a specific LTV estimator and the estimation procedure can be done by any appropriate LTV estimator. The following LPV

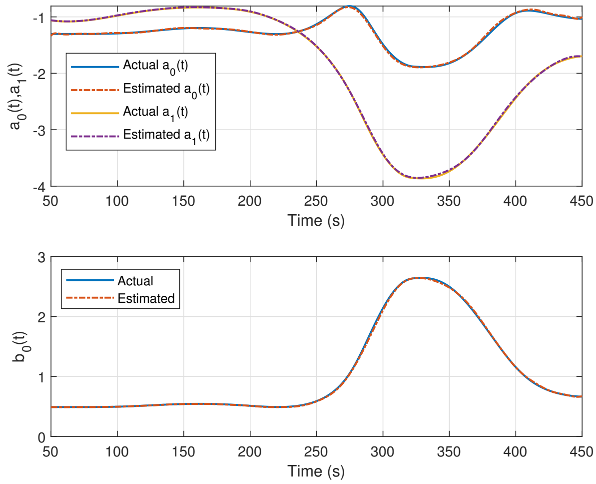

Figure 4 Estimated time-varying coefficients. Top: $a_0(t)$, $a_1(t)$. Bottom: $b_0(t)$.

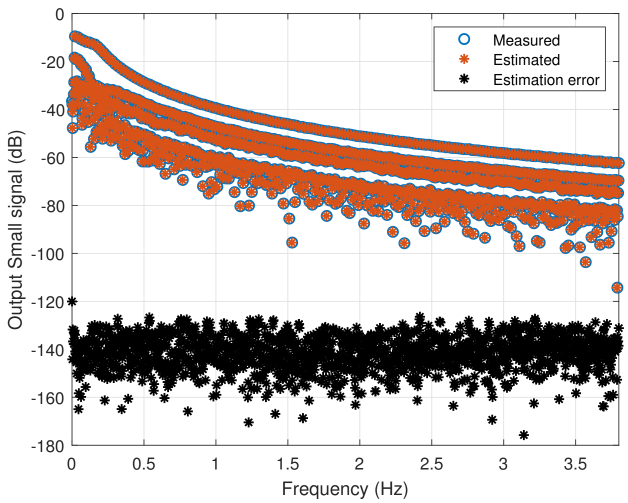

Figure 5 Performance of the frequency domain LTV estimator.

modeling is done according to the condition (16). This condition guarantees the uniqueness of the reconstruction result. The core part of the post processing step is the nonlinear reconstruction which relies on the gradient theorem in calculus to convert a linear model to a nonlinear model. In (29), it is shown that the quality of the final nonlinear model highly depends on the quality of the estimated linear model. At the end, the proposed algorithm is tested on a simulation example. It is shown that the estimated nonlinear system mimics the system output to an acceptable level. In addition, not only did the algorithm reproduce the output of the nonlinear system, but it also successfully rebuilt the nonlinear parts of the system in spite of the fact that there

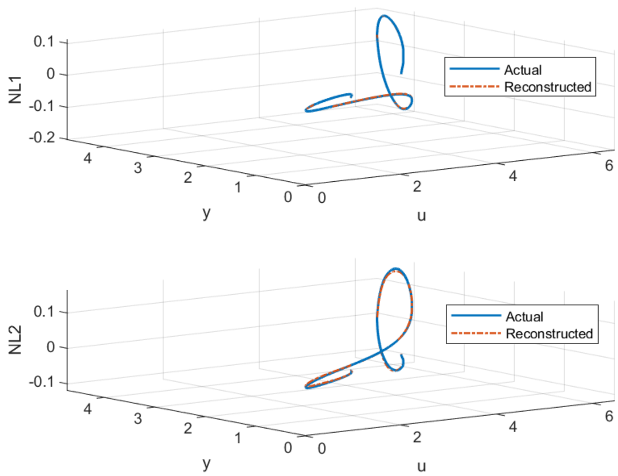

Figure 6 Reconstructed nonlinear terms. Top: NL1 $= -0.8\dot{y} - 0.1y\dot{y}u$. Bottom: NL2 $= 0.5u - 0.981\sin(y) + 0.2(-2y^2 + 2)\cos(0.286u - \pi/4)$.

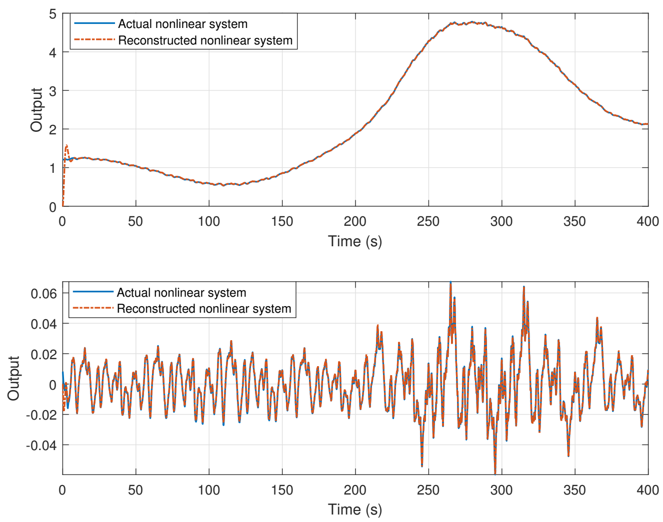

Figure 7 Performance of the reconstructed nonlinear system. Top: response to Large+small signal. Bottom: Contribution of the Small signal.

are multiple approximations during the modeling. In conclusion, this framework of algorithms can be used to model a broad class of nonlinear continuous-time systems with mild conditions.

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Conflict of interest

The authors declare no potential conflict of interests.

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