New Gamma Function to solve n fold integrals faster

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Python code on github

Abstract

We can calculate n fold integrals and derivative exponentially faster using fractional calculus. Integral of some functions are NP problem meaning it will take long time (Non polynomial time). By calculating n fold integrals faster we can achieve real time simulations.we can also solve hyper dimensional problems and to achieve hyper dimensional computing.we can reduce space and time complexity of an algorithm using this method.we also compare results with normal n fold integrals.we got 100 % accurate result.

Keywords: fractional calculus;New gamma function;Negative integer factorials;Gamma function for variables;N fold integrals;N fold chain rule;fractional permutations and combinations;fractional differential equations;P VS NP Problem;natural logarithm

1 Introduction

Fractional Calculus is extension of calculus to complex number order or quaternions (4D number) order. we are going to extend gamma function to find negative integer factorial.we will prove that it is not ∞ using the result we can find n fold integrals of some functions faster.

2 Laplace Transform and Inconsistency of gamma function

$$\frac{d^n f(x)}{dx^n} = \begin{cases} \textbf{n fold integral} & \text{if } n < 0 \\ f(x) & \text{if } n = 0 \\ n^{th} \textbf{ order derivative} & \text{if } n > 0 \end{cases}$$

2.1 Euler form

$$\frac{d^m x^n}{dx^m} = \frac{n!}{(n-m)!} x^{(n-m)} = ^n P_m \cdot x^{(n-m)} \tag{1}$$

[1] from (1) take n=m=-1

$$\frac{d^{-1} x^{-1}}{dx^{-1}} = \frac{(-1)!}{(0)!} x^{(0)} \tag{2}$$

$$= \int x^{-1} dx \tag{3}$$

$$= \ln x + c = (-1)! = \Gamma(0) \tag{4}$$

But we know $\Gamma(0) = \infty$ which is not accurate

$$\Gamma(z+1) = z\Gamma(z) = z!$$

2.2 Recursive formula

Assuming all factorial have this property

$$n! = (n-1)!n$$

at n =0

$$\begin{aligned} 0! &= (0-1)!0 \\ &= (-1)!0 \qquad 0! = 1 \end{aligned}$$

By solving

$$(-1)! = \frac{1}{0} \tag{5}$$

Taking limits on both sides on (5)

$$\lim_{x\to 0^+} (-1)! = \lim_{x\to 0^+} \frac{1}{x} = \infty \tag{6}$$

$$\lim_{x\to 0^-} (-1)! = \lim_{x\to 0^-} \frac{1}{x} = -\infty \tag{7}$$

$$\lim_{x\to 0^+} \frac{1}{x} \neq \lim_{x\to 0^-} \frac{1}{x}$$

limit doesn't exist so (-1)! has 2 functions depending on right or left limit

$$(-1)! = \begin{cases} \ln x + C1 & \text{if } x < 0 \\ -\ln x + C2 & \text{if } x > 0 \end{cases} \tag{8}$$

we will prove (8) is piecewise natural logarithm

Take (4) we have to find constant c if

$$(-1)! = \ln x + c$$

$$\frac{d^m x^n}{dx^m} = \frac{n!}{(n-m)!} x^{(n-m)} =^n P_m \cdot x^{(n-m)}$$

fix n=-1 and gives values of m=-2,-3,-4

at m=-2

$$\frac{d^{-2}x^{-1}}{dx^{-2}} = \frac{(-1)!}{(-1+2)!} x =^{(-1)} P_{-2} \cdot x \tag{9}$$

$$= \int\int x^{-1} dx = x(lnx - 1) \tag{10}$$

$$= x(lnx + c) \tag{11}$$

we are using (-1)!=lnx + c (12)

at m=-1 we see that c=-1

by applying machine learning program we will see that c increases by $\frac{1}{x}$ each time when we integrate

$$\frac{d^{-3}x^{-1}}{dx^{-3}} = \frac{(-1)!}{(-1+3)!}x^{-1+3} =^{(-1)} P_{-3} \cdot x^2 \tag{13}$$

