Neighbouring mode-dependent dynamic event-triggered control for Markov jump interconnected systems with unknown interconnections

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Abstract

This work is absorbed in the neighboring mode-dependent event-triggered control scheme for Markov jump systems with unknown interconnections. A dynamic event-triggered mechanism composed of neighboring modes parameters is provided to save communication resources and reduce the number of solution parameters. Distinguished from existing version, the global operating modes are required to be unknown for each local subsystem. Resorting to neighboring modes information, an event-triggered state feedback controller is established to assure system stability. The cyclic-small-gain criterion is used to tackle the unknown interconnections so that novel stability criteria are obtained for the closed-loop systems with H_∞ performance. Finally, an illustrative example is employed to confirm the proposed method in the aspect of validity.

Keywords: Markov jump systems, neighboring mode-dependent event-triggered control, interconnected systems

1. Introduction

Markov jump systems (MJSs) have significant advantages in describing physical models encountering abrupt environmental changes or stochastic disturbances [1, 2, 3]. Therefore, many scholars are absorbed in the analysis and synthesis

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for MJSs [4, 5, 6, 7, 8, 9]. [10] discusses the H_∞ finite-time stabilization for MJSs with unmeasurable state by sliding mode control approach. [11] designs a quantized feedback controller to assure the stability and L_2-L_∞ performance for the MJSs with time-varying delay. In [12], uncertain singular MJSs are investigated for stabilization using a dynamic output feedback control strategy. It is noted that the aforementioned results focus on centralized control strategies. Interconnected systems usually have strong coupling and high dimensions. Thus, the existing centralized control strategies are rocky to handle the synthetical problems of interconnected systems. Recently, decentralized control methods have been applied to interconnected systems and numerous research results have emerged [13, 14, 15]. The work in [16] introduces a decentralized state feedback approach aimed at ensuring the stability of Markov jump interconnected systems. In [17], a decentralized tracking control method is devised to attain finite-time stability for MJSs subject to both actuator with saturation and without saturation. It should be noted that unknown uncertainties are likely to occur between subsystems, which have a significant impact on system performance. Hence, it is essential to explore the control problem of systems with unknown interconnections.

Recently, some progress on Markov jump interconnected systems with unidentified interconnections has been made [18, 19, 20]. In [21], a decentralized adaptive sliding mode control strategy is introduced to stabilize semi-MJSs featuring unknown interconnections. A decentralized observer-based controller is designed and sufficient criteria are given to stabilize MJSs with unknown time delay and nonlinear interconnection in [20]. However, the above-decentralized control results rely on global modes, where the controller accesses the modes information of each subsystem. The timeliness and accuracy of mode information cannot be guaranteed in the process of transferring information. This makes it difficult to use a global mode-dependent controller. [22] presents a decentralized control scheme depending on local modes information to relax the requirement of global operational modes availability. Based on [22], [23] proposes a neighboring mode-dependent control method for uncertain MJSs, where each local controller accesses modes information of its neighboring subsystems. [24] studies a neighboring mode-dependent control scheme for MJSs with measurement errors. Compared with the global mode-dependent control approach, the neighboring mode-dependent control mechanism eliminates the need to broadcast mode information between subsystems and saves costs. To the author's knowledge, until now the control methods related to neighboring modes for Markov jump interconnected systems with unknown interconnections have not been thoroughly reported.

On the other hand, many needless data packets are dispatched to the communication channel, which causes a waste of communication resources. Therefore, an event-triggered mechanism (ETM) for the control system is proposed and developed [25, 26, 27, 28, 29]. [30] designs an event-triggered output feedback controller to ensure stochastic stability of MJSs with external disturbances. A memory-based adaptive event-triggered control (ETC) scheme is used to guarantee the security issues of Markov jump neural networks under deception attacks in [31]. A method combining a PI controller with an event-triggered state feedback controller is proposed to control the power system with Dos attacks in [32]. Observing that a neighboring mode-dependent ETC strategy doesn't only save communication resources, but also reduces costs and is easier to install. How to design a neighboring mode-dependent ETC scheme for Markov jump interconnected systems with unidentified interconnections is a meaningful and challenging work, which stimulates the author's research interest.

Inspired by previous studies, this article considers the neighboring mode-dependent dynamic ETC scheme for Markov jump interconnected systems with unidentified interconnections. Cyclic-small-gain conditions are used to tackle unknown interconnections. The stability conditions are obtained. Finally, the effectiveness of the put forward control method is verified through a digital example. The main benefits of the proposed method are summarized as follows.

1) Different from [7, 33], where ETM is global mode-dependent, the neighboring mode-dependent dynamic ETM is proposed for each subsystem

in this work. It saves communication resources, reduces costs and easily installed.

2) Based on the neighboring mode-dependent dynamic ETM, this work proposes a neighboring mode-dependent event-triggered state feedback controller. Compared with results in [19, 32], the proposed scheme avoids the need of global mode information.

3) A cyclic-small-gain condition is utilized to address the unknown interconnections in order to guarantee the stability and the H_∞ performance of the resultant closed-loop system.

The rest of the paper consists of five sections. Section 2 introduces the system model and formulates the neighboring mode-dependent ETC issues. A dynamic neighboring mode-dependent ETC design methodology and stability analysis conditions are provided in Section 3. Section 4 provides one simulation result to verify the theoretical result. Finally, Section 5 summarises the whole paper.

Notations: In this work, For a matrix A, A^{-1} and A^T denote its inverse and transpose , respectively. $He\left(M\right) = M^T + M$. $P_r\left(\cdot\right)$ is the probability measure and $\varepsilon\left(\cdot\right)$ denotes the mathematical expectation operator. The symbol $*$ denotes the symmetric structure.

2. Problem statement and preliminaries

2.1. System description

Explore the following Markov jump interconnected systems with N subsystems, where i-th subsystem is characterized as:

$$\chi_i : \begin{cases} \dot{x}_i\left(t\right) = A_i\left(r_i\left(t\right)\right)x_i\left(t\right) + B_i\left(r_i\left(t\right)\right)u_i\left(t\right) + D_i\left(r_i\left(t\right)\right)w_i\left(t\right) \\ \qquad\qquad + H_i\left(r_i\left(t\right)\right)\Phi_i\left(y\left(t\right)\right), \\ z_i\left(t\right) = C_i\left(r_i\left(t\right)\right)x_i\left(t\right) + G_i\left(r_i\left(t\right)\right)w_i\left(t\right), \\ y_i\left(t\right) = E_i\left(r_i\left(t\right)\right)x_i\left(t\right), \end{cases} \tag{1}$$

in which $i \in \mathfrak{N} = \{1, ..., N\}$, $x_i(t) \in \mathbb{R}^{n_i}$ depicts the system state; $u_i(t) \in \mathbb{R}^{r_i}$ denotes the control input; $y_i(t) \in \mathbb{R}^{m_i}$ stands for output; $w_i(t) \in \mathcal{L}_2[0, \infty)$ describes the exogenous disturbance; $A_i(r_i(t))$, $B_i(r_i(t))$, $D_i(r_i(t))$, $E_i(r_i(t))$, $C_i(r_i(t))$, $G_i(r_i(t))$ and $H_i(r_i(t))$ represent known matrices; $\Phi_i(\cdot) : \mathbb{R}^n \to \mathbb{R}^{n_i}$ is the unknown interconnection and is met as follows:

$$\Phi_i^T(y(t))\,\Phi_i(y(t)) \leqslant d_i^2 y^T(t)\,y(t), \tag{2}$$

where $r = \sum_{i=0}^{N} r_i$, $y(t) = \left[y_1^T(t), y_2^T(t), ..., y_N^T(t)\right]^T$ and $d_i > 0$; $r_i(t) \in \mathcal{M}_i = \{1, 2, 3, ..., M_i\}$ represents the local Markov process. The vector $[r_1(t)\ \ r_2(t), ..., r_N(t)]$ represents a global Markov process and assumes it belonging to a set $\mathcal{M}_l$ with M elements. Then, we denote $\mathcal{M}_j \triangleq \{1, ..., M\}$ and a bijective function $\varphi : \mathcal{M}_l \to \mathcal{M}_j$ with $\varsigma = \varphi\left([\varsigma_1, \varsigma_2, ...\varsigma_N]^T\right)$, where $\varsigma \in \mathcal{M}_j$, $\varsigma_i \in \mathcal{M}_i$. Let $\varphi^{-1} : \mathcal{M}_j \to \mathcal{M}_l$ be the inverse function given by $\varphi^{-1}(\varsigma) = [\varsigma_1, \varsigma_2, ...\varsigma_N]^T$. It is similar to the work of [23] and [24]. Hence, $\varsigma_i = \varphi^{-1}(\varsigma)$. From these, we can know the association between the local modes and the global modes.

Denote $r(t) \triangleq \varphi\left([r_1(t), r_2(t), ..., r_N(t)]^T\right)$. The transition probability of $\{r(t)\}$ is expressed by the following:

$$Pr\{r(t+\triangledown) = v | r(t) = \varsigma\} = \begin{cases} \pi_{\varsigma v}\triangledown + o(\triangledown), & v \neq \varsigma \\ 1 + \pi_{\varsigma v}\triangledown + o(\triangledown), & v = \varsigma, \end{cases} \tag{3}$$

where $\triangledown > 0$ and $\lim_{\triangledown \to 0}(o(\triangledown))/\triangledown = 0$, $\pi_{\varsigma v} \geqslant 0$ shows the transition rate and $\pi_{\varsigma\varsigma} = -\sum_{v=1, v\neq u}^{M} \pi_{\varsigma v}$.

2.2. The event-triggered control scheme

To address an issue in which global modes information may be unavailable to each subsystem, we consider an event-triggered state feedback controller associated with adjacent modes. We first define a binary matrix $\mathbb{E} = [e_{ij}] \in \mathbb{R}^{N \times N}$, $e_{ij} = 1$, indicating that the modes information of the j-th subsystem can be used for the i-th local controller, otherwise $e_{ij} = 0$. Thus, the adjacent modes information for each subsystem can be described as $[e_{i1}r_1(t), e_{i2}r_2(t), ..., e_{iN}r_N(t)]^T$. Assume it belonging to a set $\mathcal{M}_{li}$ with $\mathcal{M}_{oi}$ elements and indicate $\mathcal{M}_{gi} \triangleq$

$\{1, 2, ..., M_{oi}\}$. Describe a bijective mapping $\psi_i : \mathcal{M}_{li} \rightarrow \mathcal{M}_{gi}$ with $\iota_i = \psi_i\left([e_{i1}\varsigma_1, e_{i2}\varsigma_2, ..., e_{iN}\varsigma_N]^T\right)$ and $\iota_i \in \mathcal{M}_{gi}$. It can be seen that $\iota_i = \psi_i\left(\mathrm{diag}\left[e_{i1}, e_{i2}, ..., e_{iN}\right]\varphi^{-1}(\varsigma)\right)$. Let $\xi_i(t) \triangleq \psi\left([e_{i1}r_1(t), e_{i2}r_2(t), ..., e_{iN}r_N(t)]^T\right)$, we use the following novel ETM:

$$\begin{aligned}&\varrho_i\left([x_i(kT) - x_i(t_kT)]^T \mathit{\Omega}_i(\xi_i(t))[x_i(kT) - x_i(t_kT)]\right)\\ &\geqslant \varrho_i\left(\varepsilon_i(\xi_i(t))x_i(kT)^T \mathit{\Omega}_i(\xi_i(t))x_i(kT)\right) + h_i(kT),\end{aligned} \tag{4}$$

where $\varrho_i > 0$, $\varepsilon_i(\xi_i(t)) \in [0, 1)$ is the detection threshold; $\mathit{\Omega}_i(\xi_i(t)) > 0$ stands for weighting matrix; $x_i(kT)$ denotes the current sampled state; $x_i(t_kT)$ means the latest transmitted data; $h_i(t)$ is the dynamic variable and supposed to satisfied

$$\begin{aligned}\dot{h}_i(t) = &-\eta_i h_i(t) + \varepsilon_i(\xi_i(t))x_i(kT)^T \mathit{\Omega}_i(\xi_i(t))x_i(kT)\\ &- [x_i(kT) - x_i(t_kT)]^T \mathit{\Omega}_i(\xi_i(t))[x_i(kT) - x_i(t_kT)],\end{aligned} \tag{5}$$

with the initial conditions $h_i(0) \geqslant 0$ and $\eta_i \geqslant 0$.

According to [7], defining the time-varying as $\tau(t) = t - t_kT - cT$ with a maximum value of τ_M. The sampling instant from t_kT to $t_{k+1}T$can be expressed as $s_kT = t_kT + cT$, the ETM (4) is described as:

$$\begin{aligned}\varrho_i\left(e_i^T(s_kT)\mathit{\Omega}_i(\xi_i(t))e_i(s_kT)\right) \geqslant &\varrho_i\left(\varepsilon_i(\xi_i(t))x_i(s_kT)^T \mathit{\Omega}_i(\xi_i(t))x_i(s_kT)\right)\\ &+ \tilde{h}_i(t),\end{aligned} \tag{6}$$

where $\tilde{h}_i(t)$ meets $\dot{\tilde{h}}_i(t) = -\eta_i h_i(t) + \varepsilon_i(\xi_i(t))x_i(s_kT)^T \mathit{\Omega}_i(\xi_i(t))x_i(s_kT) - e_i^T(s_kT)\mathit{\Omega}_i(\xi_i(t))e_i(s_kT)$, t_kT and $t_{k+1}T$ are used to represent the current sampling moment and the next sampling moment, respectively, $e_i(s_kT) = x_i(s_kT) - x_i(t_kT)$.

The neighboring-mode dependent event-triggered state feedback controller is designed as follows:

$$u_i(t) = K_i(\xi_i(t))x_i(t_kT), \tag{7}$$

where $t \in \left[t_kT + \tau_{t_k}, t_{k+1}T + \tau_{t_{k+1}}\right)$, and $K_i(\xi_i(t))$ represents the controller gain.

Remark 1. *Compared with the global mode-dependent ETM in [32], this paper puts forward the ETM related to neighboring modes and designs the neighboring mode-dependent ETC scheme (7). The modes information accessed by the neighboring mode-dependent event-triggered controllers is determined by local modes and neighboring modes. This approach relieves the need for all subsystems to have accessible modes of operation.*

Definition 1. *[24] The system (1) with* $w_i(t) = 0$ *and* $u_i(t) \equiv 0$ *is steady if*

$$\lim_{t\to\infty} \varepsilon \left\{ \sum\nolimits_{i=1}^{N} \|x_i(t)\|^2 \right\} = 0$$

under incipient conditions $x_0 = \left[x_1^T(0), x_2^T(0), ..., x_N^T(0)\right]^T$ *and* $r_0 = \left[r_1^T(0),\right.$ $\left. r_2^T(0), ..., r_N^T(0)\right]^T$.

2.3. Neighboring mode-dependent control problem

This article considers Markov jump interconnected systems and designs neighboring mode-dependent event-triggered state feedback controller such that
(i) The resultant closed-loop system is stochastically stable when $w_i(t) = 0$.
(ii) For given $\gamma > 0$ and $\jmath_i > 0$, the inequality
$\varepsilon \left\{ \sum_{i=1}^{N} \int_t^{\infty} \jmath_i z_i^T(t) z_i(t) dt \right\} \leqslant \varepsilon \left\{ \sum_{i=1}^{N} \int_t^{\infty} \jmath_i \gamma^2 w_i^T(t) w_i(t) dt \right\}$
holds for any nonzero $w_i(t) \in \mathcal{L}_2$ under the zero initial conditions.

3. Design method

Our goal is to design event-triggered state feedback controllers related to neighboring modes. Firstly, introduce an auxiliary MJS $\bar{\chi}_i$:

$$\bar{\chi}_i : \begin{cases} \dot{\bar{x}}_i(t) = \bar{A}_i(r(t))\bar{x}_i(t) + \bar{B}_i(r(t))\bar{u}_i(t) + \bar{D}_i(r(t))\bar{w}_i(t) \\ \qquad\qquad + \bar{H}_i(r(t))\Phi_i(\bar{y}(t)), \\ \bar{z}_i(t) = \bar{C}_i(r(t))\bar{x}_i(t) + \bar{G}_i(r(t))\bar{w}_i(t), \\ \bar{y}_i(t) = \bar{E}_i(r(t))\bar{x}_i(t), \end{cases} \tag{8}$$

where $\bar{A}_i(\varsigma) = A_i(\varsigma_i)$, $\bar{B}_i(\varsigma) = B_i(\varsigma_i)$, $\bar{D}_i(\varsigma) = D_i(\varsigma_i)$, $\bar{H}_i(\varsigma) = H_i(\varsigma_i)$, $\bar{C}_i(\varsigma) = C_i(\varsigma_i)$, $\bar{E}_i(\varsigma) = E_i(\varsigma_i)$, and $\bar{G}_i(\varsigma) = G_i(\varsigma_i)$, for all $\varsigma \in \mathcal{M}_j$, $\varsigma_i = \varphi^{-1}(\varsigma) \in \mathcal{M}_i$ and $\|\Phi_i(\bar{y}(t))\| \leqslant d_i \|\bar{y}(t)\|$.

In addition, the ETM and controller are constructed as follows:

$$\begin{aligned}\bar{\varrho}_i\left(\bar{e}_i^T\left(s_kT\right)\mathit{\Omega}_i\left(r\left(t\right)\right)\bar{e}_i\left(s_kT\right)\right)\geqslant&\bar{\varrho}_i\left(\bar{\varepsilon}_i\left(r\left(t\right)\right)\bar{x}_i\left(s_kT\right)^T\mathit{\Omega}_i\left(r\left(t\right)\right)\bar{x}_i\left(s_kT\right)\right)\\&+\bar{h}_i\left(t\right),\end{aligned}\tag{9}$$

$$\bar{u}_i\left(t\right)=K_i\left(r\left(t\right)\right)\bar{x}_i\left(t_kT\right),\tag{10}$$

Where $\bar{h}_i\left(t\right)$ meeets $\dot{\bar{h}}\left(t\right)=-\eta_i\bar{h}\left(t\right)+\bar{\varepsilon}_i\left(\xi_i\left(t\right)\right)\bar{x}_i\left(s_kT\right)^T\mathit{\Omega}_i\left(r\left(t\right)\right)\bar{x}_i\left(s_kT\right)-\bar{e}_i^T\left(s_kT\right)\mathit{\Omega}_i\left(r\left(t\right)\right)\bar{e}_i\left(s_kT\right)$ and $\bar{\varrho}_i>0$. Specially, we have $K_i\left(\varsigma\right)=\bar{K}_i\left(\varsigma\right)+\mathit{\Delta}K_i\left(\varsigma\right)$ and $\mathit{\Omega}_i\left(\varsigma\right)=\bar{\mathit{\Omega}}_i\left(\varsigma\right)+\mathit{\Delta}\mathit{\Omega}_i\left(\varsigma\right)$, where $\bar{K}_i\left(\varsigma\right)$ and $\bar{\mathit{\Omega}}_i\left(\varsigma\right)$ indicate the gain of the controller and weighting matrices, respectively, $\mathit{\Delta}K_i\left(\varsigma\right)$ and $\mathit{\Delta}\mathit{\Omega}_i\left(\varsigma\right)$ are variations with the following from:

$$\begin{aligned}&\mathit{\Delta}K_i\left(\varsigma\right)=\left[\mathit{\Delta}K_{idb}\left(\varsigma\right)\right]_{m_i\times n_i},\left|\mathit{\Delta}K_{idb}\left(\varsigma\right)\right|\leqslant\delta_{kidb}\left(\varsigma\right),d=1,2,...,m_i,b=1,2,...,n_i,\\&\mathit{\Delta}\mathit{\Omega}_i\left(\varsigma\right)=\left[\mathit{\Delta}\mathit{\Omega}_{idb}\left(\varsigma\right)\right]_{n_i\times n_i},\left|\mathit{\Delta}\mathit{\Omega}_{idb}\left(\varsigma\right)\right|\leqslant\delta_{\mathit{\Omega}idb}\left(\varsigma\right),d,b=1,2,...,n_i.\end{aligned}$$

Augmenting (8) with (10) yields as:

$$\bar{\mathscr{H}}_i:\begin{cases}\dot{\bar{x}}_i\left(t\right)=&\bar{A}_i\left(\varsigma\right)\bar{x}_i\left(t\right)+\bar{B}_i\left(\varsigma\right)K_i\left(\varsigma\right)\bar{x}_i\left(t-\tau\left(t\right)\right)+\bar{D}_i\left(\varsigma\right)\bar{w}_i\left(t\right)\\&-\bar{B}_i\left(\varsigma\right)K_i\left(\varsigma\right)\bar{e}_i\left(s_kT\right)+\bar{H}_i\left(\varsigma\right)\mathit{\Phi}_i\left(\bar{y}\left(t\right)\right),\\\bar{z}_i\left(t\right)=&\bar{C}_i\left(\varsigma\right)x_i\left(t\right)+\bar{G}_i\left(\varsigma\right)w_i\left(t\right),\\\bar{y}_i\left(t\right)=&\bar{E}_i\left(\varsigma\right)x_i\left(t\right),\end{cases}\tag{11}$$

Remark 2. *Since system (1) depends on $r_i\left(t\right)$, ETM (6) and controller (7) depends on $\xi_i\left(t\right)$, this controller cannot be designed directly. To solve the above problem, we propose the auxiliary system (8) that relies on $r\left(t\right)$. According to the bijective function $\varsigma_i=\varphi_i^{-1}\left(\varsigma\right)$ and $\iota_i=\psi_i\left(\mathrm{diag}\left[e_{i1},e_{i2},...,e_{iN}\right]\varphi^{-1}\left(\varsigma\right)\right)$, it can be found that the auxiliary system (8) includes the system (1). Therefore, a controller that is effective for the system (8) is also useful for the system (1). The controller (10) is given that can be used to derive the neighboring mode-dependent the controller (7).*

3.1. Stability Analysis

Below we will give the conditions for system (11) stability and give the strategy for designing the neighboring mode-dependent event-triggered state feedback controller (7).

Lemma 1. *For $\bar{\varrho}_i > 0$, $\bar{\varepsilon}_i\left(r\left(t\right)\right) \in [0, 1)$ and $\bar{h}_i\left(0\right) \geqslant 0$, $\bar{h}_i\left(t\right)$ satisfies*

$$\bar{h}_i\left(t\right) \geqslant 0 \tag{12}$$

Proof: Inspired by [34], from inequality (9), we can know

$$\begin{aligned}\dot{\bar{h}}_i\left(t\right) + \eta_i \bar{h}_i\left(t\right) = & -\bar{e}_i^T\left(s_k T\right) \varOmega_i\left(r\left(t\right)\right) \bar{e}_i\left(s_k T\right) \\ & + \bar{\varepsilon}_i\left(r\left(t\right)\right) \bar{x}_i\left(s_k T\right)^T \varOmega_i\left(r\left(t\right)\right) \bar{x}_i\left(s_k T\right) \\ \geqslant & -\frac{1}{\varrho_i} \bar{h}_i\left(t\right)\end{aligned}$$

with $\bar{h}_i\left(0\right) \geqslant 0$. Then, get the following:

$$\bar{h}_i\left(t\right) \geqslant \bar{h}_i\left(0\right) e^{-\left(\eta_i + \frac{1}{\varrho_i}\right)t}$$

Thus, inequality (12) hold. This completes the proof.