$$= \int\int\int x^{-1}dx = \frac{x^2}{2}(lnx - 1.5) \tag{14}$$

$$= \frac{x^2}{2}(lnx + c) \tag{15}$$

at m=-2 we see that $c = -1 - \frac{1}{2} = -\frac{3}{2}$

This process continues next c or c at m=-3

$$c = 1 - \frac{1}{2} - \frac{1}{3} = -\frac{11}{6}$$

constant c is -$H_n(x)$

$$H_n(x) = \sum_{k=0}^{n} \frac{1}{k}$$

2.3 $\ln x$

n^{th} derivative ln(x) is found using derivative

$$\frac{d^n}{dx^n}lnx = (-1)^{n-1}(n-1)!x^{-n}$$

put n=0 we get

$$\frac{d^0}{dx^0}lnx = (-1)^{0-1}(0-1)!x^{-0} \tag{16}$$

$$= lnx + c = (-1)(-1)! \tag{17}$$

Therefore,

$$(-1)! = -\ln x + C2$$

from integral

$$\frac{d^{-n+1}}{dx^{-n+1}}lnx = \frac{(-1)!}{(n-1)!}x^{n-1} \tag{18}$$

$$= (lnx + c)\frac{x^{n-1}}{(n-1)!} \tag{19}$$

This is due to $(-1)! = \frac{1}{0}$ so we have to choose right or limit depending on problem

$\int ln(x)\,dx$ $\qquad\qquad$ $\frac{d(ln(x))}{dx}$

$$-\infty \longleftarrow\quad 0 \quad\longrightarrow \infty$$

$ln(x) + c$ $\qquad\qquad$ $-lnx + C2$

$$\frac{d^n}{dx^n} lnx = \begin{cases} (\ln x + c)\frac{x^n}{n!} & \text{if } n < 0 \qquad \text{(integral)} \\ \ln x & \text{if } n = 0 \\ (-1)^{n-1}(n-1)!x^{-n} & \text{if } n > 0 \qquad \text{(derivative)} \end{cases} \tag{20}$$

3 Laplace transform

3.1 Derivative of Transforms

$$\mathcal{L}\{t^n f(t)\} = (e^{i\pi})^n \frac{d^n}{ds^n} F(s) \tag{21}$$

[3]

(21) using this we can make sure whether (-1)! is positive or not

proof

$$\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} F(s) \tag{22}$$

let f(t)=1 then $F(s) = \frac{1}{s}$

n=-1

$$\mathcal{L}\{t^{-1}\} = (-1)^{-1} \frac{d^{-1}}{ds^{-1}} (\frac{1}{s}) \tag{23}$$

$$= (-1)! = (-1)[\ln s + C2] \tag{24}$$

So in laplace transform $\Gamma(0) = (-1)[\ln s + C2]$

(21) let f(t)=1 then $F(s) = \frac{1}{s}$

$$n = \frac{1}{2}$$

$$\mathcal{L}\{t^{\frac{1}{2}}\} = (e^{i\pi})^{\frac{1}{2}} \frac{d^{\frac{1}{2}}}{ds^{\frac{1}{2}}} (\frac{1}{s}) \tag{25}$$

$$= i \frac{d^{\frac{1}{2}}}{ds^{\frac{1}{2}}} (\frac{1}{s}) \tag{26}$$

$$\mathcal{L}\{D^{-m} t^n\} = \frac{\Gamma(n+1)}{s^{m+n+1}} \tag{27}$$

[2]

(27) let m=1,n=-1

$$\mathcal{L}\{D^{-1} t^{-1}\} = \frac{\Gamma(0)}{s} \tag{28}$$

$$\mathcal{L}\{D^{-1} t^{-1}\} = \mathcal{L}\{\ln t\} \tag{29}$$

$$\tag{30}$$

(8) and (24)

$$\Gamma(0) = (-1)[\ln s + C2]$$

$$\mathcal{L}\{\ln t\} = \frac{-1}{s}(\ln s + C2) \tag{31}$$

By using definition of laplace to find $\mathcal{L}\{\ln t\}$ we will get same result C2 is found to be euler mascheroni constant γ

4 Fractional Differential Equation

solve

$$D^{-\frac{1}{2}} \ln x = 2 \tag{32}$$

we can solve this equation using (20)

$$D^{-n} \ln x = \frac{x^n}{n!}(\ln x + c)$$
$$\frac{x^{\frac{1}{2}}}{\frac{1}{2}!}(\ln x + c) = 2$$
$$x^{\frac{1}{2}}(\ln x + c) = \frac{4}{\sqrt{\pi}}$$

Assuming $\Gamma\left(\frac{3}{2}\right) = \frac{1}{2}! = \frac{\sqrt{\pi}}{2}$.