Remark 3. *To facilitate the design, ETM (9) is designed for the auxiliary system (8), where ETM relies on r(t). According to the bijective function $\varsigma_i = \varphi_i^{-1}\left(\varsigma\right)$, we can see that $\tilde{h}_i\left(t\right)$ is a special form of $\bar{h}_i\left(t\right)$. So if $\bar{h}_i\left(t\right) \geqslant 0$, then $\tilde{h}_i\left(t\right) \geqslant 0$.*

Lemma 2. *[35] For all $\mathcal{W} = \mathcal{W}^\top$, if $\mathscr{N}$ exists, the following statement is equivalent:*

(1)*: $\mathscr{Q}^\top \mathcal{W} \mathscr{Q} < 0$, for all $\mathscr{Q} \neq 0$, $\mathscr{N}\mathscr{Q} = 0$;*

(2)*: $\hat{\mathscr{N}^\perp}^\top \mathcal{W} \hat{\mathscr{N}^\perp} < 0$;*

(3)*: Existence of $\mathscr{E}$ to $\mathcal{W} + He\left(\mathcal{F}\mathscr{N}\right) < 0$*

Lemma 3. *[36] Let matrices $\varLambda = \varLambda^T$, $\boldsymbol{L}$ and a compact subset of real matrices $\boldsymbol{M}$ be given. Then the sentences listed below are equal:*

(1) *For $M \in \boldsymbol{M}$, $\xi^T \varLambda \xi < 0$ such that $M\boldsymbol{L}\xi = 0$ for all $\xi \neq 0$;*

(2) *Exists $\mathbb{N} = \mathbb{N}^T$ such that $\varLambda + \boldsymbol{L}^T \mathbb{N} \boldsymbol{L} < 0$, $\boldsymbol{N}_H^T \mathbb{N} \boldsymbol{N}_H \geqslant 0$, for all $M \in \boldsymbol{M}$.*

Lemma 4. *[37] For $\tau(t) \in [0, \tau_m]$, any real symmetric matrices $F > 0$ and G which satisfy $\begin{bmatrix} F & G \\ * & F \end{bmatrix} \geqslant 0$, the following inequality holds:*

$$-\int_{t-\tau_m}^{t} x^T(t) F x(t) dt \leqslant \xi^T(t) \varXi \xi(t),$$

where $\xi(t) = col\{x(t), x(t-\tau(t)), x(t-\tau_m)\}$, and

$$\varXi = \begin{bmatrix} -F & F-G & G \\ * & -2F + He[G] & F-G \\ * & * & -F \end{bmatrix}.$$

Lemma 5. *For given scalars γ, λ_i and a_i, if there exist $0 < \epsilon_i < 1$, $\delta_i > 0$, $P_i(u) > 0$, $Q_i(u) > 0$, $R_i > 0$, $Q_i > 0$ and $\varOmega_i(u) > 0$ of appropriate dimensions such that the following (13) -(15) are satisfied*

$$\begin{bmatrix} R_i & M_i \\ * & R_i \end{bmatrix} > 0, \tag{13}$$

$$\varPi_i(u) = \begin{bmatrix} \theta_{i11} & \theta_{i12} & \theta_{i13} & \theta_{i14} & \theta_{i15} & \theta_{i16} & \theta_{i17} \\ * & \theta_{i22} & \theta_{i23} & \theta_{i24} & 0 & 0 & 0 \\ * & * & \theta_{i33} & 0 & 0 & 0 & 0 \\ * & * & * & \theta_{i44} & \theta_{i45} & \theta_{i46} & 0 \\ * & * & * & * & \theta_{i55} & 0 & \theta_{i57} \\ * & * & * & * & * & \theta_{i66} & 0 \\ * & * & * & * & * & * & \theta_{i77} \end{bmatrix} < 0, \tag{14}$$

$$\sum_{v=1}^{M} \pi_{\varsigma v} Q_i(u) - Q_i < 0, \tag{15}$$

where

$$\begin{aligned} \theta_{i11} =& \sum_{v=1}^{M} \pi_{\varsigma v} P_i(\varsigma) + He\left[P_i(\varsigma) \bar{A}_i(\varsigma)\right] + Q_i(\varsigma) + d_i^2 P_i(\varsigma) \bar{H}_i(\varsigma) \bar{H}_i^T(\varsigma) P_i(\varsigma), \\ & + \tau_m Q_i + \left(1 + a_i^{-2}\right)\left(1 + \beta_i^{-1}\right) \bar{E}_i^T(\varsigma) \bar{E}_i(\varsigma) + \beta_i^{-1} \delta_i I - R_i, \\ \theta_{i12} =& P_i(\varsigma) \bar{B}_i(\varsigma) K_i(\varsigma) + R_i - M_i, \theta_{i13} = M_i, \theta_{i14} = \bar{A}_i^T(\varsigma) P_i(\varsigma), \\ \theta_{i15} =& P_i(\varsigma) \bar{D}_i(\varsigma), \theta_{i16} = -P_i(\varsigma) \bar{B}_i(\varsigma) K_i(\varsigma), \theta_{i17} = \bar{C}_i^T(\varsigma), \end{aligned}$$

$$\theta_{i22} = \bar{\varepsilon}_i(\varsigma)\,\Omega_i(\varsigma) - R_i + He\left[M_i\right], \theta_{i23} = R_i - M_i, \theta_{i24} = K_i^T(\varsigma)\,\bar{B}_i^T(\varsigma)\,P_i(\varsigma),$$
$$\theta_{i33} = -\bar{Q}_i(\varsigma) - R_i, \theta_{i44} = \tau_m R_i - 2\lambda_i P_i(\varsigma) + \lambda_i^2 a_i^2 d_i^2 P_i(\varsigma)\,\bar{H}_i(\varsigma)\,\bar{H}_i^T(\varsigma)\,P_i(\varsigma),$$
$$\theta_{i45} = \lambda_i P_i(\varsigma)\,\bar{D}_i(\varsigma), \theta_{i46} = -\lambda_i P_i(\varsigma)\,\bar{B}_i(\varsigma)\,K_i(\varsigma), \theta_{i55} = -\gamma^2 I, \theta_{i57} = \bar{G}_i^T(\varsigma),$$
$$\theta_{i66} = -\Omega_i(\varsigma), \theta_{i77} = -I$$

Then, we have

$$\begin{aligned}\mathcal{L}V_i \leqslant & -\beta_i^{-1}\delta_i \bar{x}_i^T(t)\,\bar{x}_i(t) - \left(1 + a_i^{-2}\right)\beta_i^{-1}\bar{y}_i^T(t)\,\bar{y}_i(t) \\ & + \left(1 + a_i^{-2}\right)\sum_{j=1, j\neq i}^{N} \bar{y}_j^T(t)\,\bar{y}_j(t).\end{aligned} \tag{16}$$

Proof: Construct the following function

$$V_i = \sum_{k=1}^{4} V_{ik} + \bar{h}_i(t), \tag{17}$$

where

$$\begin{aligned}V_{i1} &= x_i^T(t)\,P_i(u)\,x_i(t), \\ V_{i2} &= \int_{t-\tau_m}^{t} x_i^T(s)\,Q_i(u)\,x_i(s)ds, \\ V_{i3} &= \int_{-\tau_m}^{0}\int_{t+\theta}^{t} x_i^T(s)\,Q_i x_i(s)\,dsd\theta, \\ V_{i4} &= \int_{-\tau_m}^{0}\int_{t+\theta}^{t} x_i^T(s)\,R_i x_i(s)\,dsd\theta,\end{aligned}$$

and define

$$\bar{\upsilon}_i^T(t) = \left[\bar{x}_i^T(t), \bar{x}_i^T(t - \tau(t)), \bar{x}_i^T(t - \tau_m), \dot{\bar{x}}_i^T(t), \bar{w}_i^T(t), \bar{e}_i^T(s_k T)\right],$$

one has

$$\begin{aligned}\mathcal{L}V_{i1} =& 2x_i^T(t)\,P_i(u)\,\dot{x}_i(t) + x_i^T(t)\sum_{v=1}^{M}\pi_{\varsigma v}P_i(v)\,x_i(t), \\ \mathcal{L}V_{i2} =& x_i^T(t)\,Q_i(u)\,x_i(t) - x_i^T(t-\tau_m)\,Q_i(u)\,x_i(t-\tau_m) \\ & + \int_{t-\tau_m}^{t} x_i^T(s)\sum_{v=1}^{M}\pi_{\varsigma v}Q_i(v)\,x_i(s)ds, \\ \mathcal{L}V_{i3} =& \tau_m x_i^T(t)\,Q_i x_i(t) - \int_{t-\tau_m}^{t} x_i^T(s)\,Q_i x_i(s)ds, \\ \mathcal{L}V_{i4} =& \tau_m x_i^T(t)\,R_i x_i(t) - \int_{t-\tau_m}^{t} x_i^T(s)\,R_i x_i(s)ds.\end{aligned}$$

Combining Lemma 2, it follows that

$$-\int_{t-\tau_m}^{t} x_i^T(s)\,R_i x_i(s)ds < \xi_i^T(t)\,\varXi_i \xi_i(t)\,,$$

where $\bar{\xi}_i(t) = col\left\{\bar{x}_i(t)\,, \bar{x}_i(t-\tau(t))\,, \bar{x}_i(t-\tau_m)\right\}$, *and*

$$\varXi_i = \begin{bmatrix} -R_i & R_i - M_i & M_i \\ * & -2R_i + He\left[M_i\right] & R_i - M_i \\ * & * & -R_i \end{bmatrix}.$$

Taking (9) into account, we have

$$\begin{aligned}
\mathcal{L}V_i \leqslant & 2\bar{x}_i^T(t)\,P_i(\varsigma)\left(\bar{A}_i(\varsigma)\,\bar{x}_i(t) + \bar{B}_i(\varsigma)\,K_i(\varsigma)\,\bar{x}_i(t-\tau(t))\right.\\
& \left.-\bar{B}_i(\varsigma)\,K_i(\varsigma)\,\bar{e}_i(s_kT) + \bar{D}_i(\varsigma)\,\bar{w}_i(t) + \bar{H}_i(\varsigma)\,\phi_i(\bar{y})\right)\\
& + \tau_m \bar{x}_i^T(t)\,Q_i \bar{x}_i(t) + \tau_m \bar{x}_i^T(t)\,R_i \bar{x}_i(t) + \bar{z}_i^T(t)\,\bar{z}_i(t)\\
& + 2\bar{x}_i^T(t)\left(\lambda_i P_i(\varsigma)\right)\left(\bar{A}_i(\varsigma)\,\bar{x}_i(t) + \bar{B}_i(\varsigma)\,K_i(\varsigma)\,\bar{x}_i(t-\tau(t))\right.\\
& \left.-\bar{B}_i(\varsigma)\,K_i(\varsigma)\,\bar{e}_i(s_kT) + \bar{D}_i(\varsigma)\,\bar{w}_i(t) + \bar{H}_i(\varsigma)\,\phi_i(\bar{y}) - \dot{\bar{x}}_i(t)\right)\\
& + \bar{\varepsilon}_i(\varsigma)\,\bar{x}_i(s_kT)^T\,\varOmega_i(\varsigma)\,\bar{x}_i(s_kT) + \bar{x}_i^T(t)\sum_{v=1}^{M}\pi_{\varsigma v}P_i(v)\,\bar{x}_i(t)\\
& - \bar{x}_i^T(t-\tau_m)\,Q_i(\varsigma)\,\bar{x}_i(t-\tau_m) - \bar{e}_i^T(s_kT)\,\varOmega_i(\varsigma)\,\bar{e}_i(s_kT) - \eta_i\bar{h}_i(t)\\
& + \bar{x}_i^T(t)\,Q_i(\varsigma)\,\bar{x}_i(t) - \gamma^2\bar{w}_i^T(t)\,\bar{w}_i(t) + \bar{\xi}_i^T(t)\,\varXi_i\bar{\xi}_i(t)
\end{aligned}$$

$$\begin{aligned}
\leqslant & 2\bar{x}_i^T(t)\,P_i(\varsigma)\left(\bar{A}_i(\varsigma)\,\bar{x}_i(t) + \bar{B}_i(\varsigma)\,K_i(\varsigma)\,\bar{x}_i(t-\tau(t))\right.\\
& \left.-\bar{B}_i(\varsigma)\,K_i(\varsigma)\,\bar{e}_i(s_kT) + \bar{D}_i(\varsigma)\,\bar{w}_i(t)\right) + \bar{x}_i^T(t)\,Q_i(\varsigma)\,\bar{x}_i(t)\\
& + 2\bar{x}_i^T(t)\left(\lambda_i P_i(\varsigma)\right)\left(\bar{A}_i(\varsigma)\,\bar{x}_i(t) + \bar{B}_i(\varsigma)\,K_i(\varsigma)\,\bar{x}_i(t-\tau(t))\right.\\
& \left.-\bar{B}_i(\varsigma)\,K_i(\varsigma)\,\bar{e}_i(s_kT) + \bar{D}_i(\varsigma)\,\bar{w}_i(t) - \dot{\bar{x}}_i(t)\right) - {\gamma_i}^2\bar{w}_i^T(t)\,\bar{w}_i(t)\\
& - \bar{x}_i^T(t-\tau_m)\,Q_i(\varsigma)\,\bar{x}_i(t-\tau_m) + \bar{\varepsilon}_i(\varsigma)\,\bar{x}_i(s_kT)^T\,\varOmega_i(\varsigma)\,\bar{x}_i(s_kT)\\
& - \bar{e}_i^T(s_kT)\,\varOmega_i(\varsigma)\,\bar{e}_i(s_kT) + \bar{x}_i^T(t)\sum_{v=1}^{M}\pi_{\varsigma v}P_i(v)\,\bar{x}_i(t) + \bar{z}_i^T(t)\,\bar{z}_i(t)\\
& + \bar{\xi}_i^T(t)\,\varXi_i\bar{\xi}_i(t) + \left(1 + a_i^{-2}\right)\bar{y}^T(t)\,\bar{y}(t) + \tau_m\bar{x}_i^T(t)\,Q_i\bar{x}_i(t)\\
& + d_i^2\bar{x}_i^T(t)\,P_i(\varsigma)\,\bar{H}_i(\varsigma)\,\bar{H}_i^T(\varsigma)\,P_i(\varsigma)\,\bar{x}_i(t) + \tau_m\bar{x}_i^T(t)\,R_i\bar{x}_i(t)\\
& + \lambda_i^2 a_i^2 d_i^2\bar{x}_i^T(t)\,P_i(\varsigma)\,\bar{H}_i(\varsigma)\,\bar{H}_i^T(\varsigma)\,P_i(\varsigma)\,\bar{x}_i(t)\,.
\end{aligned}$$

From inequality (14) and (15), it follows that

$$\mathcal{L}V_i \leqslant -\beta_i^{-1}\delta_i\bar{x}_i^T(t)\,\bar{x}_i(t) - \left(1+a_i^{-2}\right)\beta_i^{-1}\bar{y}_i^T(t)\,\bar{y}_i(t) + \left(1+a_i^{-2}\right)\sum_{j=1,j\neq i}^{N}\bar{y}_j^T(t)\,\bar{y}_j(t).$$

This completes the proof.

Lemma 6. *Combined with the conditions provided in Lemma 5 if the cyclic-small-gain condition (18)*

$$\sum_{j=1}^{N-1} j \sum_{1\leqslant i_i<i_2<\cdots<i_{j+1}} \beta_{i_1}\beta_{i_2}\cdots\beta_{i_j+1} < 1 \tag{18}$$

holds and (18) can ensure that the following formula is solvable,

$$\begin{bmatrix} -\left(1+a_1^{-2}\right) & \left(1+a_2^{-2}\right)\beta_2 & \cdots & \left(1+a_N^{-2}\right)\beta_N \\ \left(1+a_1^{-2}\right)\beta_1 & -\left(1+a_2^{-2}\right) & \cdots & \left(1+a_N^{-2}\right)\beta_N \\ \vdots & \vdots & \ddots & \vdots \\ \left(1+a_1^{-2}\right)\beta_1 & \left(1+a_2^{-2}\right)\beta_2 & \cdots & -\left(1+a_N^{-2}\right) \end{bmatrix} \times \begin{bmatrix} \kappa_1 \\ \kappa_2 \\ \vdots \\ \kappa_N \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ \vdots \\ -1 \end{bmatrix},$$

then there exist positive scalars λ_i, a_i, and β_i, such that

$$\begin{aligned} &\varepsilon\left\{\frac{d}{dt}\left(\sum_{i=1}^{N}\left(\kappa_i\beta_i\ \left(\bar{x}_i^T(t)\,P_i(u)\,\bar{x}_i(t) + \int_{t-\tau_m}^{t}\bar{x}_i^T(s)\,Q_i(u)\,\bar{x}_i(t)ds\right.\right.\right.\right. \\ &+\int_{-\tau_m}^{0}\int_{t+\theta}^{t}\bar{x}_i^T(s)\,Q_i\bar{x}_i(t)\,dsd\theta + \int_{-\tau_m}^{0}\int_{t+\theta}^{t}\bar{x}_i^T(s)\,R_i\bar{x}_i(t)\,dsd\theta \\ &\left.\left.\left.\left.+\bar{z}_i^T(t)\,\bar{z}_i(t) - {\gamma_i}^2\bar{w}_i^T(t)\,\bar{w}_i(t)\right)\right)\right)\right\} \\ &< -\sum_{i=1}^{N}\kappa_i\beta_i\bar{x}_i^T(t)\,\bar{x}_i(t) + \sum_{i=1}^{N}\kappa_i\left(1+a_i^{-2}\right)\left(-\bar{y}^T(t)\,\bar{y}(t)\right) \end{aligned}$$

and κ_i is solved to

$$\kappa_i = \frac{\prod_{j=1,j\neq i}^{N}(1+\beta_i)}{\left(1+a_i^{-2}\right)\left(1-\sum_{j=1}^{N-1}j\sum_{1\leqslant i_1<i_2<\cdots<i_{j+1}\leqslant N}\beta_{i_1}\beta_{i_2}\cdots\beta_{i_{j+1}}\right)} > 0. \tag{19}$$

Proof: This proof refers to the Lemma 2 in [38].

Based on Lemma 5 and Lemma 6, the stability of the system (11) under ETM (9) is analyzed.

Theorem 1. *For given scalars $\gamma,\ \tau_m,\ \lambda_i,\ a_i$, and β_i satisfying the condition (19), the system (11) is stable and has H_∞ performance if $0 < \epsilon_i < 1,\ \delta_i > 0,\ \kappa_i > 0$ are existent, and matrices $P_i(\varsigma) > 0,\ Q_i(\varsigma) > 0,\ R_i > 0,\ Q_i > 0,\ \Omega_i(\varsigma) > 0$ with appropriate dimensions, such that the inequalities (13) -(15) hold.*

Proof: Construct Lyapunov function under a Markov process as:

$$V(t) = \sum_{i=1}^{N} \kappa_i \beta_i V_i, \tag{20}$$

where κ_i, a_i and β_i satisfy the conditions (18) and (19). Based on Lemma 5 , we obtain

$$\varepsilon \left\{ \frac{d}{dt} \left(\sum_{i=1}^{N} \kappa_i \beta_i V_i \right) + \sum_{i=1}^{N} \kappa_i \beta_i \left(\bar{z}_i^T(t)\, \bar{z}_i(t) - \gamma^2 \bar{w}_i(t)\, \bar{w}_i(t) \right) \right\} \leqslant 0.$$

Thus, we can derive that

$$\varepsilon \left\{ \frac{d}{dt} \left(\sum_{i=1}^{N} \kappa_i \beta_i V_i \right) \right\} \leqslant 0,$$

when $w_i(t) = 0$, and

$$\varepsilon \int_0^\infty \sum_{i=1}^{N} \kappa_i \beta_i \bar{z}_i^T(t)\, \bar{z}_i(t) dt \leqslant \varepsilon \int_0^\infty \sum_{i=1}^{N} \kappa_i \gamma^2 \beta_i \bar{w}_i(t)\, \bar{w}_i(t) dt$$

with initial condition. Thus, we obtain that the system (11) is steady and meets H_∞ performance index γ, which finishes the proof.

Remark 4. *A state feedback control scheme for Markov jump interconnected systems is proposed in [16]. However, the interconnections are assumed to be known. Theorem 1 of this article analyzes the stability of systems with unidentified interconnections based on the cyclic-small-gain condition (18), which is more realistic.*

Theorem 1 provides stability criteria for the global mode-dependent system (8). Then, we have the theorem 2.

Theorem 2. *If controller gain $K_i(\cdot)$ and weighting matrix $\Omega_i(\cdot)$ are selected to meet*

$$\begin{aligned}&\left|K_{idb}(\iota_i)-\bar{K}_{idb}(\varsigma)\right|\leqslant\delta_{kidb}(\varsigma),\\&\left|\Omega_{idb}(\iota_i)-\bar{\Omega}_{idb}(\varsigma)\right|\leqslant\delta_{\Omega idb}(\varsigma),\end{aligned}\tag{21}$$

for $\varsigma\in\mathcal{M}_j$, $\iota_i=\psi_i\left(\operatorname{diag}\left[e_{i1},e_{i2},...,e_{iN}\right]\varphi^{-1}(\varsigma)\right)\in\mathcal{M}_{j_i}$, *where* $\left[K_{idb}(\iota_i)\right]_{m_i\times n_i}=K_i(\iota_i)$, $\left[\bar{K}_{idb}(\iota_i)\right]_{m_i\times n_i}=\bar{K}_i(\iota_i)$, $\left[\Omega_{idb}(\iota_i)\right]_{n_i\times n_i}=\Omega_i(\iota_i)$, *and* $\left[\bar{\Omega}_{idb}(\iota_i)\right]_{n_i\times n_i}=\bar{\Omega}_i(\iota_i)$. *Then the neighboring mode–dependent controllers (7) stabilize system (1).*

Proof: Define $\Delta K_{idb}(\varsigma)=K_{idb}(\iota_i)-\bar{K}_{idb}(\varsigma)$ and $\Delta\Omega_{idb}(\varsigma)=\Omega_{idb}(\iota_i)-\bar{\Omega}_{idb}(\varsigma)$, inequality (21) implies $|\Delta K_{idb}(\varsigma)|\leqslant\delta_{kidb}(\varsigma)$ and $|\Delta\Omega_{idb}(\varsigma)|\leqslant\delta_{\Omega idb}(\varsigma)$. We can obtain that

$$\begin{aligned}&\bar{K}_i(r(t))+\Delta K_i(r(t))=\bar{K}_i(r(t))+K_i(\xi_i(t))-\bar{K}_i(r(t))=K_i(\xi_i(t)),\\&\bar{\Omega}_i(r(t))+\Delta\Omega_i(r(t))=\bar{\Omega}_i(r(t))+\Omega_i(\xi_i(t))-\bar{\Omega}_i(r(t))=\Omega_i(\xi_i(t)).\end{aligned}$$

Thus, by (8), (9) and (10) modeling of the auxiliary system contains by (1), (6) and (7) modeling system. If the controller (10) can achieve the stability of the system (8), it also guarantees the stability of the system (1). This accomplishes the proof.