5 Gamma Function for variables

$$y = x^{x+n} \tag{33}$$

$$\ln y = (x+n)\ln x \tag{34}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{(x+n)}{x} + \ln x \tag{35}$$

$$\frac{dy}{dx} = x^{x+n}(\ln x + \frac{x+n}{x}) \tag{36}$$

from (1)

$$\frac{dy}{dx} =^{(x+n)} P_1 \cdot x^{(x+n-1)} \tag{37}$$

$$= \frac{(x+n)!}{(x+n-1)!}x^{(x+n-1)} \tag{38}$$

from (36)

$$\frac{(x+n)!}{(x+n-1)!} = x(\ln x + \frac{(x+n)}{x}) \tag{39}$$

from (39) Let n=0

$$\frac{(x)!}{(x-1)!} = x(\ln x + 1)$$

i.e. $\frac{x!}{(x-1)!} \neq x$ This evidence suggests we need to reevaluate the current theory about gamma function

6 Results and discussion

6.1 Comparison

Here we are integrating $\frac{1}{x}$ we can also find integral of $\ln x$ using this As iteration increases Our code will become more faster

Table 1: Code Efficiency Comparison

Code	**Iterations**	**Execution Time (s)**	**Memory Usage (MB)**
Existing Code	100	4.5	62.92
Existing Code	200	11.73	70.27
Existing Code	300	21.56	76.09
Our Python Code	100	0.00022	15.07
Our Python Code	200	0.00042	15.25
Our Python Code	300	0.00065	14.98

Now divide execution time of existing code and our code

Table 2: Execution Time Ratio Comparison

Iterations	**Existing Code (s)**	**Our Python Code (s)**	**Execution Time Ratio**
100	4.5	0.00022	20454.5
200	11.73	0.00042	27928.57
300	21.56	0.00065	33169.23

6.2 Unit

we are using SI units in theses Example if we are using Degree instead of radians.
For Radians

$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1$$

For Degree

$$\lim_{x \to 0^0} \frac{\sin(x)}{x} = \frac{\pi}{180}$$

we can use calculator to prove this statement.Type $A = 10^{-9}$ means save value of 10^{-9} to A. Calculate $\frac{\sin A}{A}$. Type $B = -10^{-9}$ Calculate $\frac{\sin B}{B}$
if limit exist then

$$\frac{\sin A}{A} = \frac{\sin B}{B}$$

. Left Limit = Right Limit

6.3 Reverse of limit

for radians

$$\lim_{\frac{\sin(x)}{x}\to 1} x = 0$$

7 Conclusion

we can use this along with CUDA of nvidia GPU to accelerate machine learning task.we need machine learning project to optimize algorithms by developing new mathematical tools. In short we can use Machine learning to find better mathematical tools. It is also possible to extend permutations. This extension of permutation is called fractional permutations. Its domain is also quaternions (4D numbers).we need to find high complexity mathematics to solve many problems in science including quantum mechanics. since mathematics is language of science.if we have right mathematical tools we can make hyper dimensional computer that computes in higher dimensions.using these tools we can boost simulations,video game graphics etc. Use ai to find efficient algorithms. we don't have math for highly complex systems like biology,sociology etc. High complexity mathematics can solve high complexity problems like many body schrodinger equation,P VS NP problems. in this paper we didn't try to develop fractional permutations to solve problems in graph theory etc . we can develop fractional permutations by using their properties to extend the domain. our method works eventhough Riemann-liouville definition fails for other functions we can use this definition.

References

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