Remark 5. *Theorem 2 offers the relationship between the ETM (6) and the controller (7) dependent on $\xi_i(t)$ and the ETM (9) and the controller (10) the dependent on $r(t)$ by introducing a gain variation of $\Delta\cdot_i(\varsigma)$ and giving bounds $\delta\cdot_{iqp}(\varsigma)$. Based on this, we can get the event-triggered controller associated with adjacent modes.*

3.2. Controller design

Non-convex form cannot be directly designed in Theorem 1. Theorem 3 is given to obtain controller (10). Firstly, we denote $\ell_{i1}=n_i^3+n_im_i+r_im_i$,

$\ell_{i2} = n_i + m_i + r_i$, $\ell_{i3} = n_i^2 + n_i m_i + r_i n_i$, $\ell_{i4} = n_i^2 + n_i + m_i$, $\ell_i = \ell_{i1} + \ell_{i4}$ and

$$\begin{aligned} \mathcal{S}_{i1}(\varsigma) &= \left[\mathcal{S}_{i11}(\varsigma)\ \ \mathcal{S}_{i12}(\varsigma\varsigma)\ldots\mathcal{S}_{i1\ell_i}(\varsigma)\right], \\ \mathcal{S}_{i2}(\varsigma) &= \left[\mathcal{S}_{i21}(\varsigma);\mathcal{S}_{i22}(\varsigma);\ldots;\mathcal{S}_{i2\ell_i}(\varsigma)\right], \\ \Delta_i(\varsigma) &= \operatorname{diag}\left[\Delta_{i1}(\varsigma),\Delta_{i2}(\varsigma),\ldots,\Delta_{i\ell_i}(\varsigma)\right], \end{aligned} \tag{22}$$

where

$$\begin{aligned} \mathcal{S}_{i1k_1}(\varsigma) &= \left[\left(r_i\bar{B}_i(\varsigma)V_i(\varsigma)c_d\right)^T\ \ 0_{n_i\times(\ell_{i1}+n_i)}\right]^T, \\ \mathcal{S}_{i2k_1}(\varsigma) &= \left[0_{n_i\times n_i}\ g_b^T\ 0_{n_i\times\ell_{i2}}\ \ -g_b^T\ 0_{n_i\times\ell_{i4}}\right], \\ \Delta_{ik_1}(\varsigma) &= \Delta K_{idb}(\varsigma), k_1 = (d-1)m_i + b, \\ \mathcal{S}_{i1\Omega_1}(\varsigma) &= \left[0_{n_i\times n_i}\ \ (\bar{\varepsilon}_i c_d)^T\ \ 0_{n_i\times\ell_{i1}}\right]^T, \mathcal{S}_{i2\Omega_1}(\varsigma) = \left[0_{n_i\times n_i}\ g_b^T\ 0_{n_i\times\ell_{i1}}\right], \\ \Delta_{i\Omega_1}(\varsigma) &= \Delta\Omega_{idb}(\varsigma), \Omega_1 = n_i^2 + (d-1)n_i + b, \\ \mathcal{S}_{i1k_2}(\varsigma) &= \left[0_{n_i\times\ell_{i2}}\ \ \left(r_i\bar{B}_i(\varsigma)V_i(\varsigma)c_d\right)^T\ \ 0_{n_i\times\ell_{i2}}\ r_iV_i(\varsigma)\ \ 0_{n_i\times(\ell_{i2}-n_i)}\right]^T, \\ \mathcal{S}_{i2k_1}(\varsigma) &= \left[0_{n_i\times n_i}\ g_b^T\ 0_{n_i\times\ell_{i1}}\right], \\ \Delta_{ik_2}(\varsigma) &= \Delta K_{idb}(\varsigma), k_2 = n_i^3 + (d-1)m_i + b, \\ \mathcal{S}_{i1k_3}(\varsigma) &= \left[0_{n_i\times\ell_{i2}}\ \ \left(r_i\bar{B}_i(\varsigma)V_i(\varsigma)c_d\right)^T\ \ 0_{n_i\times\ell_{i3}}\right]^T, \\ \mathcal{S}_{i2k_3}(\varsigma) &= \left[0_{n_i\times(\ell_{i3}+n_i)}\ g_b^T\ 0_{n_i\times\ell_{i4}}\right], \\ \Delta_{ik_3}(\varsigma) &= \Delta K_{idb}(\varsigma), k_3 = n_i^3 + (d+1)m_i + b, \\ \mathcal{S}_{i1\Omega_2}(\varsigma) &= \left[0_{n_i\times(\ell_{i4}+n_i)_i}\ c_d{}^T\ 0_{n_i\times\ell_{i4}}\right]^T, \\ \mathcal{S}_{i2\Omega_2}(\varsigma) &= \left[0_{n_i\times(\ell_{i4}+n_i)_i}\ g_b^T\ 0_{n_i\times\ell_{i4}}\right], \end{aligned}$$

$$\begin{aligned} \Delta_{i\Omega_2}(\varsigma) &= \Delta\Omega_{idb}(\varsigma), \Omega_2 = n_i^4 + (d-1)n_i + b, \\ \mathcal{S}_{i1k_4}(\varsigma) &= \left[0_{n_i\times(\ell_{i1}-n_i)}\ \ (r_iV_i(\varsigma)c_d)^T\ \ 0_{n_i\times\ell_{i3}}\right]^T, \\ \mathcal{S}_{i2k_4}(\varsigma) &= \left[0_{n_i\times(\ell_{i4}+n_i)}\ g_b^T\ 0_{n_i\times\ell_{i4}}\right], \\ \Delta_{ik_4}(\varsigma) &= \Delta K_{idb}(\varsigma), k_4 = n_i^4 + (d+1)m_i + b, \end{aligned}$$

c_k and g_k are the column vector where the kth element is equal to 0.

Theorem 3. *For given positive scalars γ, λ_i, a_i, κ_i and β_i satisfying the cyclic-small-gain conditions (18) and (19), the system (11) is steady and has H_∞*

performance, if there are matrix $\Theta_i(\varsigma)$, *and* $Q_i > 0$, $Q_i(\varsigma) > 0$, $R_i > 0$, $\begin{bmatrix} R_i & M_i \\ * & R_i \end{bmatrix} > 0$, *matrix* V_i *with appropriate dimensions such that inequalities (23) and (24) hold,*

$$\begin{bmatrix} \Psi_i(\varsigma) + \mathcal{S}_{i2}^T(\varsigma)\Theta_{i1}(\varsigma)\mathcal{S}_{i2}(\varsigma) & \mathcal{S}_{i1}(\varsigma) + \mathcal{S}_{i2}^T(\varsigma)\Theta_{i2}(\varsigma) \\ * & \Theta_{i3}(\varsigma) \end{bmatrix} < 0, \tag{23}$$

$$\Theta_{i1}(\varsigma) + He(\Theta_{i2}(\varsigma)\Delta_i(\varsigma)) + \Delta_i(\varsigma)\Theta_{i3}(\varsigma)\Delta_i(\varsigma) \geqslant 0, \tag{24}$$

where

$$\Psi_i(\varsigma) = \begin{bmatrix} \bar{\theta}_{i11} & \bar{\theta}_{i12} & \bar{\theta}_{i13} & \bar{\theta}_{i14} & \bar{\theta}_{i15} & \bar{\theta}_{i16} & \bar{\theta}_{i17} & \bar{\theta}_{i18} & \bar{\theta}_{i19} & 0 \\ * & \bar{\theta}_{i22} & \bar{\theta}_{i23} & \bar{\theta}_{i24} & 0 & 0 & 0 & \bar{\theta}_{i28} & 0 & 0 \\ * & * & \bar{\theta}_{i33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \bar{\theta}_{i44} & \bar{\theta}_{i45} & \bar{\theta}_{i46} & 0 & \bar{\theta}_{i48} & 0 & \bar{\theta}_{i410} \\ * & * & * & * & \bar{\theta}_{i55} & 0 & \bar{\theta}_{i57} & 0 & 0 & 0 \\ * & * & * & * & * & \bar{\theta}_{i66} & 0 & \bar{\theta}_{i68} & 0 & 0 \\ * & * & * & * & * & * & \bar{\theta}_{i77} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \bar{\theta}_{i88} & 0 & 0 \\ * & * & * & * & * & * & * & * & \bar{\theta}_{i99} & 0 \\ * & * & * & * & * & * & * & * & * & \bar{\theta}_{i1010} \end{bmatrix}$$

with

$$\begin{aligned}
\bar{\theta}_{i11} &= \sum_{v=1}^{M} \pi_{\varsigma v} P_i(\varsigma) + He\left[P_i(\varsigma)\bar{A}_i(\varsigma)\right] + Q_i(\varsigma) + \tau_m Q_i + \tau_m R_i \\
&\quad + \left(1 + a_i^{-2}\right)\left(1 + \beta_i^{-1}\right)\bar{E}_i^T(\varsigma)\bar{E}_i(\varsigma) - R_i + \beta_i^{-1}\delta_i I, \\
\bar{\theta}_{i12} &= r_i\bar{B}_i(\varsigma)\bar{X}_i(\varsigma) + R_i - M_i, \bar{\theta}_{i13} = M_i, \bar{\theta}_{i14} = \lambda_i\bar{A}_i^T(\varsigma)P_i(\varsigma), \\
\bar{\theta}_{i15} &= P_i(\varsigma)\bar{D}_i(\varsigma), \bar{\theta}_{i16} = -r_i\bar{B}_i(\varsigma)\bar{X}_i(\varsigma), \bar{\theta}_{i17} = \bar{C}_i^T(\varsigma), \\
\bar{\theta}_{i18} &= -P_i(\varsigma)\bar{B}_i(\varsigma) + r_i\bar{B}_i(\varsigma)V_i(\varsigma), \bar{\theta}_{i19} = P_i(\varsigma)\bar{H}_i(\varsigma), \\
\bar{\theta}_{i22} &= \bar{\varepsilon}_i(\varsigma)\bar{\Omega}_i(\varsigma) + He[M_i] - R_i, \bar{\theta}_{i23} = R_i - M_i, \\
\bar{\theta}_{i24} &= \left(r_i\bar{B}_i(\varsigma)\bar{X}_i(\varsigma)\right)^T, \bar{\theta}_{i28} = r_i\bar{X}_i^T(\varsigma), \bar{\theta}_{i33} = -\bar{Q}_i(\varsigma) - R_i, \\
\bar{\theta}_{i44} &= -2\lambda_i P_i(\varsigma) + \tau_m R_i, \bar{\theta}_{i45} = \lambda_i P_i(\varsigma)\bar{D}_i(\varsigma),
\end{aligned}$$

$$\bar{\theta}_{i46} = -r_i \bar{B}_i(\varsigma)\bar{X}_i(\varsigma), \bar{\theta}_{i55} = -\gamma_i^2 I, \bar{\theta}_{i57} = \bar{G}_i^T(\varsigma),$$
$$\bar{\theta}_{i66} = -\bar{\Omega}_i(\varsigma), \bar{\theta}_{i68} = -r_i \bar{X}_i^T(\varsigma), \bar{\theta}_{i77} = -I,$$
$$\bar{\theta}_{i88} = -He[r_i V_i(\varsigma)], \bar{\theta}_{i99} = -d_i^2 I, \bar{\theta}_{i1010} = -\lambda_i^{-2} a_i^{-2} d_i^2 I.$$

Proof: $\Pi_i(\varsigma)$ in (14) can be written as

$$\Pi_i(\varsigma) = \hat{\mathcal{N}}^{\perp\top} \dot{\Pi}_i(\varsigma) \hat{\mathcal{N}}^{\perp}, \tag{25}$$

where $\left(\hat{\mathcal{N}}^{\perp}\right)^{\top} = \begin{bmatrix} I & \dot{\mathcal{N}}^{\perp} \end{bmatrix}^{\top}$ and

$$\dot{\Pi}_i(\varsigma) = \begin{bmatrix} \Pi_i(\varsigma) & 0 \\ 0 & 0 \end{bmatrix}$$

with

$$\dot{\mathcal{N}}^{\perp} = \begin{bmatrix} 0 & K_i(\varsigma) & 0 & 0 & 0 & -K_i(\varsigma) & 0 & -I \end{bmatrix}.$$

Applying Lemma 2 to (25), we can get

$$\tilde{\Pi}_i(\varsigma) = \dot{\Pi}_i(\varsigma) + He[\mathcal{F}\mathcal{N}] < 0, \tag{26}$$

where $\mathcal{N} = \begin{bmatrix} \dot{\mathcal{N}}^{\perp} & -I \end{bmatrix}$,

$$\mathcal{F} = \begin{bmatrix} F^T & 0 & 0 & F^T & 0 & 0 & 0 & (r_i V_i(\varsigma))^T \end{bmatrix}^T$$

with $F = -P_i(\varsigma)\bar{B}_i(\varsigma) + r_i \bar{B}_i(\varsigma) V_i(\varsigma)$. Using the Schuler complement to inequality (26) and denoting $\bar{X}_i(\varsigma) = V_i(\varsigma)\bar{K}_i(\varsigma)$, then the following equation is obtained.

$$\hat{\Pi}_i(\varsigma) + He\left[\Delta\hat{\Pi}_i(\varsigma)\right] < 0, \tag{27}$$

where

$$\hat{\Pi}_i(\varsigma) = \Psi_i(\varsigma)$$

$$
\Delta\hat{\Pi}_i(\varsigma)=\begin{bmatrix}
0 & \Delta\dot{\theta}_{i1} & 0 & 0 & 0 & -\Delta\dot{\theta}_{i1} & 0 & 0 & 0 & 0\\
* & \bar{\varepsilon}_i(\varsigma)\,\Delta\Omega_{iqp}(\varsigma) & 0 & \Delta\dot{\theta}_{i1}^T & 0 & 0 & 0 & \Delta\dot{\theta}_{i2}^T & 0 & 0\\
* & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
* & * & * & 0 & 0 & -\Delta\dot{\theta}_{i1} & 0 & 0 & 0 & 0\\
* & * & * & * & 0 & 0 & 0 & 0 & 0 & 0\\
* & * & * & * & * & -\Delta\Omega_{iqp}(\varsigma) & 0 & \Delta\dot{\theta}_{i2}^T & 0 & 0\\
* & * & * & * & * & * & 0 & 0 & 0 & 0\\
* & * & * & * & * & * & * & 0 & 0 & 0\\
* & * & * & * & * & * & * & * & 0 & 0\\
* & * & * & * & * & * & * & * & * & 0
\end{bmatrix}
$$

with $\Delta\dot{\theta}_{i1}=r_i\bar{B}_i(\varsigma)\,V_i(\varsigma)\,\Delta K_{iqp}(\varsigma)$, $\Delta\dot{\theta}_{i2}=r_iV_i(\varsigma)\,\Delta K_i(\varsigma)$.

From (22), it follows that

$$
\Psi_i(\varsigma)+He\left[\mathcal{S}_{i1}(\varsigma)\,\Delta_i(\varsigma)\,\mathcal{S}_{i2}(\varsigma)\right]<0 \tag{28}
$$

Then, inequality (28) is described as

$$
\zeta_i^T\begin{bmatrix}\Psi_i(\varsigma) & \mathcal{S}_{i1}(\varsigma)\\ * & 0\end{bmatrix}\zeta_i<0,\ \zeta_i=\mathcal{L}\mho_i, \tag{29}
$$

where $\mho_i$ are nonzero vectors and $\mathcal{L}^T=\begin{bmatrix}I & (\Delta_i(\varsigma)\,\mathcal{S}_{i2}(\varsigma))^T\end{bmatrix}$. We can easily find that $\mathscr{U}_iL_i\zeta_i=0$ is established, if given

$$
\mathscr{U}_i=\begin{bmatrix}\Delta_i(\varsigma) & -I\end{bmatrix},\ L_i=\begin{bmatrix}\mathcal{S}_{i2}(\varsigma) & 0\\ 0 & I\end{bmatrix}.
$$

Giving a symmetric matrix

$$
\Theta_i(\varsigma)=\begin{bmatrix}\hat{\Theta}_{i1} & \hat{\Theta}_{i2}\\ * & \hat{\Theta}_{i3}\end{bmatrix}
$$

where $\hat{\Theta}_{i1}=\mathrm{diag}\left[\Theta_{i11}(\varsigma),\Theta_{i12}(\varsigma),...,\Theta_{i1\ell_i}(\varsigma)\right]$, $\hat{\Theta}_{i2}=\mathrm{diag}\left[\Theta_{i21}(\varsigma),\Theta_{i22}(\varsigma),\right.$ $\left....,\Theta_{i2\ell_i}(\varsigma)\right]$ and $\hat{\Theta}_{i3}=\mathrm{diag}\left[\Theta_{i31}(\varsigma),\Theta_{i32}(\varsigma),...,\Theta_{i3\ell_i}(\varsigma)\right]$. Applying Lemma 3, if the inequality (29) holds, it can be concluded that conditions(23) and (24) are achieved for all $\Delta_{i\cdot}(\varsigma)\in[-\delta_{i\cdot}(\varsigma),\delta_{i\cdot}(\varsigma)]\,,\varsigma\in\mathcal{M}$. The proof is completed.

Remark 6. *Theorem 3 eliminates the coupling between the matrix* $P_i(\varsigma)$ *and the system matrix. By introducing the relaxation matrix* $\Theta_i(\varsigma)$ *and using Lemma 2, the stability conditions of the system are obtained in terms of LMIs.*

After that, the design process is introduced for acquiring desired the event-triggered state feedback controller related to neighboring modes.

Algorithm 1.

S1: Initialize $\delta_{i\cdot}(\varsigma)$ to proper values;

S2: Process the optimization question

$$\min\ \gamma \qquad \text{subject to (23), (24).}$$

Then, the event-triggered state feedback controller gain associated with the global mode is decided by

$$\bar{K}_i(\varsigma) = \bar{X}_i(\varsigma) V_i^{-1}(\varsigma)$$

S3: Check whether there exist solutions $K_i(\iota_i)$ and $\Omega_i(\iota_i)$ satisfying condition (21). If that is solvable, choose viable solutions under these conditions, otherwise, add the value of $\delta_{i\cdot}(\varsigma)$ and keep on S2.

Remark 7. *When* $\delta_{i\cdot}(\varsigma)$ *are immovable, the matrix inequalities (23), (24) and (21) turn into LMIs. Therefore, the global mode–dependent event-triggered state feedback controllers are obtained in S2. Afterwards, the neighboring mode–dependent event-triggered state feedback controller is obtained in S3.*

When $r_i(t)$, $\xi_i(t)$ and $\breve{r}(t)$ are equal and $\Delta K_i(\varsigma) = \Delta\Omega_i(\varsigma) = 0$, where $\breve{r}(t) = \breve{\varsigma} \in \breve{\mathcal{M}}$ and $\breve{\mathcal{M}} = \left\{1, ..., \breve{M}\right\}$, the problems addressed in this article will come down to global mode-dependent situation in [32]. Augment system (1) with (9) and (10) ,then, the closed-loop system can be obtained as:

$$\wp_i : \begin{cases} \dot{\bar{x}}_i\left(t\right) = & A_i\left(\breve{\varsigma}\right) x_i\left(t\right) + B_i\left(\breve{\varsigma}\right) K_i\left(\breve{\varsigma}\right) x_i\left(t-\tau\left(t\right)\right) - B_i\left(\breve{\varsigma}\right) K_i\left(\breve{\varsigma}\right) e_i\left(s_k T\right) \\ & + D_i\left(\breve{\varsigma}\right) w_i\left(t\right) + H_i\left(\breve{\varsigma}\right) \varPhi_i\left(y\left(t\right)\right), \\ \bar{z}_i\left(t\right) = & C_i\left(\breve{\varsigma}\right) x_i\left(t\right) + G_i\left(\breve{\varsigma}\right) w_i\left(t\right), \\ \bar{y}_i\left(t\right) = & E_i\left(\breve{\varsigma}\right) x_i\left(t\right). \end{cases} \tag{30}$$

Then, from Theorem 3, the corollary 1 is derived.

Corollary 1. *Given positive scalars γ, λ_i, κ_i, a_i, and β_i satisfying the modified small-gain conditions (18) and (19), system (30) is stable and meets H_∞ performance, if there exist matrices $Q_i > 0$, $Q_i\left(\breve{\varsigma}\right)$, $R_i > 0$ and matrix V_i with appropriate dimensions such that*

$$\tilde{\varPsi}_i\left(\varsigma\right) = \begin{bmatrix} \tilde{\theta}_{i11} & \tilde{\theta}_{i12} & \tilde{\theta}_{i13} & \tilde{\theta}_{i14} & \tilde{\theta}_{i15} & \tilde{\theta}_{i16} & \tilde{\theta}_{i17} & \tilde{\theta}_{i18} & \tilde{\theta}_{i19} & 0 \\ * & \tilde{\theta}_{i22} & \tilde{\theta}_{i23} & \tilde{\theta}_{i24} & 0 & 0 & 0 & \bar{\theta}_{i28} & 0 & 0 \\ * & * & \tilde{\theta}_{i33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \tilde{\theta}_{i44} & \tilde{\theta}_{i45} & \tilde{\theta}_{i46} & 0 & \tilde{\theta}_{i48} & 0 & \tilde{\theta}_{i410} \\ * & * & * & * & \tilde{\theta}_{i55} & 0 & \tilde{\theta}_{i57} & 0 & 0 & 0 \\ * & * & * & * & * & \tilde{\theta}_{i66} & 0 & \tilde{\theta}_{i68} & 0 & 0 \\ * & * & * & * & * & * & \tilde{\theta}_{i77} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \tilde{\theta}_{i88} & 0 & 0 \\ * & * & * & * & * & * & * & * & \tilde{\theta}_{i99} & 0 \\ * & * & * & * & * & * & * & * & * & \tilde{\theta}_{i1010} \end{bmatrix} \tag{31}$$

where

$$\begin{aligned} \tilde{\theta}_{i11} = & \sum_{v=1}^{M} \pi_{\breve{\varsigma}v} P_i\left(\breve{\varsigma}\right) + He\left[P_i\left(\breve{\varsigma}\right) A_i\left(\breve{\varsigma}\right)\right] + Q_i\left(\breve{\varsigma}\right) + \tau_m Q_i + \tau_m R_i \\ & + \left(1 + a_i^{-2}\right)\left(1 + \beta_i^{-1}\right) E_i^T\left(\breve{\varsigma}\right) E_i\left(\breve{\varsigma}\right) - R_i + \beta_i^{-1}\delta_i I, \\ \tilde{\theta}_{i12} = & r_i B_i\left(\breve{\varsigma}\right) X_i\left(\breve{\varsigma}\right) + R_i - M_i, \tilde{\theta}_{i13} = M_i, \tilde{\theta}_{i14} = \lambda_i A_i^T\left(\breve{\varsigma}\right) P_i\left(\breve{\varsigma}\right), \\ \tilde{\theta}_{i15} = & P_i\left(\breve{\varsigma}\right) D_i\left(\breve{\varsigma}\right), \tilde{\theta}_{i16} = -r_i B_i\left(\breve{\varsigma}\right) X_i\left(\breve{\varsigma}\right), \theta_{i17} = C_i^T\left(\breve{\varsigma}\right), \\ \tilde{\theta}_{i18} = & -P_i\left(\breve{\varsigma}\right) B_i\left(\breve{\varsigma}\right) + r_i B_i\left(\breve{\varsigma}\right) V_i\left(\breve{\varsigma}\right), \tilde{\theta}_{i19} = P_i\left(\breve{\varsigma}\right) H_i\left(\breve{\varsigma}\right), \end{aligned}$$

$$
\begin{aligned}
\tilde{\theta}_{i22} =& \varepsilon_i\left(\breve{\varsigma}\right) \mathit{\Omega}_i\left(\breve{\varsigma}\right) + He\left[M_i\right] - R_i, \tilde{\theta}_{i23} = R_i - M_i, \\
\tilde{\theta}_{i24} =& \left(r_i B_i\left(\breve{\varsigma}\right) X_i\left(\breve{\varsigma}\right)\right)^T, \tilde{\theta}_{i28} = r_i X_i^T\left(\breve{\varsigma}\right), \tilde{\theta}_{i33} = -Q_i\left(\breve{\varsigma}\right) - R_i, \\
\tilde{\theta}_{i44} =& -2\lambda_i P_i\left(\breve{\varsigma}\right) + \tau_m R_i, \tilde{\theta}_{i45} = \lambda_i P_i\left(\breve{\varsigma}\right) D_i\left(\breve{\varsigma}\right), \tilde{\theta}_{i46} = -r_i B_i\left(\breve{\varsigma}\right) X_i\left(\breve{\varsigma}\right), \\
\tilde{\theta}_{i55} =& -\gamma_i^2 I, \tilde{\theta}_{i57} = G_i^T\left(\breve{\varsigma}\right), \tilde{\theta}_{i66} = -\mathit{\Omega}_i\left(\breve{\varsigma}\right), \tilde{\theta}_{i68} = -r_i X_i^T\left(\breve{\varsigma}\right), \\
\tilde{\theta}_{i77} =& -I, \tilde{\theta}_{i88} = -He\left[r_i V_i\left(\breve{\varsigma}\right)\right], \tilde{\theta}_{i99} = -d_i^2 I, \tilde{\theta}_{i1010} = -\lambda_i^{-2} a_i^{-2} d_i^2 I.
\end{aligned}
$$

Proof: Similar to Theorem 3.

4. Simulation examples

The advantages of the proposed results are demonstrated by a numerical study. Consider a Markov jump interconnected system with 3 subsystems. Each subsystem exists two modes. The data of the subsystem is displayed as follows: Subsystem 1:

$$
\begin{aligned}
&A_1\left(1\right) = \begin{bmatrix} 0 & 1 \\ -0.5 & -0.3 \end{bmatrix}, A_1\left(2\right) = \begin{bmatrix} 0 & -0.1 \\ -1.02 & -0.1 \end{bmatrix}, B_1\left(1\right) = \begin{bmatrix} 0 & -0.1 \\ 0 & 0.1 \end{bmatrix}, \\
&B_1\left(2\right) = \begin{bmatrix} 0 & -0.2 \\ 0 & 0.1 \end{bmatrix}, H_1\left(1\right) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0.03 & 0 & 0.02 & 0.02 & 0 & 0.01 \end{bmatrix}, \\
&H_1\left(2\right) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0.03 & 0 & 0.01 & 0.01 & 0 & 0.01 \end{bmatrix}, \\
&C_1\left(1\right) = C_1\left(2\right) = 0.1 * eye\left(2\right), G_1\left(1\right) = G_1\left(2\right) = eye\left(2\right), \\
&D_1\left(1\right) = D_1\left(2\right) = 0.1 * eye\left(2\right), E_1\left(1\right) = E_1\left(2\right) = \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0 \end{bmatrix},
\end{aligned}
$$

Subsystem 2:

$$
\begin{aligned}
&A_2\left(1\right) = \begin{bmatrix} 0 & 0.3 \\ -0.21 & -0.11 \end{bmatrix}, A_2\left(2\right) = \begin{bmatrix} 0 & 0.3 \\ -1.23 & -0.11 \end{bmatrix}, B_2\left(1\right) = \begin{bmatrix} 0 & -0.12 \\ 0 & 0.12 \end{bmatrix}, \\
&B_2\left(2\right) = \begin{bmatrix} 0 & -0.21 \\ 0 & 0.11 \end{bmatrix}, H_2\left(1\right) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0.03 & 0 & 0.01 & 0.02 & 0 & 0.01 \end{bmatrix},
\end{aligned}
$$

$$H_2(2) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0.03 & 0 & 0.02 & 0.01 & 0 & 0.01 \end{bmatrix},$$

$$C_2(1) = C_2(2) = 0.1 * eye(2), G_2(1) = G_2(2) = eye(2),$$

$$D_2(1) = D_2(2) = 0.1 * eye(2), E_2(1) = E_2(2) = \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0 \end{bmatrix},$$

Subsystem 3:

$$A_3(1) = \begin{bmatrix} 0 & 0.1 \\ -1.05 & -0.3 \end{bmatrix}, A_3(2) = \begin{bmatrix} 0 & 0.1 \\ -1.05 & -0.8 \end{bmatrix}, B_3(1) = \begin{bmatrix} 0 & -1.2 \\ 0 & 0.5 \end{bmatrix},$$

$$B_3(2) = \begin{bmatrix} 0 & -1.2 \\ 0 & 0.3 \end{bmatrix}, H_3(1) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0.05 & 0 & 0.02 & 0.03 & 0 & 0.01 \end{bmatrix},$$

$$H_3(2) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0.06 & 0 & 0.02 & 0.03 & 0 & 0.01 \end{bmatrix},$$

$$C_3(1) = C_3(2) = 0.1 * eye(2), G_3(1) = G_3(2) = eye(2),$$

$$D_3(1) = D_3(2) = 0.1 * eye(2), E_3(1) = E_3(2) = \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0 \end{bmatrix}.$$

As in the work of [22], the global mode information is expressed through $r(t) = \varsigma \in \{[1,1,1], [1,1,2], [1,2,2], [2,1,2]\}$. Then, the TR matrix is described as

$$\begin{bmatrix} -0.8 & 0.2 & 0.4 & 0.2 \\ 0.1 & -0.6 & 0.3 & 0.2 \\ 0.3 & 0.4 & -0.8 & 0.1 \\ 0.1 & 0.2 & 0.4 & -0.7 \end{bmatrix}.$$

To demonstrate the effectiveness of the proposed strategy, giving $\beta_i = 0.8$, $\lambda_i = 0.01$, $d_i = 0.05$, $\varepsilon_1 = 0.02$, $\varepsilon_2 = 0.15$, $\varepsilon_3 = 0.01$, $\gamma_1 = \gamma_3 = 8.1$, $\gamma_2 = 8.5$, $\tau_{m1} = \tau_{m3} = 0.02$, $\tau_{m2} = 0.01$, $\delta_{k_{1qp}} = 0.6$, $\delta_{k_{2qp}} = \delta_{k_{3qp}} = 1.3$, and $\delta_{\Omega_{iqp}} = 1.2$, for $i = 1,2,3$. The global mode–dependent controller gains and the event-triggered parameters are obtained in the Appendix. The neighboring mode–dependent controller gains and the event-triggered parameters are obtained as follows:

$$K_1(1) = \begin{bmatrix} 0.3578 & -0.1479 \\ 0.0547 & -0.3151 \end{bmatrix}, K_1(2) = \begin{bmatrix} 0.4070 & -0.1968 \\ 0.1052 & -0.3653 \end{bmatrix},$$

$$K_1(3) = \begin{bmatrix} 0.9405 & -0.0815 \\ 0.5245 & -0.3712 \end{bmatrix}, K_2(1) = \begin{bmatrix} 0.6445 & -0.4663 \\ -0.1934 & 0.0062 \end{bmatrix},$$

$$K_2(2) = \begin{bmatrix} 1.4712 & -0.3839 \\ 0.0521 & -0.0413 \end{bmatrix}, K_3(1) = \begin{bmatrix} 1.5161 & -0.0916 \\ 0.2736 & -0.0794 \end{bmatrix},$$

$$K_3(2) = \begin{bmatrix} 0.4070 & -0.0270 \\ 3.9891 & -0.1361 \end{bmatrix}, K_3(3) = \begin{bmatrix} 4.0715 & -0.0261 \\ 3.9874 & -0.1350 \end{bmatrix},$$

$$\Omega_1(1) = 10^6 \times \begin{bmatrix} 2.827799 & 0.028799 \\ 0.028799 & 2.636799 \end{bmatrix}, \Omega_1(2) = 10^6 \times \begin{bmatrix} 2.8244 & 0.0287 \\ 0.0287 & 2.634 \end{bmatrix},$$

$$\Omega_1(3) = 10^6 \times \begin{bmatrix} 2.7677 & 0.0891 \\ 0.0891 & 2.634 \end{bmatrix}, \Omega_2(1) = 10^5 \times \begin{bmatrix} 6.183992 & 0.174792 \\ 0.174792 & 5.846592 \end{bmatrix},$$

$$\Omega_2(2) = 10^5 \times \begin{bmatrix} 5.9598 & 0.3335 \\ 0.3335 & 5.9091 \end{bmatrix}, \Omega_3(1) = 10^6 \times \begin{bmatrix} 1.3652 & 1.6183 \\ 1.6183 & 3.2606 \end{bmatrix},$$

$$\Omega_3(2) = 10^6 \times \begin{bmatrix} 1.5535 & 1.0323 \\ 1.0323 & 3.2943 \end{bmatrix}, \Omega_3(3) = 10^6 \times \begin{bmatrix} 1.553301 & 1.032001 \\ 1.032001 & 3.294001 \end{bmatrix}.$$

In this simulation, the initial conditions are given as $x_1(0) = [0.1, -0.5]$, $x_2(0) = [0.1, 0.3]$, and $x_3(0) = [-1, 2]$. The external disturbances are established as $w_1(t) = \begin{bmatrix} 0.1e^{-6t}\sin(6\pi t) & 0.2e^{-6t}\sin(6\pi t) \end{bmatrix}^T$ and $w_2(t) = w_3(t)$ $= \begin{bmatrix} e^{-6t}\cos(5\pi t) & 0.1e^{-6t}\sin(6\pi t) \end{bmatrix}^T$. Figure 1 shows the system jump modes. Under the action of the neighboring mode–dependent event-triggered controllers, the state responses of the subsystems are shown in Figures 2-4. It can be seen that the neighboring mode–dependent event-triggered control method proposed in this paper can effectively solve the robust problem of the interconnected system. The event-triggered intervals of the three subsystems are depicted in Figures 5-7, which indicate that the neighboring mode-dependent dynamic event-triggered strategy is more effective in saving communication resources compared with static event-triggered [7]. Table 1 also supports this result.

To further illustrate the advantages of the proposed method, the results are compared with the results in Corollary 1. The number of controllers and event detectors associated with global modes is $N_{gk} = N_{g\Omega} = 12$. The number of controllers and event detectors associated with adjacent modes ais $N_{nk} = N_{n\Omega} = 8$. Table 2 also shows the number of needed controllers and event detectors under the two approaches, where N_k and N_Ω denote the number of controllers and event detectors, respectively. They indicate that the proposed approach reduces the need for designing the number of controllers and event detectors without the need for each controller to access the mode of the entire system.

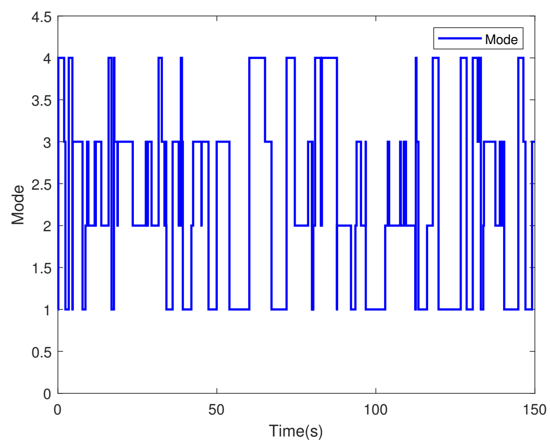

Figure 1: The jumping mode of the system.

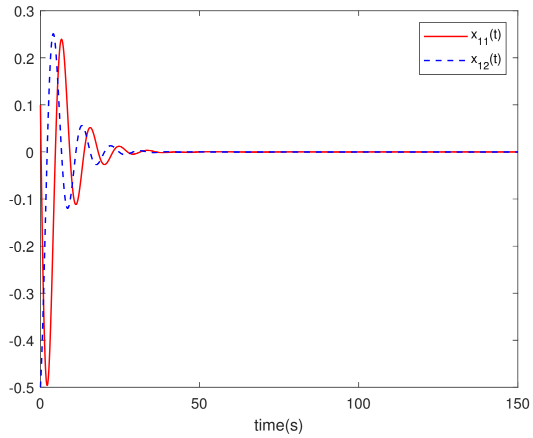

Figure 2: The state responses of subsystem 1.

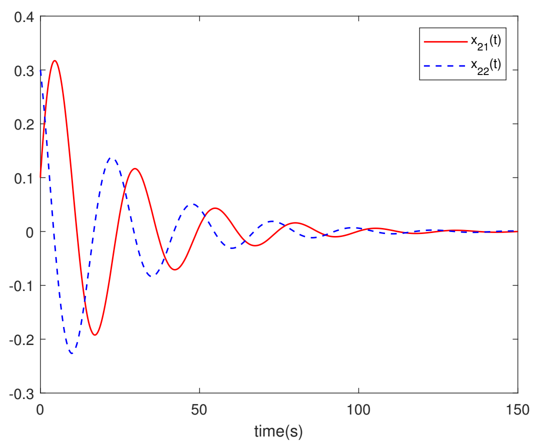

Figure 3: The state responses of subsystem 2.

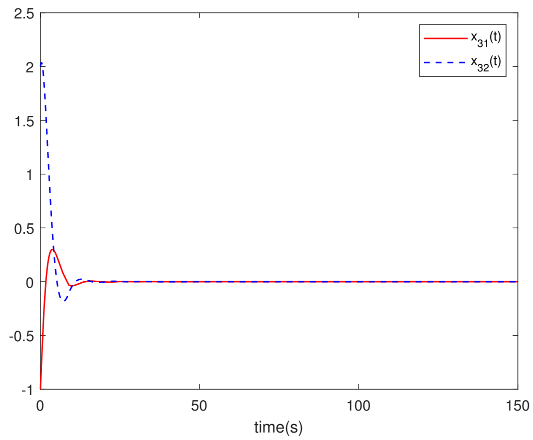

Figure 4: The state responses of subsystem 3.

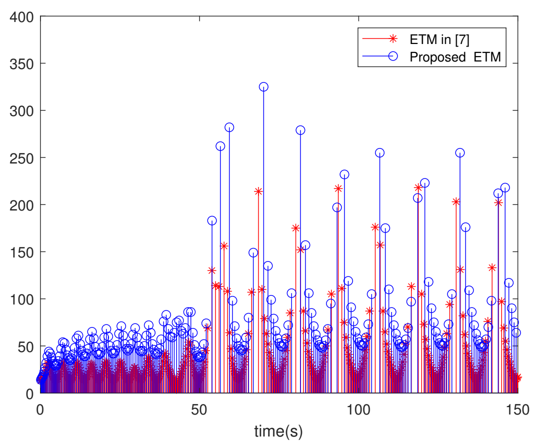

Figure 5: Dynamic ETM used in subsystem 1 of this paper and static ETM in [7].

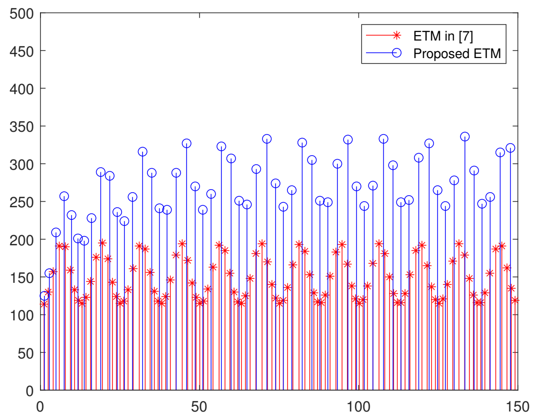

Figure 6: Dynamic ETM used in subsystem 2 of this paper and static ETM in [7].

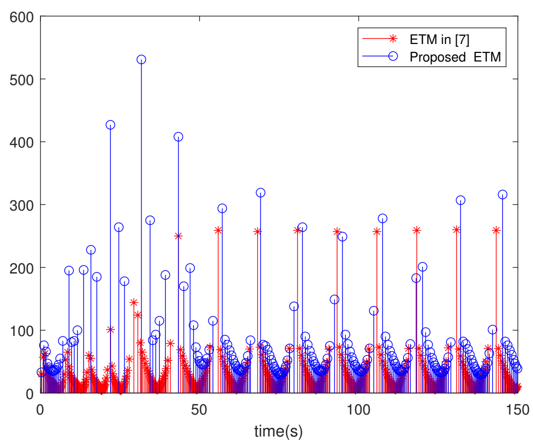

Figure 7: Dynamic ETM used in subsystem 3 of this paper and static ETM in [7].

Table 2: Controllers and event detectors numbers under different approaches.

Approach	N_k	N_Ω
Corollary 1	12	12
Theorem 3	8	8

Table 1: Triggering numbers for different ETMs.

System	Triggering numbers in [7]	Triggering numbers in this paper
subsystem 1	474	211
subsystem 2	96	55
subsystem 3	700	189

5. Conclusion

This work studies the issue of neighboring mode-dependent state feedback control for MJSs with unknown interconnections. We focus on the neighboring mode-dependent dynamic ETM, which enhances the utilization of network bandwidth in subsystems and minimizes the need for weighted matrix solutions. Assuming that the global mode information of each subsystem cannot be accessed by the local controller, which can be solved by the neighboring mode-dependent event-triggered state feedback controller used in this paper. Criteria for the system to be asymptotically steady and meet H_∞ performance under neighboring mode-dependent ETM are given by introducing the cyclic-small-gain conditions. The non-convex form of the matrix cannot be tackled using the LMI toolbox, so it is transformed into a convex condition using Fessler's Lemma. The availability of the obtained results is ultimately confirmed through a numerical demonstration.

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APPENDIX

$$\bar{K}_1(1) = \begin{bmatrix} 0.4078 & -0.1979 \\ 0.1047 & -0.3651 \end{bmatrix}, \bar{K}_1(2) = \begin{bmatrix} 0.4071 & -0.1970 \\ 0.1051 & -0.3652 \end{bmatrix},$$

$$\bar{K}_1(3) = \begin{bmatrix} 0.4070 & -0.1968 \\ 0.1052 & -0.3653 \end{bmatrix}, \bar{K}_1(4) = \begin{bmatrix} 0.9405 & -0.0815 \\ 0.5245 & -0.3712 \end{bmatrix},$$

$$\bar{K}_2(1) = \begin{bmatrix} 0.6545 & -0.4763 \\ -0.2034 & 0.0162 \end{bmatrix}, \bar{K}_2(2) = \begin{bmatrix} 0.6448 & -0.4602 \\ -0.1955 & 0.0162 \end{bmatrix},$$

$$\bar{K}_2(3) = \begin{bmatrix} 1.4712 & -0.3839 \\ 0.0521 & -0.0413 \end{bmatrix}, \bar{K}_2(4) = \begin{bmatrix} 0.6459 & -0.4618 \\ -0.1975 & 0.0184 \end{bmatrix},$$

$$\bar{K}_3(1) = \begin{bmatrix} 1.5161 & -0.0916 \\ 0.2736 & -0.0794 \end{bmatrix}, \bar{K}_3(2) = \begin{bmatrix} 4.0719 & -0.0260 \\ 3.9881 & -0.1351 \end{bmatrix},$$

$$\bar{K}_3(3) = \begin{bmatrix} 4.0715 & -0.0261 \\ 3.9874 & -0.1350 \end{bmatrix}, \bar{K}_3(4) = \begin{bmatrix} 4.0738 & -0.0260 \\ 3.9920 & -0.1354 \end{bmatrix},$$

$$\bar{\Omega}_1(1) = 10^6 \times \begin{bmatrix} 2.8278 & 0.0288 \\ 0.0288 & 2.6368 \end{bmatrix}, \bar{\Omega}_1(2) = 10^6 \times \begin{bmatrix} 2.8250 & 0.0287 \\ 0.0287 & 2.6345 \end{bmatrix},$$

$$\bar{\Omega}_1(3) = 10^6 \times \begin{bmatrix} 2.8244 & 0.0287 \\ 0.0287 & 2.6340 \end{bmatrix}, \bar{\Omega}_1(4) = 10^6 \times \begin{bmatrix} 2.7677 & 0.0891 \\ 0.0891 & 2.6743 \end{bmatrix},$$

$$\bar{\Omega}_2(1) = 10^5 \times \begin{bmatrix} 6.1840 & 0.1748 \\ 0.1748 & 5.8466 \end{bmatrix}, \bar{\Omega}_2(2) = 10^5 \times \begin{bmatrix} 6.1718 & 0.1741 \\ 0.1741 & 5.8890 \end{bmatrix},$$

$$\bar{\Omega}_2(3) = 10^5 \times \begin{bmatrix} 5.9598 & 0.3335 \\ 0.3335 & 5.9091 \end{bmatrix}, \bar{\Omega}_2(4) = 10^5 \times \begin{bmatrix} 6.1821 & 0.1745 \\ 0.1745 & 5.8978 \end{bmatrix},$$

$$\bar{\Omega}_3(1) = 10^6 \times \begin{bmatrix} 1.3652 & 1.6183 \\ 1.6183 & 3.2606 \end{bmatrix}, \bar{\Omega}_3(2) = 10^6 \times \begin{bmatrix} 1.5535 & 1.0323 \\ 1.0323 & 3.2943 \end{bmatrix},$$

$$\bar{\Omega}_3(3) = 10^6 \times \begin{bmatrix} 1.5533 & 1.0320 \\ 1.0320 & 3.2940 \end{bmatrix}, \bar{\Omega}_3(4) = 10^6 \times \begin{bmatrix} 1.5545 & 1.0329 \\ 1.0329 & 3.2955 \end{bmatrix}.$$